Recitation 14: I-V characteristics of p-n junction diode (I)

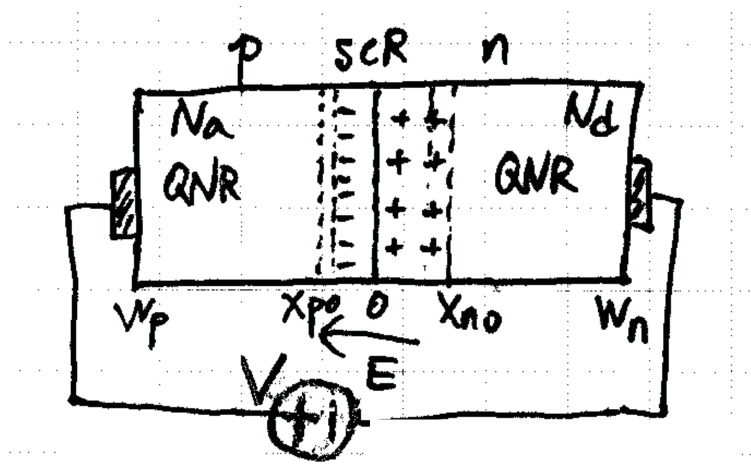
What happens to the (minority) carrier concentrations when a bias is applied?

Location-Dependence

In order to see what happens to the minority carrier concentrations at *different locations inside a diode* when a bias V is applied, let us look at some diagrams:

1. QNR & SCR (or depletion region)

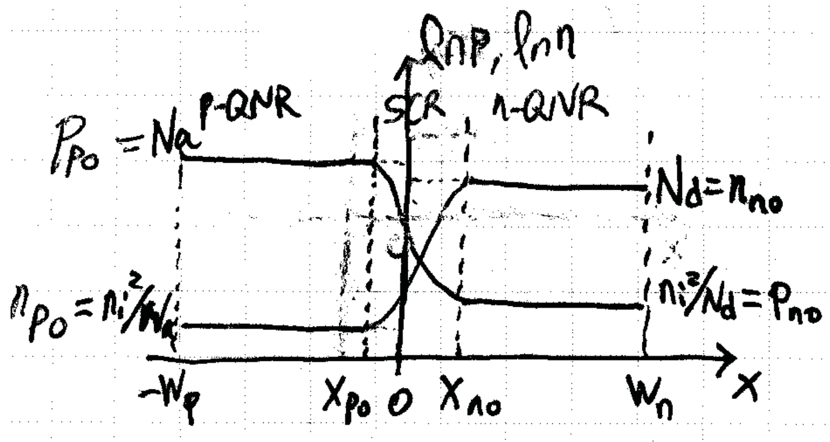
$$V = 0, \text{ SCR region edge: } x_{\text{no}} \forall x_{\text{po}}$$



2. Under thermal equilibrium:

$$\begin{aligned}
\text{Hole concentration in p-QNR:} p_{\text{po}} &= N_{\text{a}} \\
\text{Minority electron concentration in p-QNR:} n_{\text{po}} &= \frac{n_{\text{i}}^2}{N_{\text{a}}} \\
\text{Minority hole concentration in n-QNR:} p_{\text{no}} &= \frac{n_{\text{i}}^2}{N_{\text{d}}} \\
\text{Electron concentration in n-QNR:} n_{\text{no}} &= N_{\text{d}}
\end{aligned}$$

The hole concentration and minority electron concentrations are constant throughout QNR.



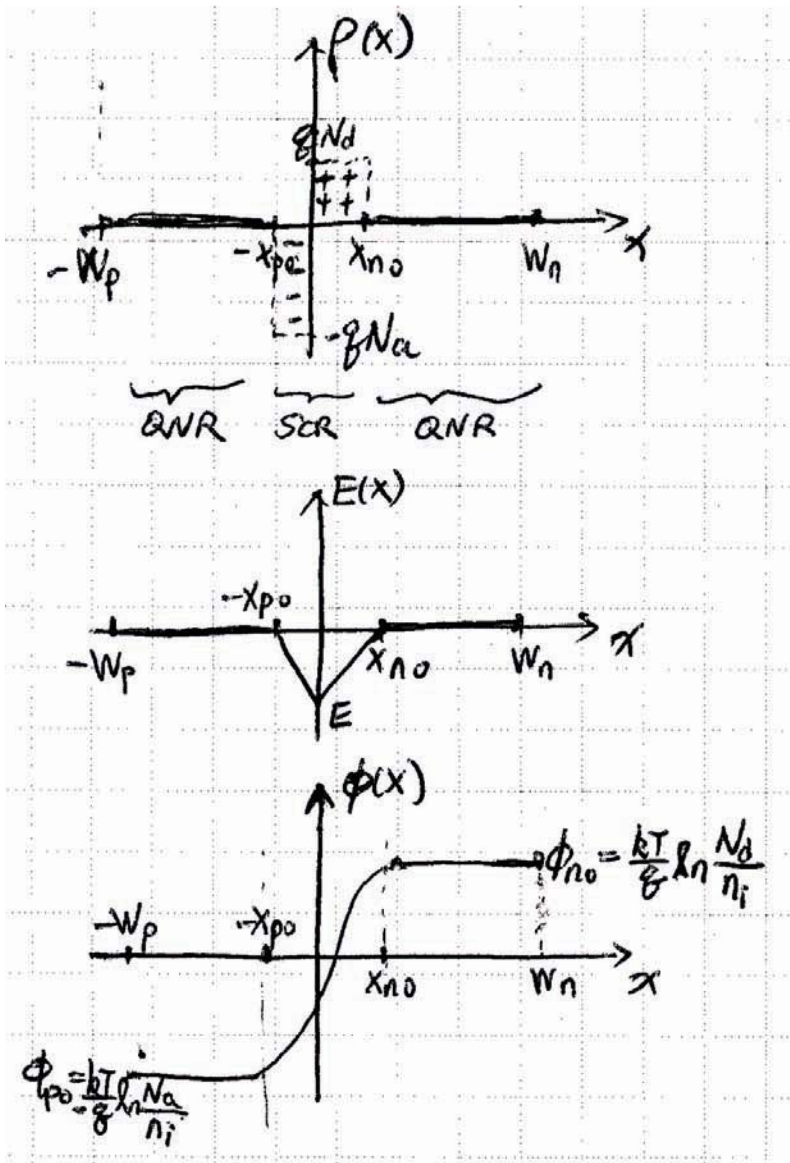
3. Built-in barrier (potential)

$$\phi_{\mathrm{B}} = \phi_{\mathrm{no}} - \phi_{\mathrm{po}} = \frac{kT}{q} \ln\left(\frac{N_{\mathrm{a}} \cdot N_{\mathrm{d}}}{n_{\mathrm{i}}^2}\right)$$

4. Is there any relationship between the hole concentration on the p-side (p_{po}) with the hole concentration on the n-side? It does not seem like at first look, but

$$\begin{aligned}
\phi_{\mathrm{B}} &= V_{\mathrm{th}} \ln\left(\frac{N_{\mathrm{a}} \cdot N_{\mathrm{d}}}{n_{\mathrm{i}}^2}\right),\ N_{\mathrm{a}} = p_{\mathrm{po}},\ \frac{N_{\mathrm{d}}}{n_{\mathrm{i}}^2} = \frac{1}{p_{\mathrm{no}}} \\
\Longrightarrow \phi_{\mathrm{B}} &= V_{\mathrm{th}} \ln\left(\frac{p_{\mathrm{po}}}{p_{\mathrm{no}}}\right) \\
\Longrightarrow p_{\mathrm{no}} &= p_{\mathrm{po}} \mathrm{e}^{-\phi_{\mathrm{B}}/\mathrm{V_{th}}} = \mathrm{N_a} \cdot \mathrm{e}^{-\phi_{\mathrm{B}}/\mathrm{V_{th}}} \\
\text{and similarly, } n_{\mathrm{po}} &= n_{\mathrm{no}} \cdot \mathrm{e}^{-\phi_{\mathrm{B}}/\mathrm{V_{th}}} = \mathrm{N_d} \mathrm{e}^{-\phi_{\mathrm{B}}/\mathrm{V_{th}}}
\end{aligned}$$

They are related to each other by ϕ_{B}


ρ(x)
qNd
-Wp
-xpo
xno
Wn
x
-qNa
QNR
SCR
QNR
E(x)
-xpo
-Wp
xno
Wn
x
E
φ(x)
φno = kT/q ln Nd/ni
-Wp
-xpo
xno
Wn
x
φpo = -kT/q ln Na/ni

What happens when $V \neq 0$?

$V > 0$

$x_{\mathrm{n}}, x_{\mathrm{p}}$ shrinks, electric field reduces, barrier reduces

$$\phi'_{\mathrm{B}} = \phi_{\mathrm{B}} - V$$

What can we say about the *minority carrier concentration at the SCR edges*?
It turns out the relationship

$$p_{\mathrm{no}} = p_{\mathrm{po}} \cdot \mathrm{e}^{-\frac{\phi_{\mathrm{B}}}{\mathrm{V_{th}}}}; \quad \mathrm{n_{po}} = \mathrm{n_{no}} \cdot \mathrm{e}^{-\mathrm{frac}\phi_{\mathrm{B}}\mathrm{V_{th}}}$$

still holds, but $\phi_{\mathrm{B}} \to \phi'_{\mathrm{B}}$.

$$\begin{aligned}
\therefore p_{\mathrm{n}} \underbrace{(x_{\mathrm{n}})}_{\text{important}} &= \underbrace{p_{\mathrm{p}}}_{\text{will talk about this later}} \cdot \mathrm{e}^{-\frac{\phi'_{\mathrm{B}}}{\mathrm{V_{th}}}} = \mathrm{N_a} \cdot \mathrm{e}^{-\frac{\phi'_{\mathrm{B}}}{\mathrm{V_{th}}}} \\
&= N_{\mathrm{a}} \cdot \mathrm{e}^{-\frac{(\phi_{\mathrm{B}} - \mathrm{V})}{\mathrm{V_{th}}}} = \underbrace{N_{\mathrm{a}}\mathrm{e}^{-\phi_{\mathrm{B}}/\mathrm{V_{th}}}}_{\mathrm{p_{no}}} \cdot \mathrm{e}^{\frac{\mathrm{V}}{\mathrm{V_{th}}}} \\
\Longrightarrow \quad & p_{\mathrm{n}}(x_{\mathrm{n}}) = p_{\mathrm{no}} \cdot \mathrm{e}^{\frac{\mathrm{V}}{\mathrm{V_{th}}}}
\end{aligned}$$

Law of the junction

$$\begin{aligned}
p_{\mathrm{n}}(x_{\mathrm{n}}) &= (N_{\mathrm{a}} \cdot \mathrm{e}^{-\frac{\phi_{\mathrm{B}}}{\mathrm{V_{th}}}}) \cdot \mathrm{e}^{\frac{\mathrm{V}}{\mathrm{V_{th}}}} = \mathrm{p_{no}}\mathrm{e}^{\frac{\mathrm{V}}{\mathrm{V_{th}}}} \\
n_{\mathrm{p}}(-x_{\mathrm{p}}) &= (N_{\mathrm{d}} \cdot \mathrm{e}^{-\frac{\phi_{\mathrm{B}}}{\mathrm{V_{th}}}}) \cdot \mathrm{e}^{\frac{\mathrm{V}}{\mathrm{V_{th}}}} = \mathrm{n_{po}}\mathrm{e}^{\frac{\mathrm{V}}{\mathrm{V_{th}}}}
\end{aligned}$$

The "law of the junction" relates the *minority* carrier concentration $\underbrace{\textit{at the SCR edge}}_{\text{important}}$ with the equilibrium minority carrier concentration and the applied bias (60 mV/dec)

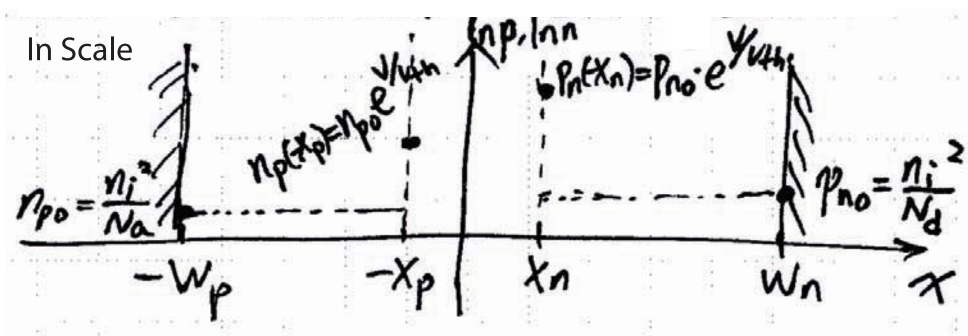
Some notes:

1. This is so only for the concentration at the SCR edge
2. For $V > 0$, this is usually called "minority carrier injection" because from the "law of the junction" relationship, by lowering the barrier, more majority carriers from one side move across the SCR and add on to the minority carriers on the other side
3. This can also be understood from the fact that, when $V > 0, E \downarrow, J_{\mathrm{drift}} < J_{\mathrm{diff}}$, more holes can diffuse from $p \to n$, and more e^- can diffuse from $n \to p$
4. *Low Level Injection*: $p_{\mathrm{n}} < n_{\mathrm{no}}$ and $n_{\mathrm{p}} < p_{\mathrm{po}}$ (minority < majority) in order to have valid law of the junction

5. What about the minority carrier concentration at other locations inside the QNR?

    - At the contact $(w_\mathrm{n}, -w_\mathrm{p})$: since recombination/generation occurs very fast, carrier concentration in equilibrium
    - Inside QNR: we will talk about this on Thursday

    So now we have two data-points



Ratio remains the same:

$$\frac{n_\mathrm{p}(-x_\mathrm{p})}{p_\mathrm{n}(-x_\mathrm{n})} = \frac{n_\mathrm{po}}{p_\mathrm{no}}$$

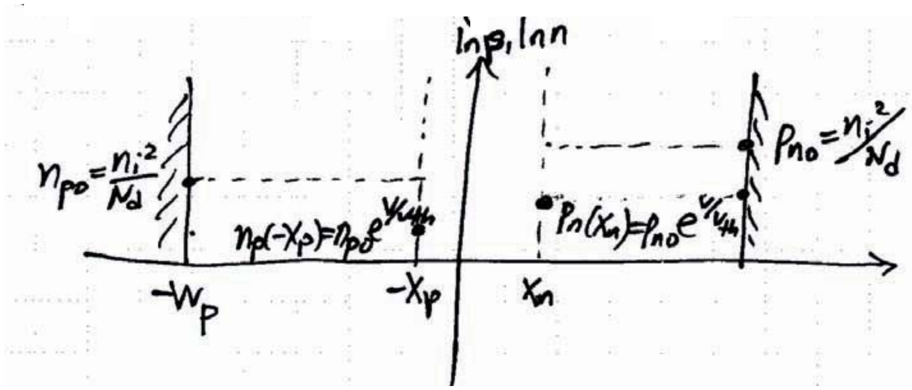
$V < 0$

Applying law of the junction:

$$\begin{aligned} p_\mathrm{n}(x_\mathrm{n}) &= N_\mathrm{a}\mathrm{e}^{-(\phi_\mathrm{B}-\mathrm{V})/\mathrm{V_{th}}} = \mathrm{N_a}\mathrm{e}^{-\frac{\phi_\mathrm{B}}{\mathrm{V_{th}}}} \cdot \mathrm{e}^{\frac{\mathrm{V}}{\mathrm{V_{th}}}} \\ &= p_\mathrm{no}\mathrm{e}^{\frac{\mathrm{V}}{\mathrm{V_{th}}}} \\ n_\mathrm{p}(-x_\mathrm{p}) &= n_\mathrm{po}\mathrm{e}^{\frac{\mathrm{V}}{\mathrm{V_{th}}}} \end{aligned}$$

Equations are the same, but since $V < 0$ this time, instead of having "minority carrier injection", we have "minority carrier extraction".

The picture regarding minority carrier concentration:



Exercise

$N_a = 10^{18}\,\text{cm}^{-3}, \text{N}_\text{d} = 10^{16}\,\text{cm}^{-3}$. Calculate minority carrier concentration at the SCR edge for $V = 0, V = -0.12\,\text{V}, \text{V} = 0.6\,\text{V}$ First,

$$\begin{aligned} p_{no} &= \frac{n_i^2}{N_d} = 10^4\,\text{cm}^{-3} \\ n_{po} &= \frac{n_i^2}{N_a} = 10^2\,\text{cm}^{-3} \end{aligned}$$

Law of the junction as power of 10:

$$n_p(-x_p) = n_{po}\cdot \text{e}^{\text{V}/\text{V}_\text{th}} = \text{n}_\text{po}\cdot 10^{\text{V}/60\,\text{mV}}$$

$$\begin{aligned} V &= 0, n_{po}, p_{no} \\ V &= -0.12\,\text{V}, \implies \text{n}_\text{p}(-\text{x}_\text{p}) = \text{n}_\text{po}\cdot 10^{-0.12/+0.06} = 1\,\text{cm}^{-3}, \text{p}_\text{n}(\text{x}_\text{n}) = 10^2\,\text{cm}^{-3} \\ V &= 0.6\,\text{V}, \implies \text{n}_\text{p}(-\text{x}_\text{p}) = \text{n}_\text{po}\cdot 10^{0.6/0.06} = 10^{10}\cdot 10^2 = 10^{12}\,\text{cm}^{-3} \ll \text{N}_\text{a} = 10^{18}\,\text{cm}^{-3} \\ & \quad p_n(x_n) = p_{no}\cdot 10^{10} = 10^{14}\,\text{cm}^{-3} \ll \text{N}_\text{d} = 10^{16}\,\text{cm}^{-3} \end{aligned}$$

At what point is Low Level Injection valid?

$$\text{Let}\, p_n(x_n) = N_d = 10^{16},\;\; \text{e}^{\text{V}/\text{V}_\text{th}} = 10^{12},\;\; \implies \text{V} = 12\times 60 = 720\,\text{mV}$$