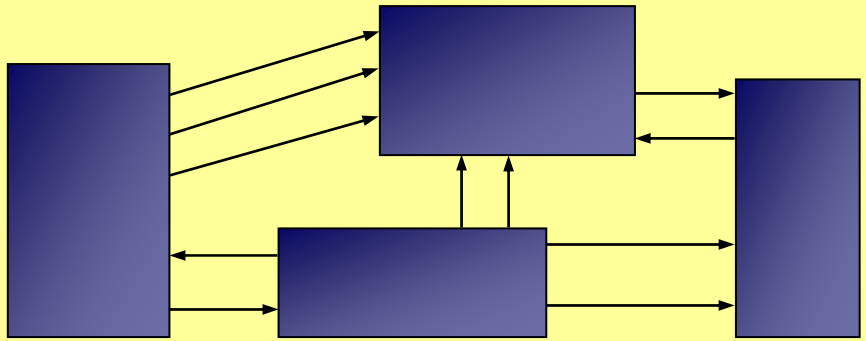
TRANSIENT SIGNALS IN COMPUTERS

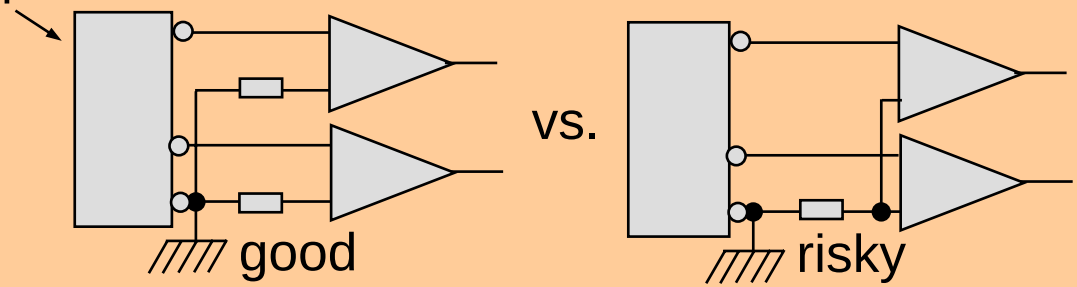
Ideal World:

- Only 1's and 0's
- Instantaneous links



Reality:

- Voltages exhibit propagation delay, decay, reflections
- Spurious transients can superimpose to flip bits erroneously
- RFI generated and picked up by wires can flip bits
- Ground loops matter

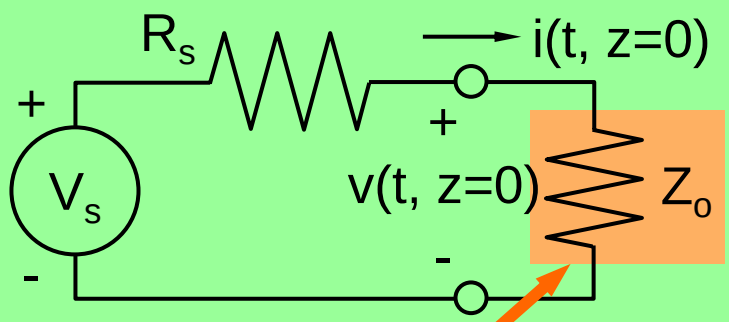


TEM LINE THEVENIN EQUIVALENT

Example: Given $V_s(t)$:

$$v(z,t) = v_+(t - \frac{z}{c}) + v_-(t + \frac{z}{c})$$

$$i(z,t) = \frac{1}{Z_o}[v_+(t - \frac{z}{c}) - v_-(t + \frac{z}{c})$$



Assume $v_- = 0$ (no other sources)

Then $v(t, z=0) = Z_o\, i(t, z=0)$ yields equivalent circuit

Solution at terminals:

$$v_+(t, z=0) = V_s(t)\,\frac{Z_o}{Z_o + R_s}$$ (voltage divider equation)

Solution for all t,z:

$$v_+(t,z) = V_s(t - \frac{z}{c})\,\frac{Z_o}{Z_o + R_s}$$ (forward propagating wave only)

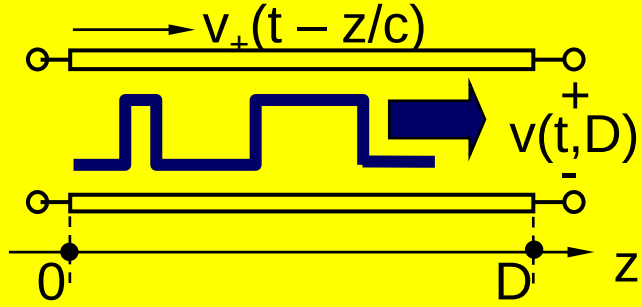
TEM LINE THEVENIN EQUIVALENT

Voltages at an Open Circuit:

$i(t,D) = Y_o[v_+(t,D) - v_-(t,D)] = 0$ (open circuit)

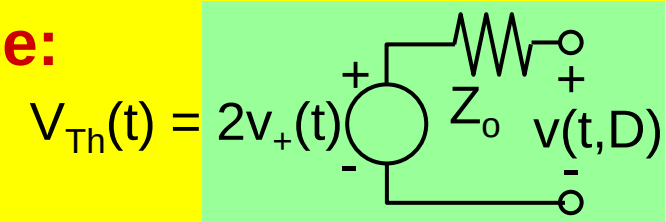
$\Rightarrow \quad v_-(t,D) = v_+(t,D)$

$\Rightarrow \quad v(t,D) = v_+(t,D) + v_-(t,D) = 2v_+(t,D)$



Thevenin Equivalent for TEM source:

$v_{\text{open circuit}}(t) = 2v_+(t) = V_{Th}(t)$

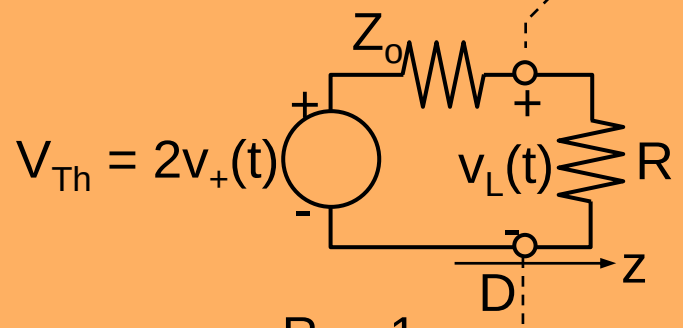


Example—Resistive Load:

At z = D: $v_-(t) = v_L(t) - v_+(t)$

But: $v(t) = 2v_+(t)\dfrac{R}{R+Z_o}$

Thus: $v_-(t) = v_+(t)\dfrac{R-Z_o}{R+Z_o} = v_+(t)\,\Gamma$

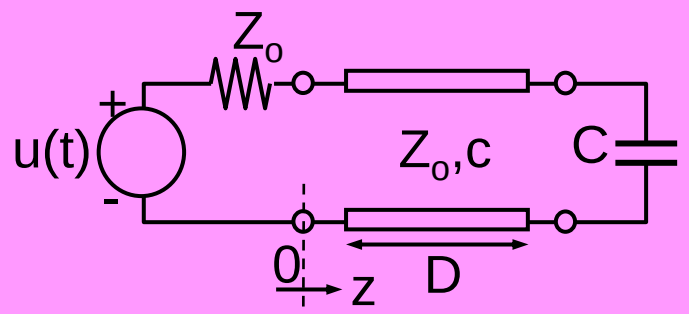


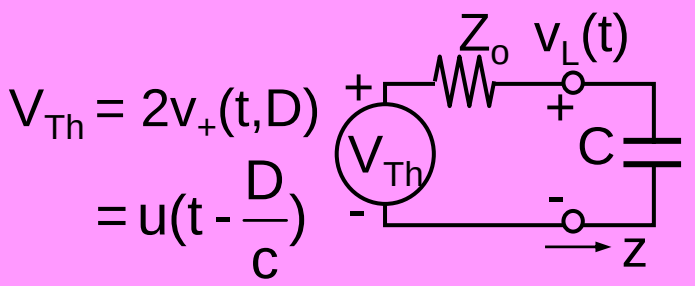
Define: Γ = "reflection coefficient" = v_-/v_+ at load = $\dfrac{R_n - 1}{R_n + 1}$

(= 0 if $R = Z_0$, = -1 if $R = 0$, and = +1 if $R = \infty$)

CAPACITIVELY TERMINATED TEM LINE

Example: Capacitive Load

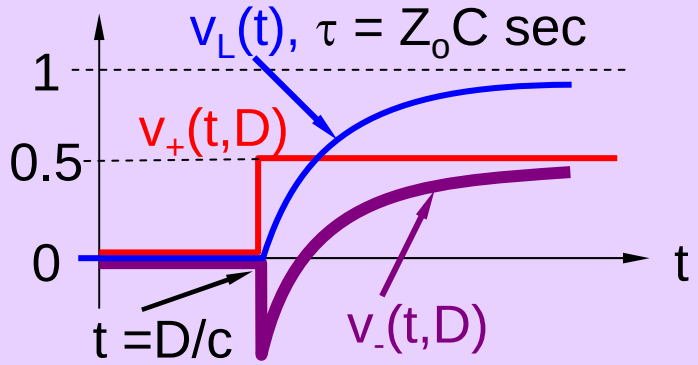




$$i(z,t) = Y_o[v_+(t - \frac{z}{c}) - v_-(t + \frac{z}{c})]$$

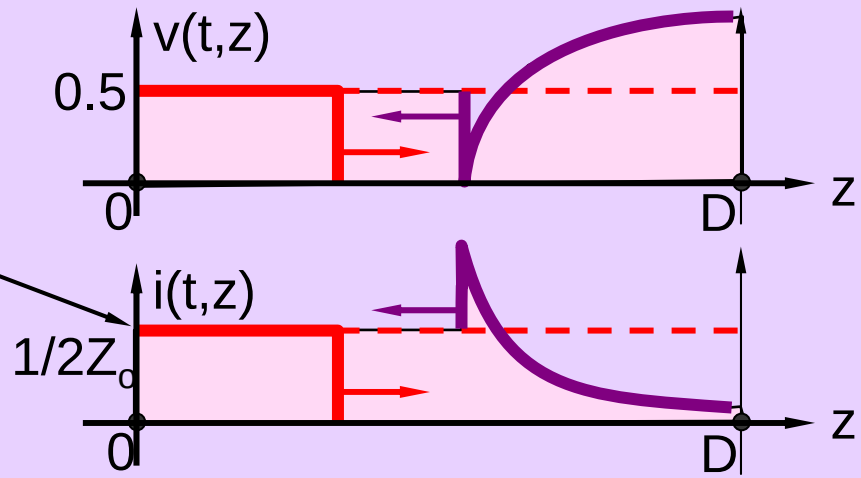
t = 0+, short-circuit response

$t \to \infty$, open-circuit response



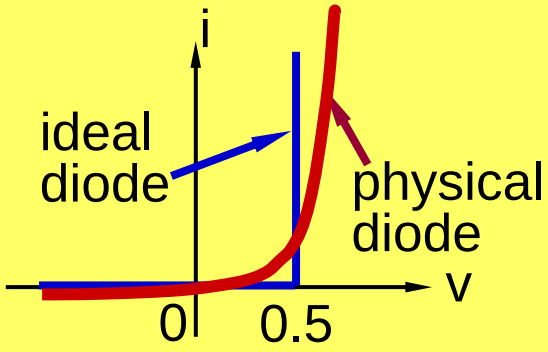
$v_L = v_- + v_+$ at the load $\Rightarrow$

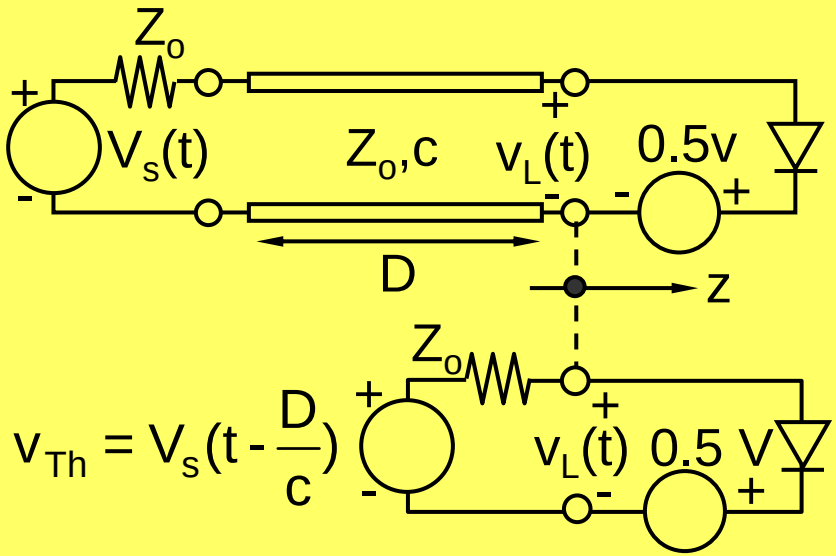
$v_-(t,D) = v_L(t,D) - v_+(t,D)$



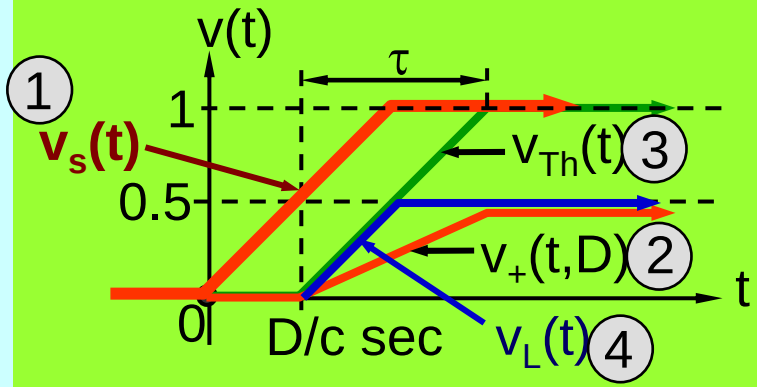
DIODE-TERMINATED TEM LINE

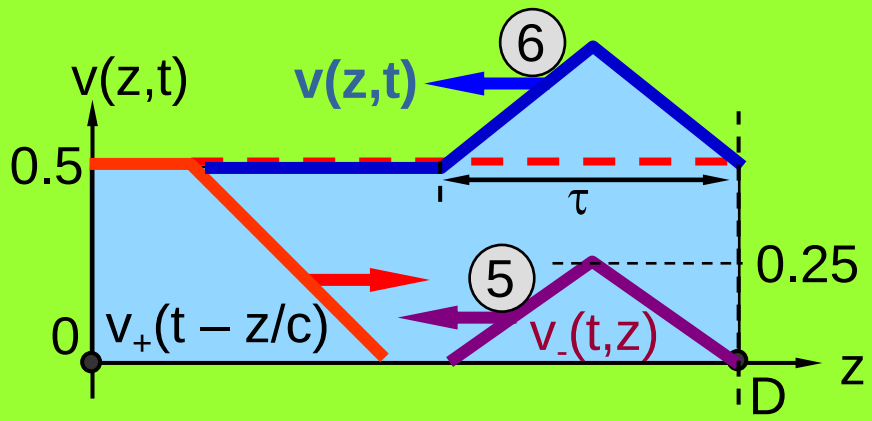
Example -- Logic Circuit:





Let $v_s(t)$ = 1-volt ramped step





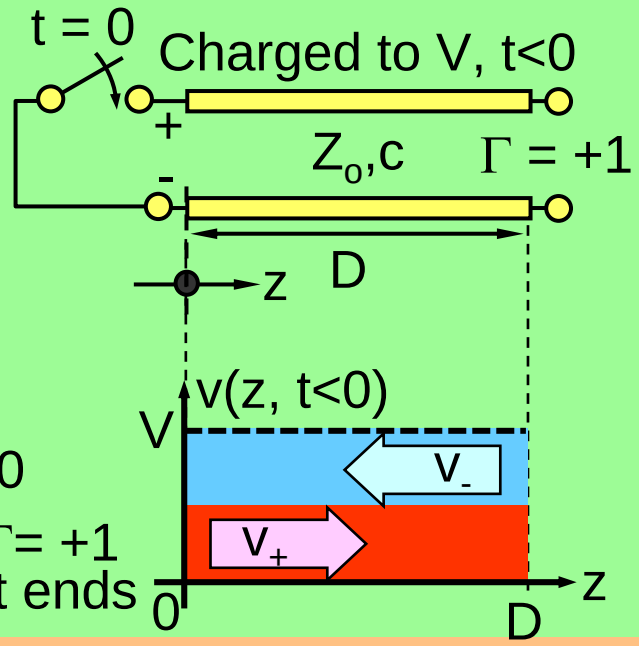
$v_L(t) = v_+(t,D) + v_-(t,D)$

INITIAL CONDITIONS

Example – charged TEM line:

$$v(z,t) = v_+(z - ct) + v_-(z + ct)$$

$$i(z,t) = \frac{1}{Z_o}[v_+(z - ct) - v_-(z + ct)]$$



Initial voltage and currents:

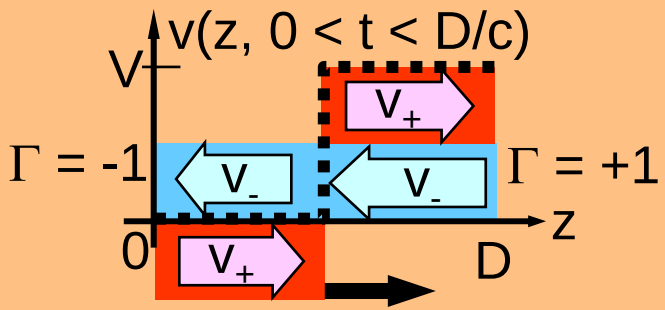
$$v_+(z,t) = \frac{1}{2}[v(z,t) + Z_o\, i(z,t)]$$

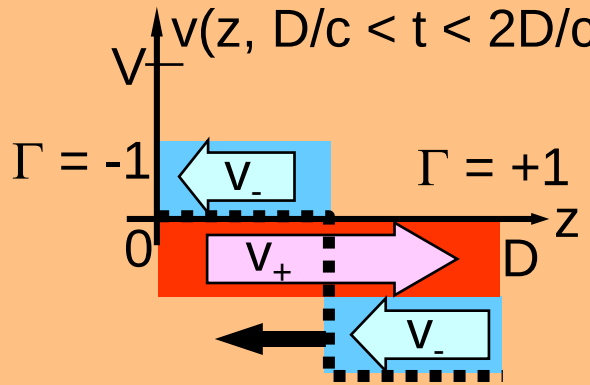
$$v_-(z,t) = \frac{1}{2}[v(z,t) - Z_o\, i(z,t)]$$

$$\Rightarrow \quad i(t < 0) = 0$$

Γ= +1
at ends

Subsequent voltages and currents:

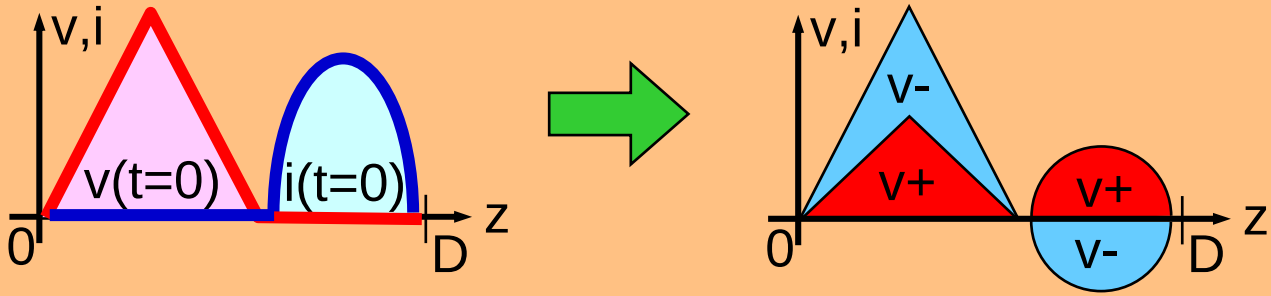




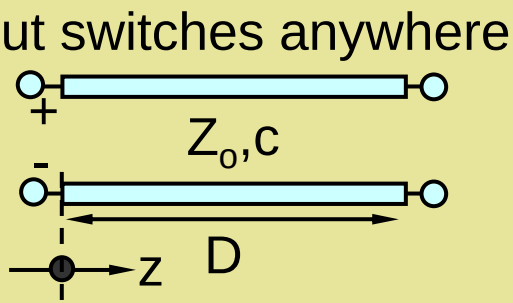
Lossless system, rattles forever

INITIAL CONDITIONS

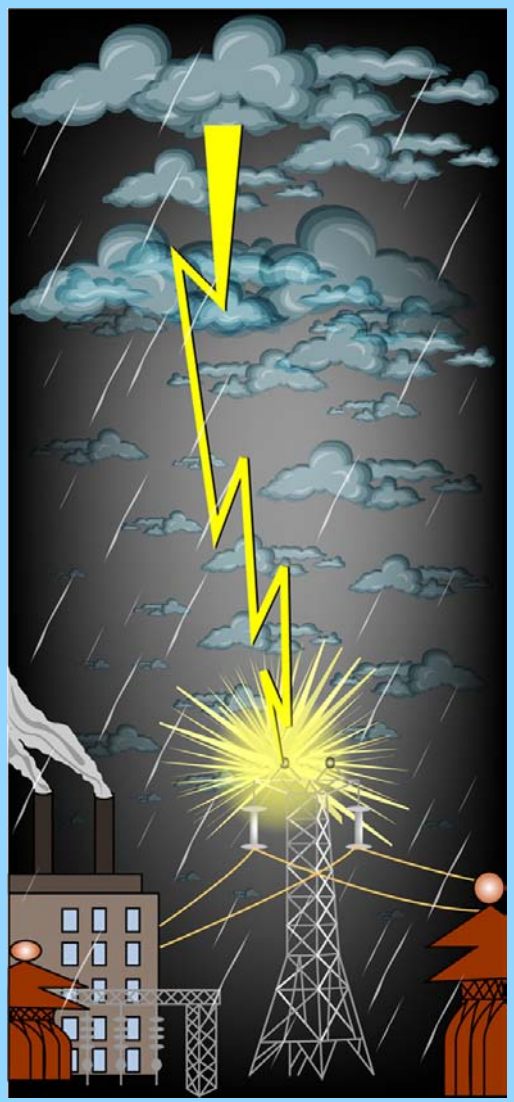
Arbitrary initial conditions:



Arbitrary circuits:



LIGHTNING STRIKE EXAMPLE



Lightning strikes the midpoint

What happens?

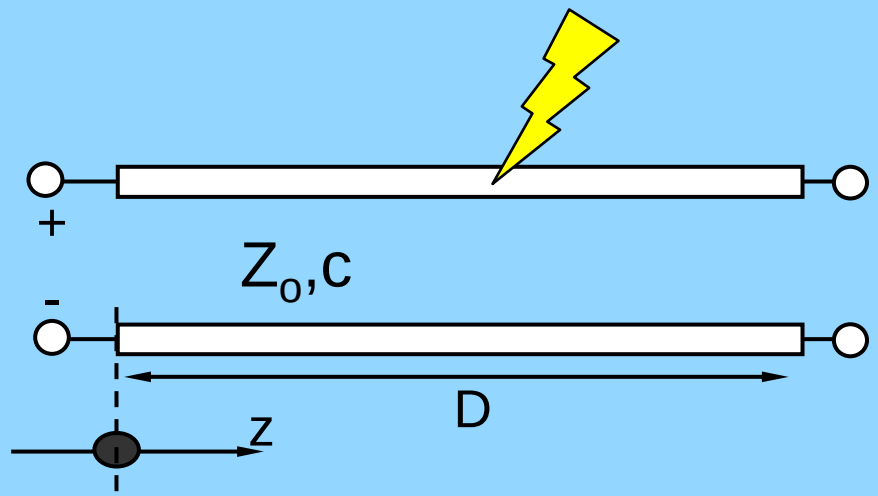
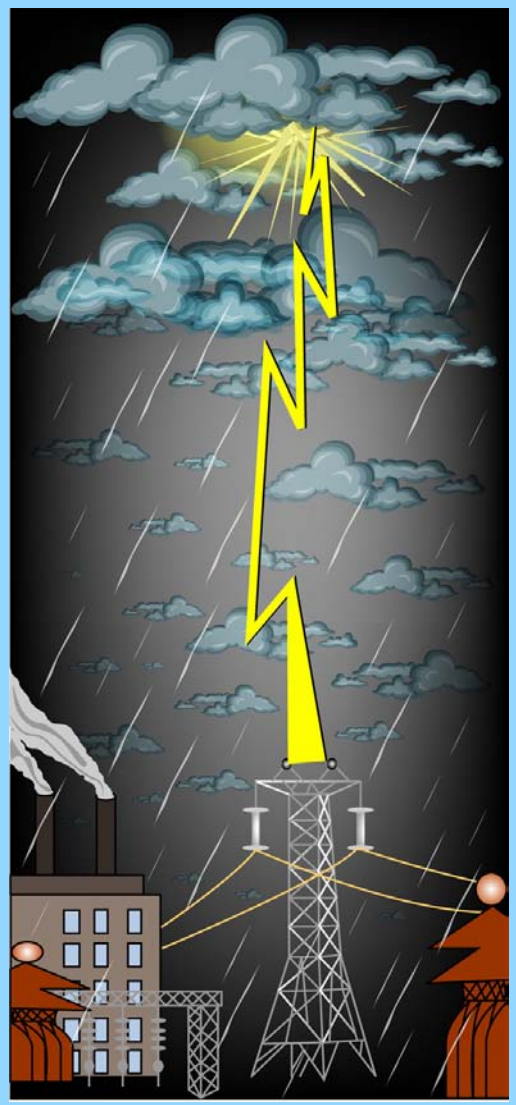


Figure by MIT OpenCourseWare.

LIGHTNING STRIKE EXAMPLE



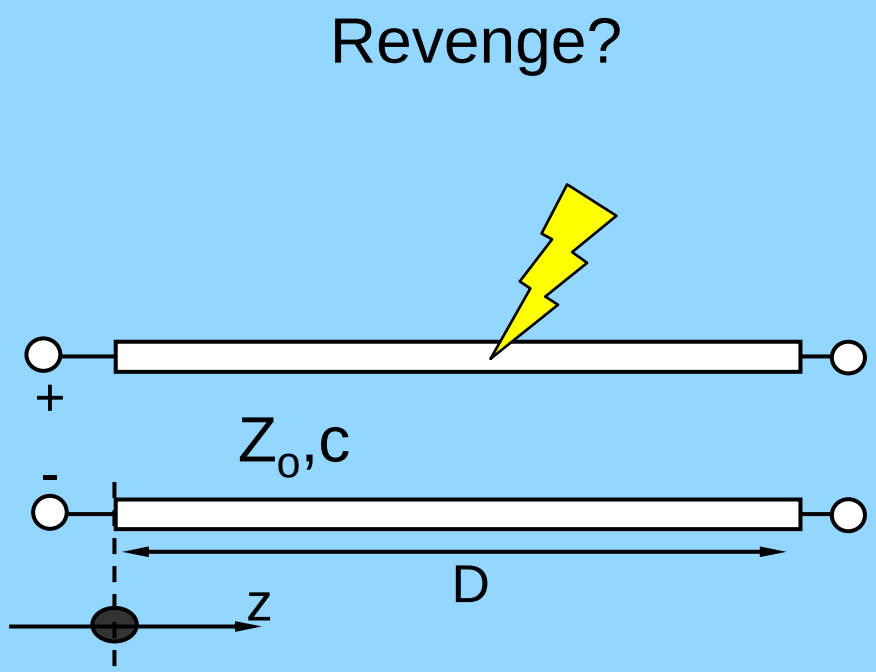
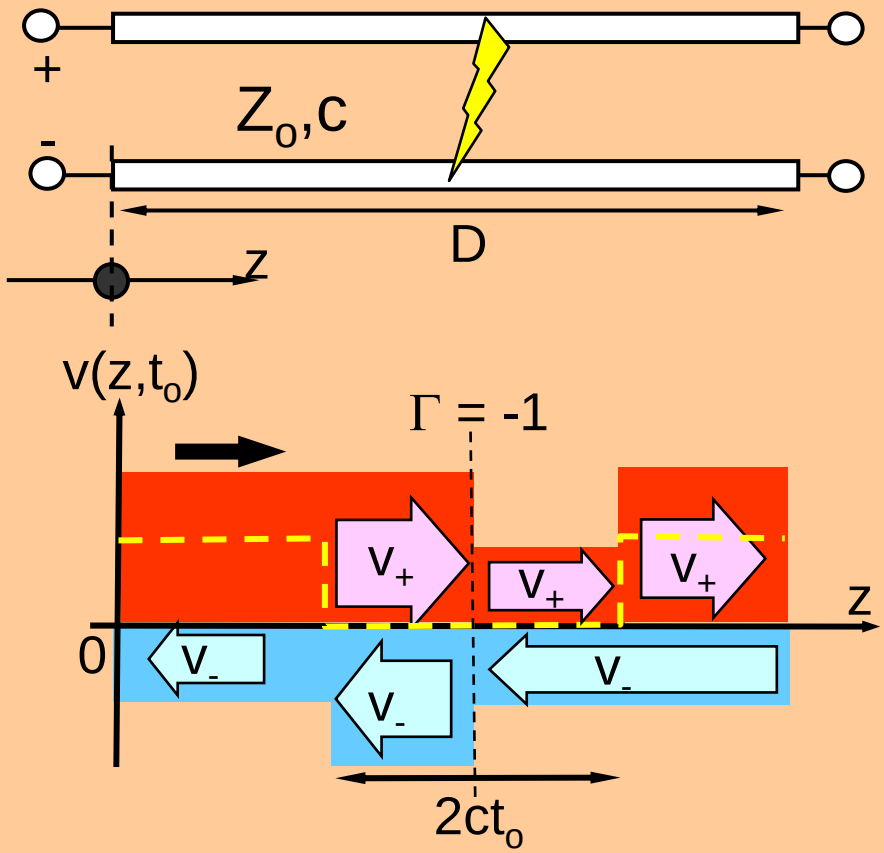


Figure by MIT OpenCourseWare.

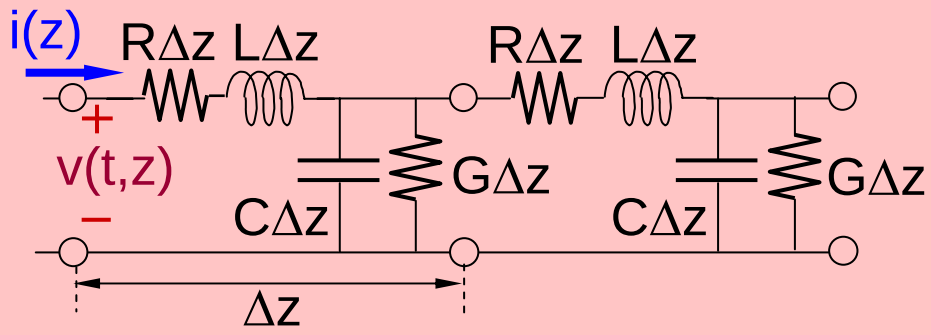
LIGHTNING STRIKE EXAMPLE

One possibility:
Temporary short through ionized channel



LOSSY TRANSMISSION LINES

Equivalent Circuit:



L [Hy m^{-1}], C [F m^{-1}],
R[Ω m^{-1}], G[S m^{-1}]

Equations for $\Delta z \to 0$:

$$\left.\begin{aligned}\frac{d\underline{V}(z)}{dz} &= -(R+j\omega L)\underline{I}(z)\\ \frac{d\underline{I}(z)}{dz} &= -(G+j\omega C)\underline{V}(z)\end{aligned}\right\} \Rightarrow \begin{cases}\dfrac{d^2\underline{V}(z)}{dz^2} = (R+j\omega L)(G+j\omega C)\underline{V}(z)\\ \underline{V}(z) = \underline{V}_+ e^{-j\underline{k}z} + \underline{V}_- e^{+j\underline{k}z}\\ \underline{k} = \sqrt{-(R+j\omega L)(G+j\omega C)} = k' \underbrace{- jk''}_{\text{decay}}\end{cases}$$

$$\left.\begin{aligned}\underline{V}(z) &= \underline{V}_+ e^{-jk'z - k''z}\\ \underline{I}(z) &= Y_o\,\underline{V}_+ e^{-jk'z - k''z}\end{aligned}\right\} \text{ for forward wave: } \underline{Y}_o = \sqrt{(G+j\omega C)/(R+j\omega L)}$$

Propagation $e^{-j\underline{k}z}$: decay rate ($e^{-k''z}$) and phase velocity (v_p) = f(ω)

(Exception: "distortionless line" for which R/L = G/C)

6.013 Electromagnetics and Applications
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