

**PROFESSOR:** Last time we talked about op-amps and the fact that they allowed us to abstract away certain components of a circuit and sample particular voltages from a circuit and then modify those voltages as we would like in an LTI fashion. Today I'd like to talk to you about some other interesting things that fall out of the fact that we're only dealing with LTI systems. In particular, Thevenin/Norton equivalence and superposition.

Thevenin/Norton equivalence is the idea that if you have a very complex circuit, but it's still a LTI circuit, you can express it using a linear curve. And superposition is the idea that because you're also dealing with LTI components, if you're trying to solve a circuit, you can take the individual contributions of independent sources to that circuit and sum them in order to find out either the total current flowing through a particular component or the total voltage drop across a particular component. At this point, I'll walk you through both.

Thevenin/Norton equivalence is an important concept in that you may have a very complicated circuit. And you don't really want to talk about the entire complicated circuit. You just want to sample the voltage drop or current in a very particular location.

Because we're dealing with LTI systems we can actually express that particular sample as its relationship between  $I$  and  $V$  and possibly whatever resistive component is associated with the voltage drop across that sample. We can solve for this curve by looking at the location that were sampling and finding the open circuit voltage associated with that position. Or if we were to leave the two terminals from where we are sampling open, what is the voltage drop across that section of our circuit?

That's the point right here, where the current flowing through the system is 0. And there's a given voltage associated with it. Likewise if we want to find the other intercept, we can close those two terminals by running a wire across them and then

look at the current that flows across that wire. That's the close circuit current.

The slope is going to tell us about our resistance, if any. If we're only dealing with a voltage source or a current source, then we're only going to be dealing with a straight line.

Once we've solved for these values, we tend to express them either as a Thevenin equivalent circuit or a Norton equivalent circuit. And you can convert between the two. Let's walk through an example.

Here I've got a simple circuit. And I would like to find the Thevenin equivalent of that circuit, given that I'm sampling from above this resistor and below this resistor. The first thing that I'm going to do is find the open circuit voltage. Or if I were sampling from this point to this point, what is the voltage drop across this section of the circuit?

Well, I've got 36 Volts. There's 12 Ohms of resistance in the section that I'm sampling. And there's 6 Ohms of resistance outside the section that I'm sampling. So  $\frac{2}{3}$  of my total voltage drop across this voltage divider is going to be accounted for inside my Thevenin equivalent circuit. So my Thevenin equivalent voltage is 24 Volts.

Now I want to go after the short circuit current. If I were to draw a wire from this terminal to this terminal, I'm curious what this value would be. As a consequence of connecting these two terminals, these resistors are going to be completely bypassed. We don't actually have to drop any voltage or put any current through these resistors because there's actually an infinitely easier way for the current to go.

In this case, we're just going to follow Ohm's law. 36 divided by 6 is 6 Amperes. But because of the directionality on this short circuit current, current in this system is going to flow in this direction. So our short circuit current is actually negative 6 Amperes.

That just leaves  $R_{th}$ . Once again we're going to rely on Ohm's law. If we divide our voltage by our current-- you can't have a negative resistance. We're actually going

to divide by the negative of this. We'll get out 4 Ohms.

Let's do it again, this time with a slightly different sample point and find the Norton equivalent of our circuit. So if I'm only sampling across the 9 Ohm resistor-- I personally still solve for the Thevenin equivalent voltage first because I think it's easiest. I'll just make note of it over here.

The voltage drop across this resistor is determined by the fact that this is a simple voltage divider. The other resistors in this circuit-- 7, 2, 9 Ohms. So the voltage drop across this resistor is going to represent half of the total voltage in our circuit. That's 18 Volts.

The short circuit current associated with connecting these two terminals is 36 Volts divided by the sum of these two resistors. Once again we're going to end up bypassing this resistor entirely because, think about it this way, if there was a resistor across here with 0 resistance, all the current would sink through it, or it would represent the location for which infinite current would flow.

36 divided by 9 is 4. And because we're following in the opposite direction we're going to say this is negative 4 Amperes. In this orientation, the Norton equivalent amperage is actually the negative of this value. You sometimes see the arrow pointing downwards.

This preserves our property of short circuit current. If this was short circuited,  $I_{sc}$  would still be negative 4 Amperes. And our  $V_{th}$ -- if our  $V_{th}$  was the voltage drop across these two points-- ha, I haven't solved for  $R_{th}$  yet.

There are a few ways to do this. But I'm going to use  $V_{th}$  and  $I_n$ . 18 over 4, we can reduce it to 9 over 2. And if I were to solve back out for the voltage drop across this resistor just as a consequence of I Norton, I would get 18 Volts, which is also the Thevenin equivalent voltage for this circuit.

That covers Thevenin/Norton equivalence. And we're going to talk about superposition really quickly.

Superposition is yet another circuit-solving strategy. It falls out as a consequence of only working with LTI components. And it means that in order to solve for a given component current or component voltage, you can actually find it by taking the linear combination of every contribution as a consequence of independent sources. What do I mean when I say that?

Well, if you have a circuit like this-- and you might recognize it from the NVCC lecture earlier-- you can actually express it as a linear combination of just the voltage source and just the current source. Note what when I remove the current source, I end up with no connection because if you have 0 current flowing through a system, it's the same as if the connection has been removed. Note that when I remove the voltage source, I no longer have a voltage drop across that connection, but I maintain the connection.

If in this example I was just looking for  $I_1$ .  $I_1$  would be the linear combination of the contribution of  $I_1$  in this particular circuit plus  $I_1$  in this particular circuit. So this is the expression that I'm looking for. In the first circuit, I'm just going to use  $V$  over  $IR$  -- or  $V$  equals  $IR$  -- and I'm going to have 3 Amperes in this direction.

In this circuit I'm actually going to have to use a current divider. Identify that I have five parts total to distribute among these two branches. Identify that this side is going to get the inverse of this side's contribution to the total parts, or this side is going to get the proportion of parts equivalent to this side's contribution. And vice versa.

That this side is the side that I'm interested in. And so  $\frac{2}{5}$  of the total current flowing through this current divider is going to end up in this branch.  $\frac{2}{5}$  of 10 is 4. It's in the opposite direction of this current flow. So I'm actually going to contribute negative 4 Amperes from this subcircuit.

This should match the results from the NVCC lecture. That's my super fast coverage of superposition and also Thevenin/Norton equivalence. Next time we're going to start a whole new module.

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