6.012 - Microelectronic Devices and Circuits

Lecture 6 - p-n Junctions: I-V Relationship - Outline

- **Announcements**

  First Hour Exam - Oct. 7, 7:30-9:30 pm; thru 10/2/09, PS #4

- **Review**

  Minority carrier flow in QNRs: 1. $L_{min} << w$, 2. $L_{min} >> w$

- **I-V relationship for an abrupt p-n junction**

  Assume: 1. Low level injection
  2. All applied voltage appears across junction:
  3. Majority carriers in quasi-equilibrium with barrier
  4. Negligible SCL generation and recombination

  Relate minority populations at QNR edges, $-x_p$ and x_n, to v_{AB}
  Use $n'(-x_p)$, $p'(x_n)$ to find hole and electron currents in QNRs
  Connect currents across SCL to get total junction current, i_D

- **Features and limitations of the model**

  Engineering the minority carrier injection across a junction
  Deviations at low and high current levels
  Deviations at large reverse bias

QNR Flow: Uniform doping, non-uniform LL injection

We use the 5 QNR flow conditions* to simplify our 5 equations...

(assuming a p-type sample)

$$\cancel{\frac{\partial p(x,t)}{\partial t}}_{\text{Quasi-static}} + \frac{1}{q}\frac{\partial J_h(x,t)}{\partial x} = \cancel{\frac{\partial n(x,t)}{\partial t}}_{\text{Quasi-static}} - \frac{1}{q}\frac{\partial J_e(x,t)}{\partial x} \approx g_L(x,t) - \left[\cancel{n(x,t)p(x,t) - n_i^2}\right]r(t)$$

LLI

$$\Rightarrow \frac{1}{q}\frac{dJ_h(x,t)}{dx} = \boxed{-\frac{1}{q}\frac{dJ_e(x,t)}{dx} \approx g_L(x,t) - \frac{n'(x,t)}{\tau_e}}$$

LLI

$$J_h(x,t) = q\mu_h p(x,t)E(x,t) - qD_h\frac{\partial p(x,t)}{\partial x} \approx q\mu_h p_o(x)E(x,t) - qD_h\frac{dn'(x,t)}{dx}$$

Quasi-neutrality

Negligible minority drift

$$J_e(x,t) = \cancel{q\mu_e n(x,t)E(x,t)} + qD_e\frac{\partial n(x,t)}{\partial x} \Rightarrow \boxed{J_e(x,t) \approx qD_e\frac{dn'(x,t)}{dx}}$$

Uniform doping

$$\frac{q}{\varepsilon(x)}\left[p(x,t) - n(x,t) + N_d^+(x) - N_a^-(x)\right] = \frac{dE_x(x,t)}{dx} \approx \frac{q}{\varepsilon}\left[p'(x,t) - n'(x,t)\right]$$

..and end up with one equation in n': the static diffusion equation!

$$\frac{d^2n'(x,t)}{dx^2} - \frac{n'(x,t)}{D_e\tau_e} = -\frac{1}{D_e}g_L(x,t)$$

* Five assumptions that define flow problems AND should be validated at the end.

QNR Flow, cont.: Solving the steady state diffusion equation

The steady state diffusion equation in p-type material is:

$$\frac{d^2 n'(x)}{dx^2} - \frac{n'(x)}{L_e^2} = -\frac{1}{D_e} g_L(x)$$

and for n-type material it is:

$$\frac{d^2 p'(x)}{dx^2} - \frac{p'(x)}{L_h^2} = -\frac{1}{D_h} g_L(x)$$

In writing these expressions we have introduced L_e and L_h, the minority carrier diffusion lengths for holes and electrons, as:

$$L_e \equiv \sqrt{D_e \tau_e} \qquad L_h \equiv \sqrt{D_h \tau_h}$$

We'll see that the minority carrier diffusion length tells us how far the average minority carrier diffuses before it recombines.

In a basic p-n diode, we have $g_L = 0$ which means we only need the homogenous solutions, i.e. expressions that satisfy:

n-side: $$\frac{d^2 p'(x)}{dx^2} - \frac{p'(x)}{L_h^2} = 0$$

p-side: $$\frac{d^2 n'(x)}{dx^2} - \frac{n'(x)}{L_e^2} = 0$$

QNR Flow, cont.: Solving the steady state diffusion equation

We seldom care about this general result. Instead, we find that most diodes fall into one of two cases:

Case I - Long-base diode: $w_n >> L_h$

Case II - Short-base diode: $L_h >> w_n$

Case I: When $w_n >> L_h$, which is the situation in an LED, for example, the solution is

$$p'(x) \approx p'(x_n) e^{-(x-x_n)/L_h} \quad \text{for} \quad x_n \le x \le w_n$$

This profile decays from $p'(x_n)$ to 0 exponentially as $e^{-x//L_h}$.

The corresponding hole current for $x_n \le x \le w_n$ in Case I is

$$J_h(x) \approx -qD_h \frac{dp'(x)}{dx} = \frac{qD_h}{L_h} p'(x_n) e^{-(x-x_n)/L_h} \quad \text{for} \quad x_n \le x \le w_n$$

The current decays to zero also, indicating that all of the excess minority carriers have recombined before getting to the contact.

QNR Flow, cont.: Solving the steady state diffusion equation

Case II: **When $L_h >> w_n$, which is the situation in integrated Si diodes, for example, the differential equation simplifies to:**

$$\frac{d^2 p'(x)}{dx^2} = \frac{p'(x)}{L_h^2} \approx 0$$

We see immediately that p'(x) is linear: $p'(x) = A\,x + B$

Fitting the boundary conditions we find:

$$p'(x) \approx p'(x_n)\left[1 - \left(\frac{x - x_n}{w_n - x_n}\right)\right] \quad \text{for} \quad x_n \leq x \leq w_n$$

This profile is a straight line, decreasing from $p'(x_n)$ at x_n to 0 at w_n.

In Case II the current is constant for $x_n \leq x \leq w_n$:

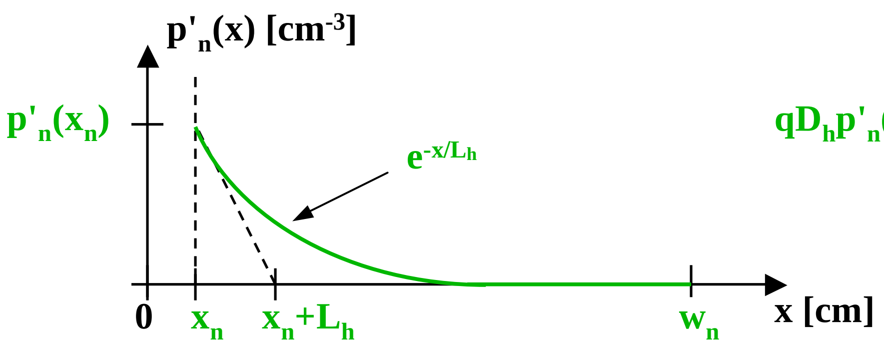
$$J_h(x) \approx -qD_h \frac{dp'(x)}{dx} = \frac{qD_h}{w_n - x_n} p'(x_n) \quad \text{for} \quad x_n \leq x \leq w_n$$

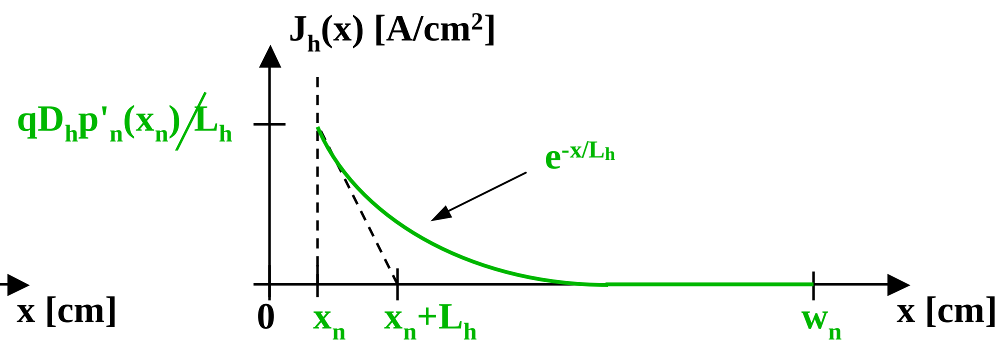
The constant current indicates that no carriers recombine before reaching the contact.

QNR Flow, cont.: Uniform doping, non-uniform LL injection

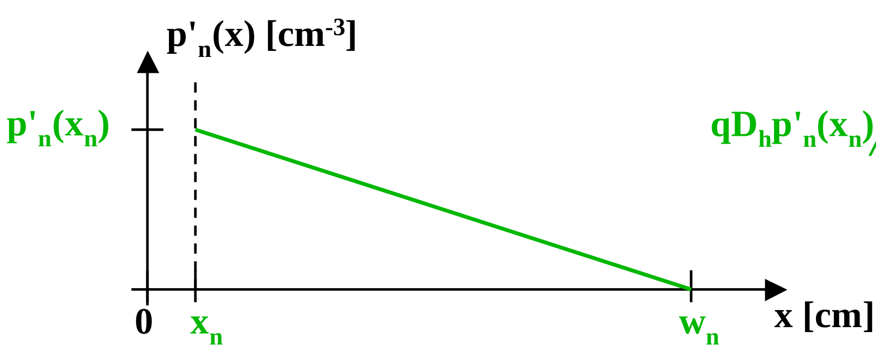
Sketching and comparing the limiting cases: $w_n >> L_h$, $w_n << L_h$

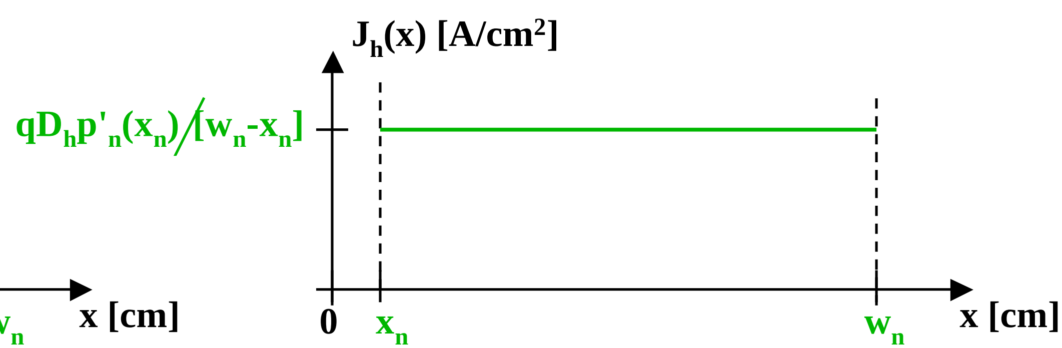
Case I - Long base: $w_n >> L_n$ (the situation in LEDs)





Case II - Short base: $w_n << L_n$ (the situation in most Si diodes and transistors)





QNR Flow, cont.: Uniform doping, non-uniform LL injection

The four other unknowns

- ⇒ **In n-type the steady state diffusion equation gives p'.**
- ⇒ **Knowing p', we can easily get n', J_e, J_h, and E_x:**

Note: In Lec 5 we saw this for a p-type sample.

First find J_h:

$$J_h(x) \approx -qD_h \frac{dp'(x)}{dx}$$

Then find J_e:

$$J_e(x) = J_{Tot} - J_h(x)$$

Next find E_x:

$$E_x(x) \approx \frac{1}{q\mu_e n_o}\left[J_e(x) + \frac{D_e}{D_h} J_h(x)\right]$$

Then find n':

$$n'(x) \approx p'(x) - \frac{\varepsilon}{q}\frac{dE_x(x)}{dx}$$

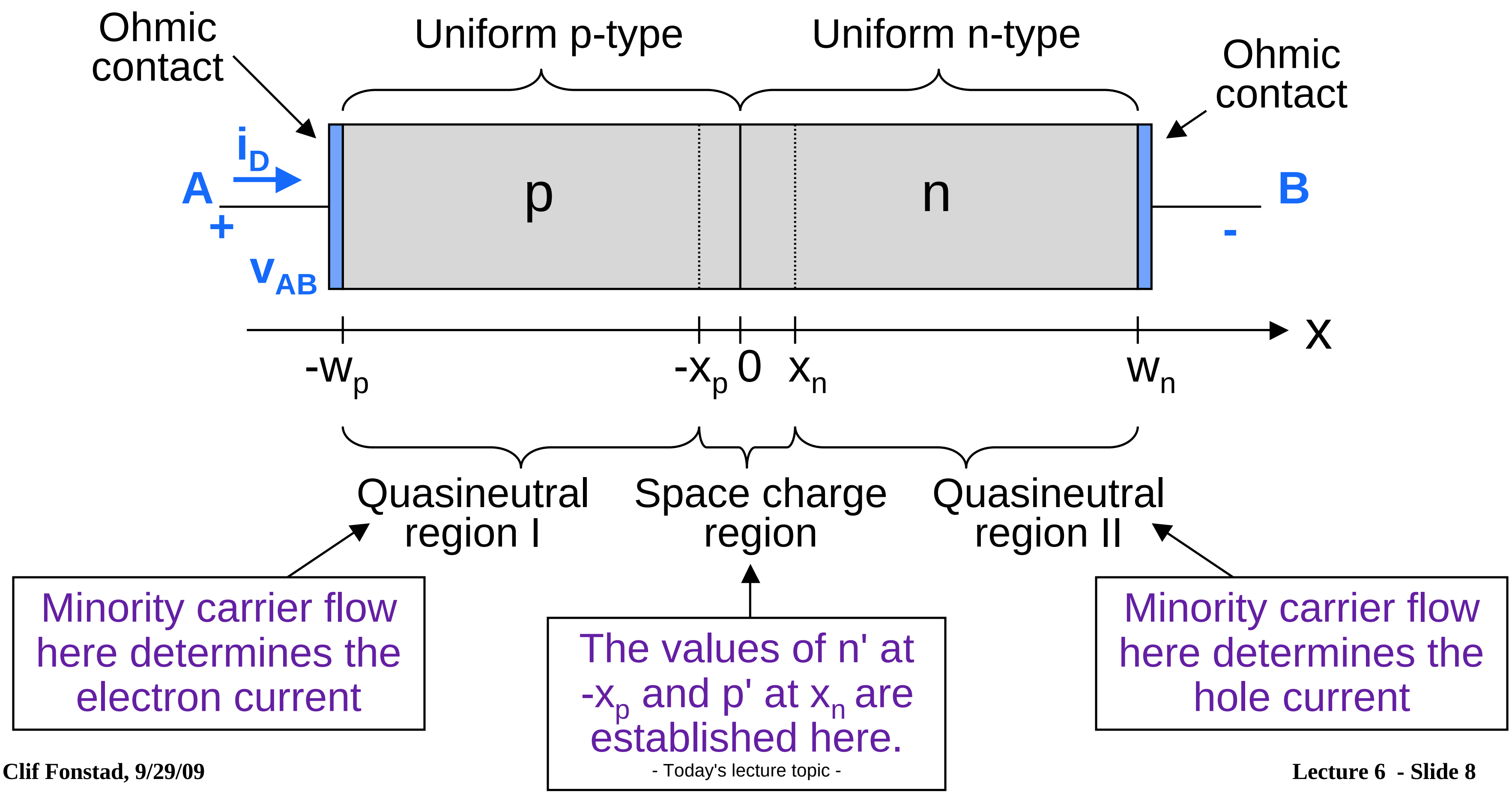
Finally, go back and check that all of the five conditions are met by the solution.

✓ **Once we solve the diffusion equation and get the minority carrier excess we know everything.**

Current flow: finding the relationship between i_D and v_{AB}

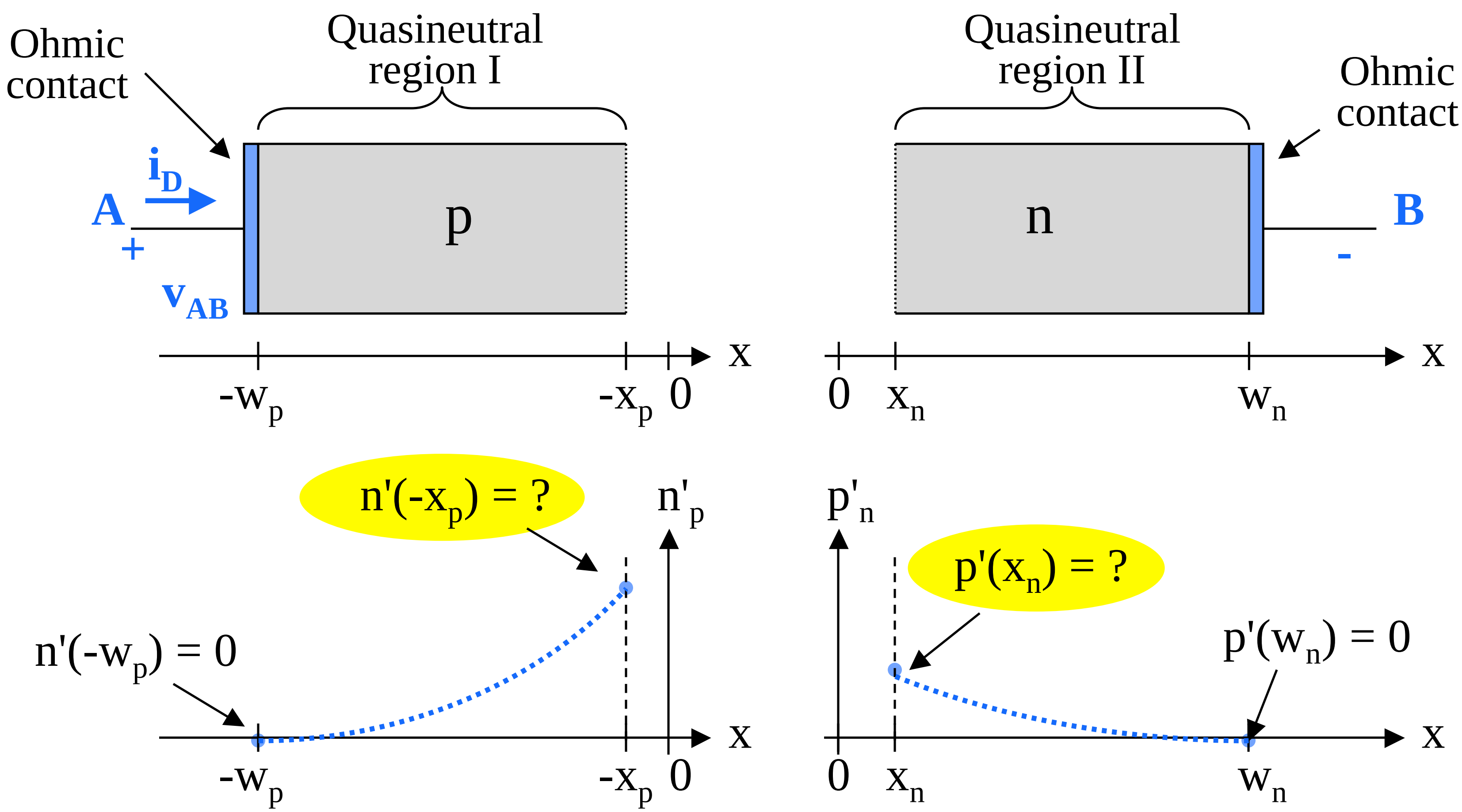
There are two pieces to the problem:

- Minority carrier flow in the QNRs is what limits the current.
- Carrier equilibrium across the SCR determines $n'(-x_p)$ and $p'(x_n)$, the boundary conditions of the QNR minority carrier flow problems.



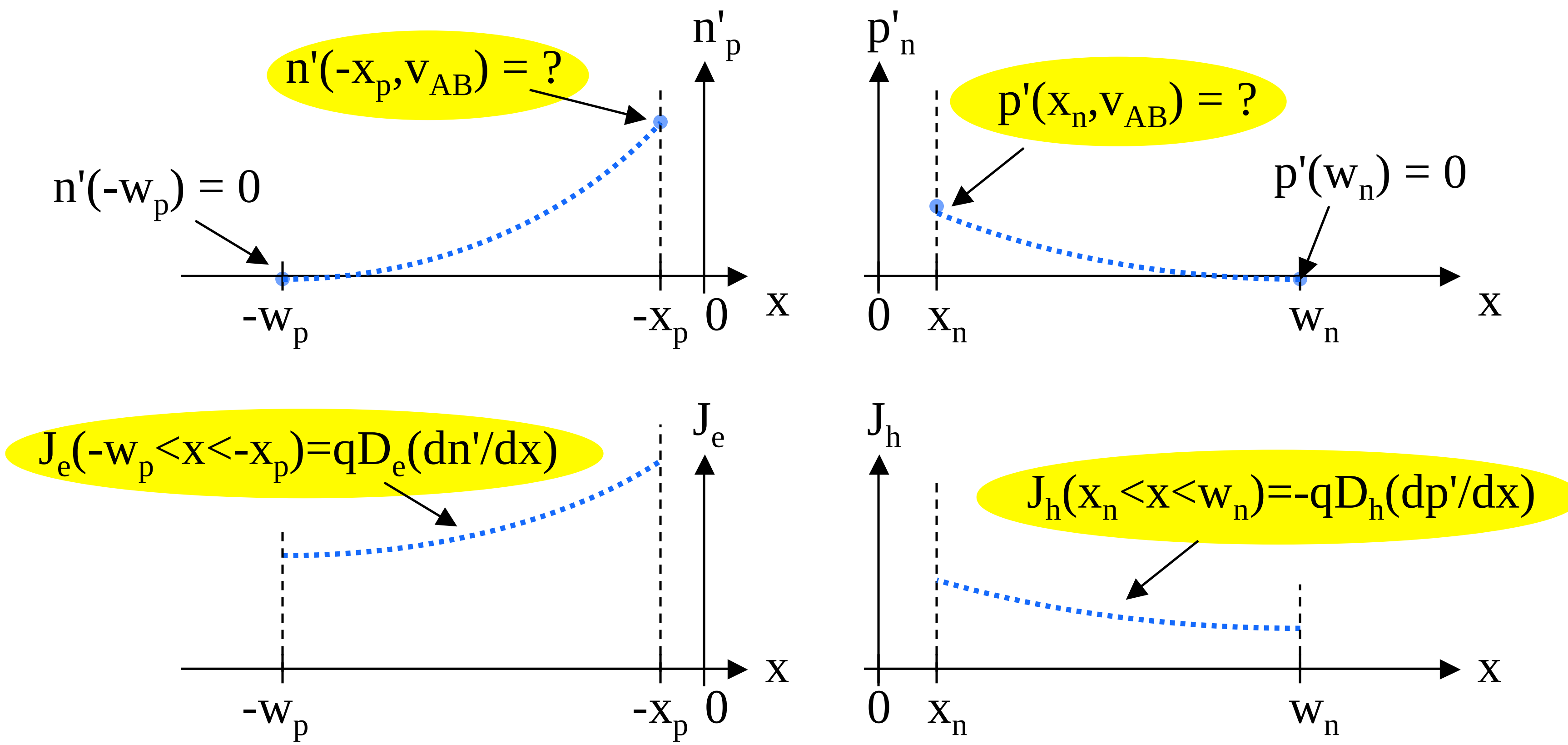
The p-n Junction Diode: *the game plan for getting $i_D(v_{AB})$*

We have two QNR's and a flow problem in each:



If we knew $n'(-x_p)$ and $p'(x_n)$, we could solve the flow problems and we could get $n'(x)$ for $-w_p<x<-x_p$, and $p'(x)$ for $x_n<x<w_n$ …

....and knowing $n'(x)$ for $-w_p<x<-x_p$, and $p'(x)$ for $x_n<x<w_n$, we can find $J_e(x)$ for $-w_p<x<-x_p$, and $J_h(x)$ for $x_n<x<w_n$.



Having $J_e(x)$ for $-w_p<x<-x_p$, and $J_h(x)$ for $x_n<x<w_n$, we can get i_D because we will argue that $i_D(v_{AB}) = A[J_e(-x_p,v_{AB})+J_h(x_n,v_{AB})]$...

...but first we need to know $n'(-x_p,v_{AB})$ and $p'(x_n,v_{AB})$.

We will do this now.

The impact of the barrier height change on the carrier populations and fluxes:

Unbiased junction

Population in equilibrium with barrier

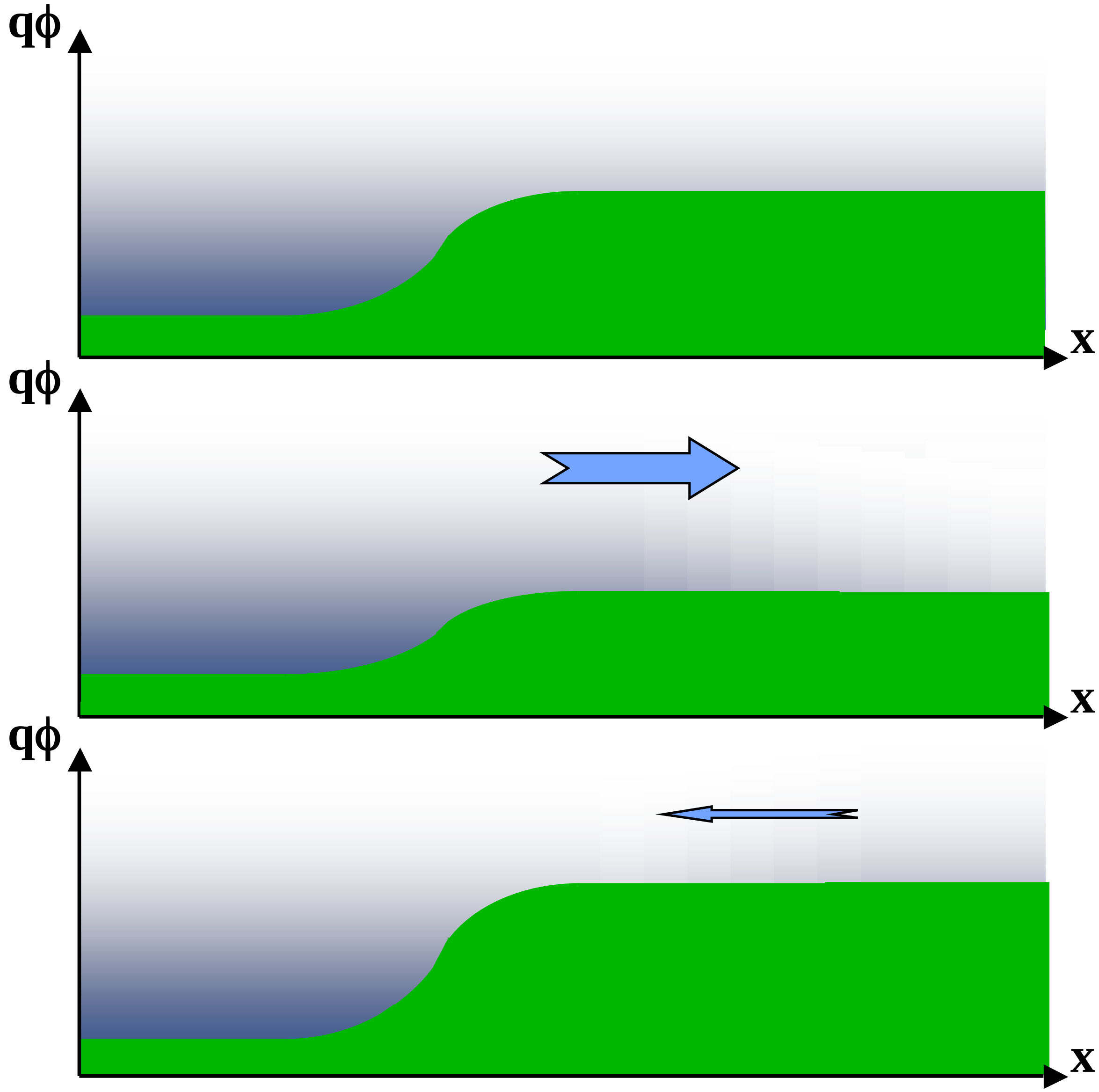
Forward bias on junction

Barrier lowered so carriers to left can cross over it.

The flux is limited by how fast they diffuse in the QNR.

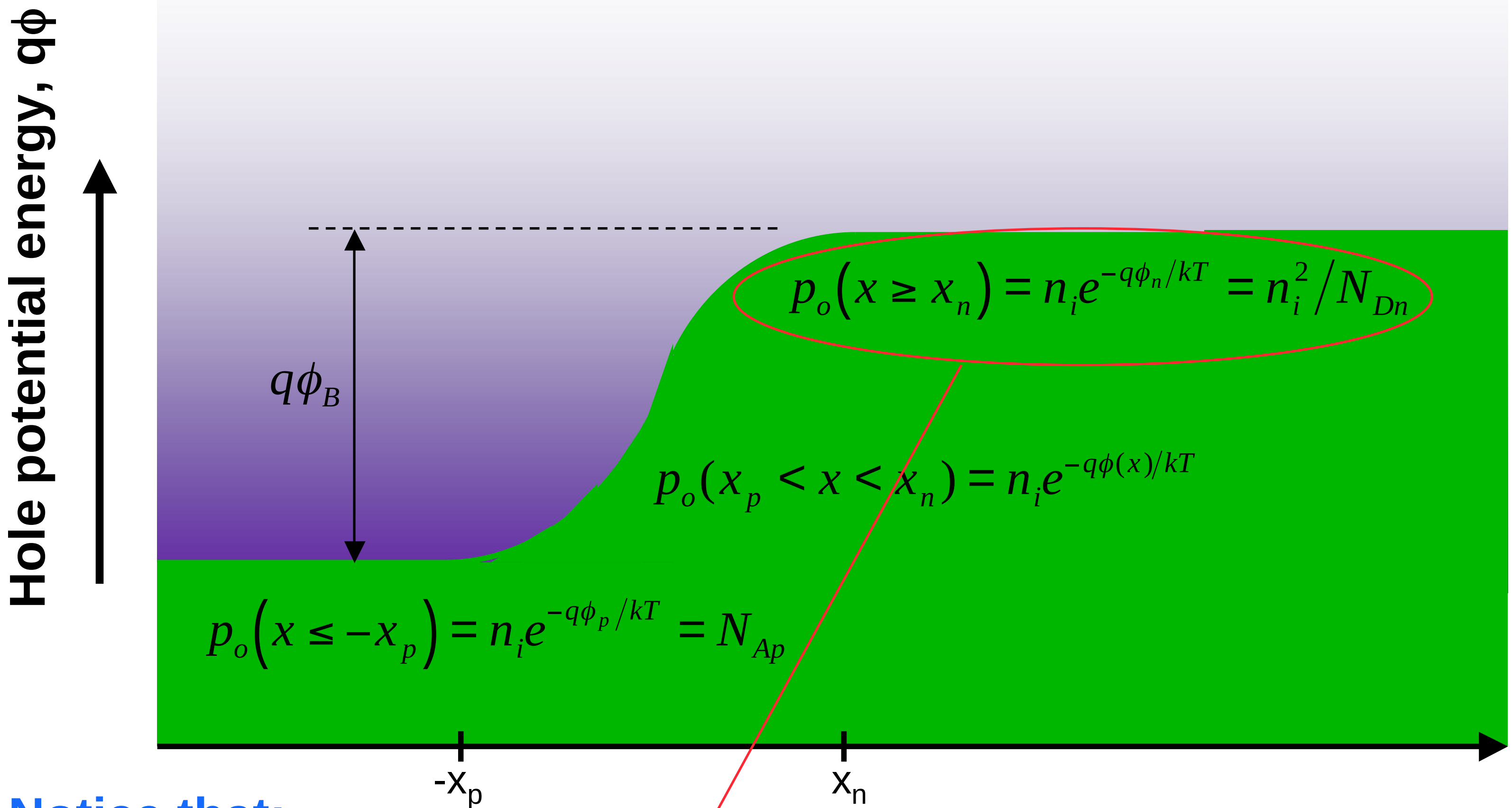
Reverse bias on junction

Barrier raised so the few carriers on top spill back down.



Majority carriers against the junction barrier

Zero applied bias, $v_{AB} = 0$; thermal equilibrium barrier



Notice that:

$$p_o(x \geq x_n) = n_i e^{-q\phi_n/kT} = N_{Ap} e^{-q(\phi_n - \phi_p)/kT} = N_{Ap} e^{-q\phi_B/kT}$$

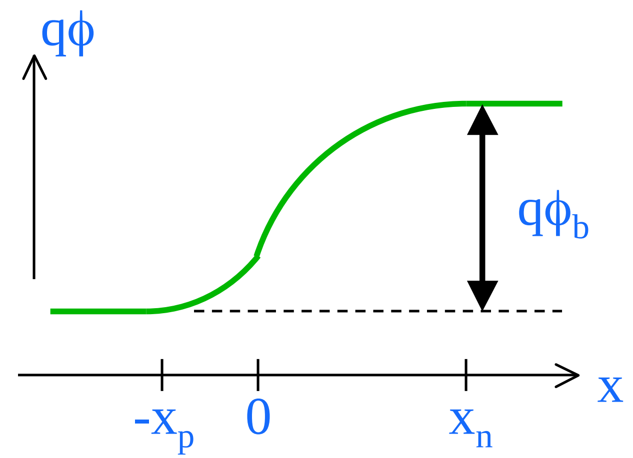
"The holes are in equilibrium with the barrier."

Boundary condtions at the edges of the space charge layer:

What are $n'(-x_p)$ and $p'(x_n)$?

Begin by looking at the situation in thermal equilibrium, where we have:

$$p_o(-x_p) = N_{Ap} \text{ and } p_o(x_n) = n_i^2 / N_{Dn}$$



If the population of holes at the top of the potential "hill" is related to the population at the bottom by a Boltzman factor, then we should also find that:

$$p_o(x_n) = p_o(-x_p) e^{-q\phi_b / kT}$$

Do we?

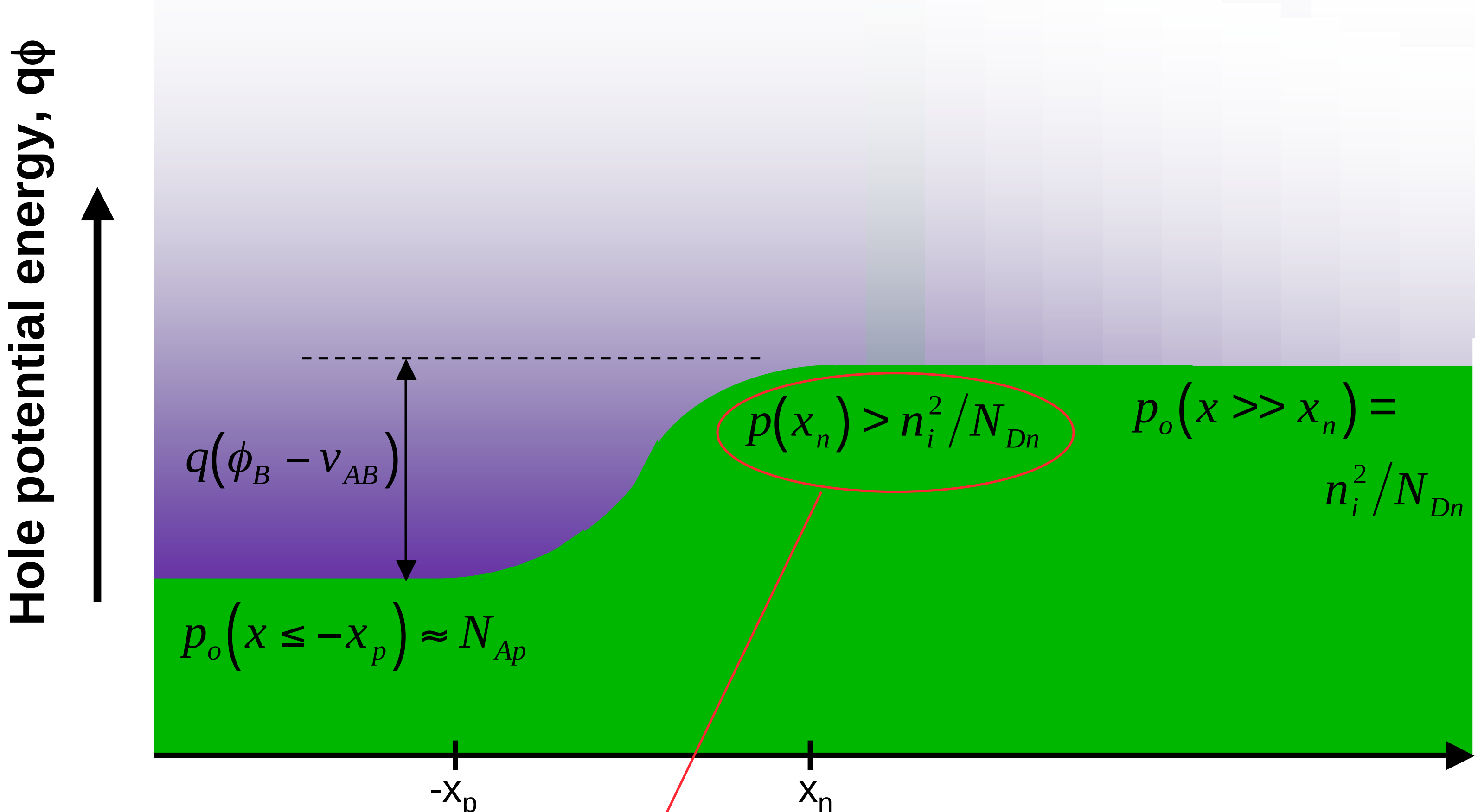
$$\phi_b = \frac{kT}{q} \ln \frac{N_{Ap} N_{Dn}}{n_i^2} \quad \Rightarrow \quad \frac{n_i^2}{N_{Dn}} = N_{Ap} e^{-q\phi_b / kT}$$

$$\text{Thus:} \quad p_o(x_n) = \frac{n_i^2}{N_{Dn}} = N_{Ap} e^{-q\phi_b / kT} = p_o(-x_p) e^{-q\phi_b / kT}$$

YES, we do, and the Boltzman relationship holds.

Majority carriers against the junction barrier

Forward bias, $v_{AB} > 0$; barrier lowered, carriers spill over

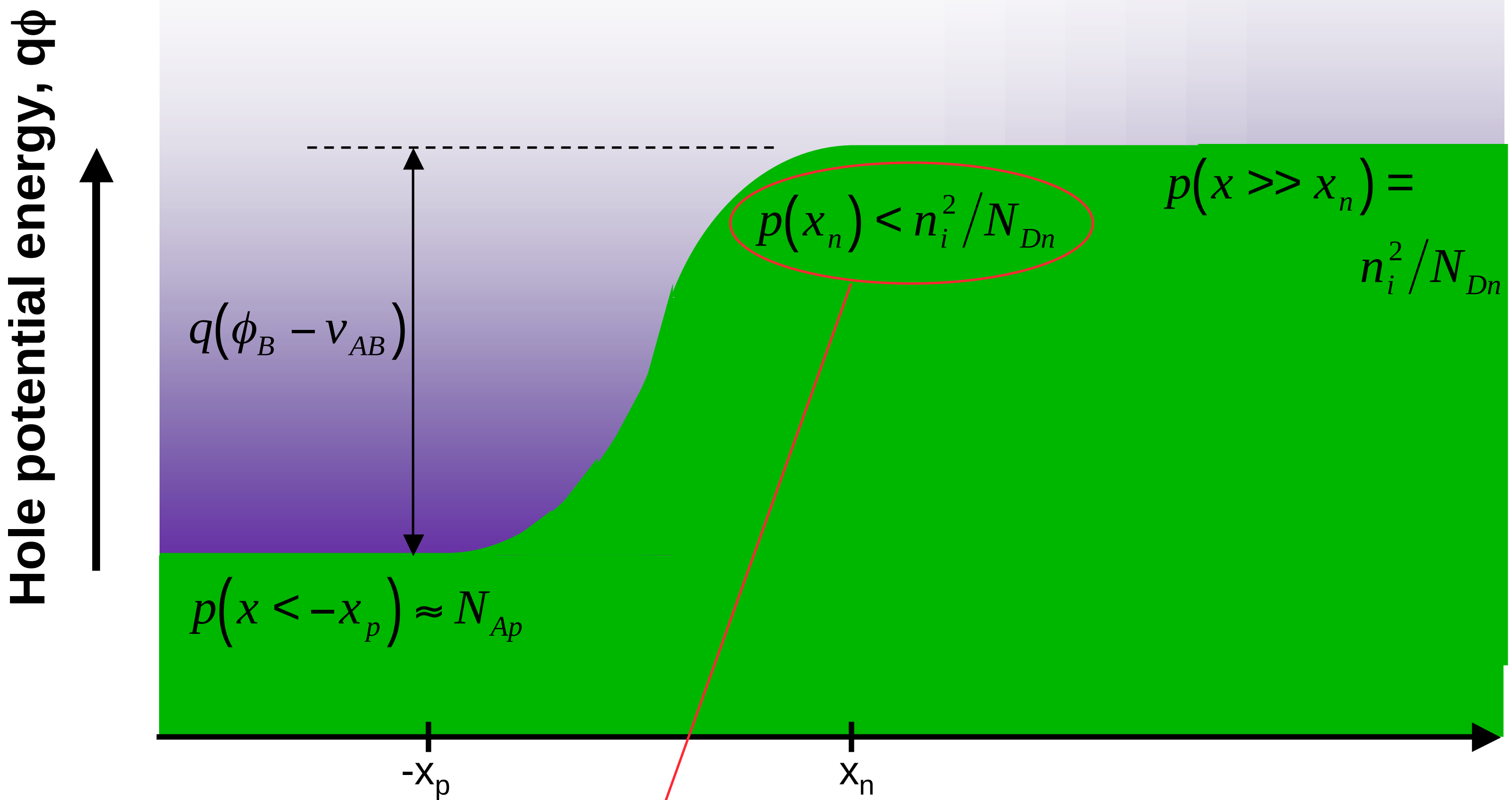


We say the holes are <u>still in equilibrium</u> with the barrier at x_n:

$$p(x_n) = N_{Ap} e^{-q(\phi_B - v_{AB})/kT} = \frac{n_i^2}{N_{Dn}} e^{+qv_{AB}/kT}$$

Majority carriers against the junction barrier

Reverse bias, v_{AB} < 0; barrier raised, carriers spill back



Again the holes maintain equilibrium with the barrier until x_n:

$$p(x_n) = N_{Ap} e^{-q(\phi_B - v_{AB})/kT} = \frac{n_i^2}{N_{Dn}} e^{+qv_{AB}/kT}$$

And we have the same expression for $p(x_n)$.

What are n'(-x_p) and p'(x_n) with v_{AB} applied?

We propose that the majority carrier populations on either side are still related by the Boltzman factor,* which is now: exp[-q(ϕ_b-v_{AB})/kT]

Thus:

$$p(x_n) = p(-x_p)e^{-q[\phi_b - v_{AB}]/kT}$$

Under low level injection conditions, the majority carrier population is unchanged, so p(-x_p) remains N_{Ap}, so:

$$p(x_n) = N_{Ap}\, e^{-q[\phi_b - v_{AB}]/kT} = \frac{n_i^2}{N_{Dn}}\, e^{qv_{AB}/kT}$$

And the excess population we seek is:

$$p'(x_n) = p(x_n) - p_{on} = \frac{n_i^2}{N_{Dn}}\left(e^{qv_{AB}/kT} - 1\right)$$

Similarly at -x_p:

$$n'(-x_p) = \frac{n_i^2}{N_{Ap}}\left(e^{qv_{AB}/kT} - 1\right)$$

* We are assuming that the majority carriers can get across the SCL much faster than they can diffuse away as minority carriers, i.e., that diffusion is the bottleneck!

Biased p-n junctions: current flow, cont.

What is the current, i_D?

Knowing p'(x_n) and n'($-x_p$), we know:

$$\begin{array}{ll} & J_h(x) \quad \text{for} \quad x_n < x < w_n \\ \text{and} & \\ & J_e(x) \quad \text{for} \quad -w_p < x < -x_p \end{array}$$

But we still don't know the total current because we don't know both currents at the same position, x:

$$i_D = A\,J_{TOT} = A\left[J_h(x) + J_e(x)\right]$$

Have to be at same "x"

To proceed we make the assumption that there is negligible recombination of holes and electrons in the depletion region, so what goes in comes out and:

$$J_h(x_n) = J_h(-x_p) \qquad \text{and} \qquad J_e(x_n) = J_e(-x_p)$$

With this assumption, we can write:

$$i_D = A\,J_{TOT} = A\left[J_h(x_n) + J_e(-x_p)\right]$$

Values at edges of SCL

Biased p-n junctions: current flow, cont.

What is the current, i_D, cont,?

Both $J_h(x_n)$ and $J_e(-x_p)$, are proportional to $p'(x_n)$ and $n'(-x_p)$, respectively, which in turn are both proportional to $(e^{qv/kT}-1)$:

$$J_h(x_n) \propto p'(x_n) \propto \left[e^{qv_{AB}/kT} - 1\right] \quad \text{and} \quad J_e(-x_p) \propto n'(x_p) \propto \left[e^{qv_{AB}/kT} - 1\right]$$

Thus the diode current is also proportional to $(e^{qv/kT}-1)$:

$$i_D = A\left[J_h(x_n) + J_e(-x_p)\right] \propto \left[e^{qv_{AB}/kT} - 1\right] \Rightarrow \boxed{i_D = I_s\left[e^{qv_{AB}/kT} - 1\right]}$$

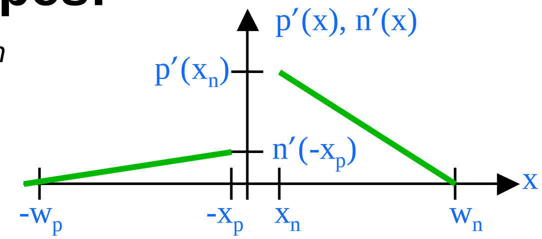
(I_S is called the reverse saturation current of the diode.)

**** Notice:** The non-linearity, i.e., the exponential dependence of the diode current on voltage, arises because of the exponential dependence of the minority carrier populations the edges of the space charge layer (depletion region). The flow problems themselves are linear.

<u>Biased p-n junctions</u>: current flow, cont.

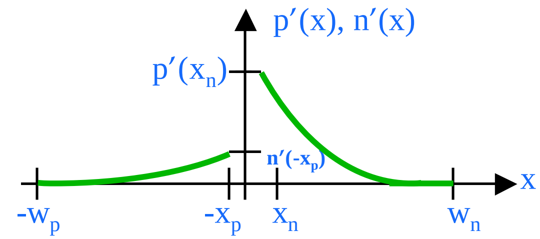
The saturation current of three diode types:

I_S's dependence on the relative sizes of w and L_{min}



<u>Short-base diode</u>, $w_n << L_h$, $w_p << L_e$:

$$\left.\begin{aligned} J_h(x_n) &= q\frac{n_i^2}{N_{Dn}}\frac{D_h}{(w_n - x_n)}\left[e^{qv_{AB}/kT} - 1\right] \\ J_e(-x_p) &= q\frac{n_i^2}{N_{Ap}}\frac{D_e}{(w_p - x_p)}\left[e^{qv_{AB}/kT} - 1\right] \end{aligned}\right\}\quad i_D = Aqn_i^2\left[\frac{D_h}{N_{Dn}(w_n - x_n)} + \frac{D_e}{N_{Ap}(w_p - x_p)}\right]\left[e^{qv_{AB}/kT} - 1\right]$$



<u>Long-base diode</u>, $w_n >> L_h$, $w_p >> L_e$:

$$\left.\begin{aligned} J_h(x_n) &= q\frac{n_i^2}{N_{Dn}}\frac{D_h}{L_h}\left[e^{qv_{AB}/kT} - 1\right] \\ J_e(-x_p) &= q\frac{n_i^2}{N_{Ap}}\frac{D_e}{L_e}\left[e^{qv_{AB}/kT} - 1\right] \end{aligned}\right\}\quad i_D = Aqn_i^2\left[\frac{D_h}{N_{Dn}L_h} + \frac{D_e}{N_{Ap}L_e}\right]\left[e^{qv_{AB}/kT} - 1\right]$$

<u>General diode</u>:

$$i_D = Aqn_i^2\left[\underbrace{\frac{D_h}{N_{Dn}w_{n,eff}}}_{\text{Hole injection into n-side}} + \underbrace{\frac{D_e}{N_{Ap}w_{p,eff}}}_{\text{Electron injection into p-side}}\right]\left[e^{qv_{AB}/kT} - 1\right]$$

$$\text{Note:}\quad w_{n,eff} \equiv L_h\tanh(w_n - x_n),\quad w_{p,eff} \equiv L_e\tanh(w_p - x_p)$$

Biased p-n junctions: current flow, cont.

The ideal exponential diode

- **General expression:**

$$i_D = I_S(e^{qv_{AB}/kT} - 1)$$

- **Forward bias, $|v_{AB}| > kT/q$:**

$$i_D \approx I_S e^{qv_{AB}/kT}$$

Current increases 10x for every 60 mV increase in v_{AB}.

- **Reverse bias, $|v_{AB}| > kT/q$:**

Current saturates at I_S.

$$i_D = -I_S$$

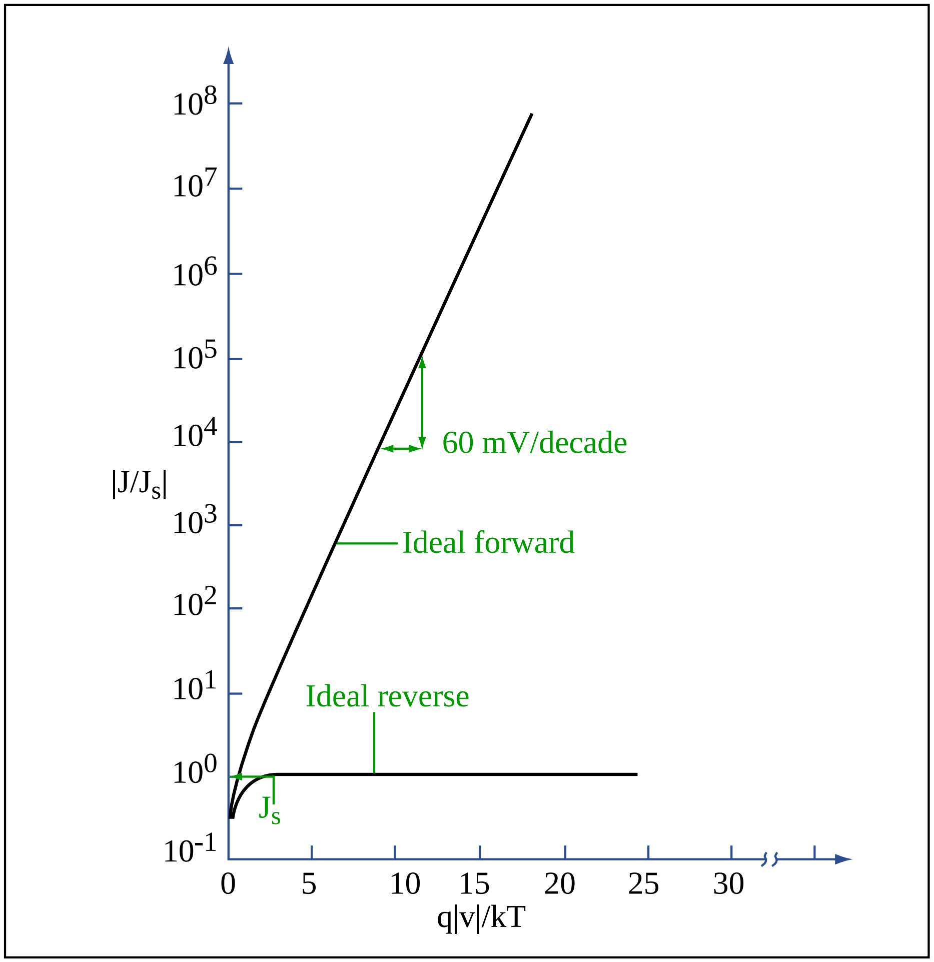


Figure by MIT OpenCourseWare.

Ref: Adapted from Figure 18 in S. M. Sze, "Physics of Semiconductor Devices" 1st. Ed (Wiley, 1969)

Biased p-n junctions: current flow, cont.

Limitations of the model

NOTE: This figure is a bit exagerated, but it makes the point.

- **Large forward bias:** Sub-exponential increase
  - High level injection **(c)**
  - Series voltage drop **(d)**
- **Large reverse bias:** Abrupt, rapid increase
  - Non-destructive break-down
- **Very low bias levels:** Excess current seen
  - SCL generation and recombination **(a, e)**

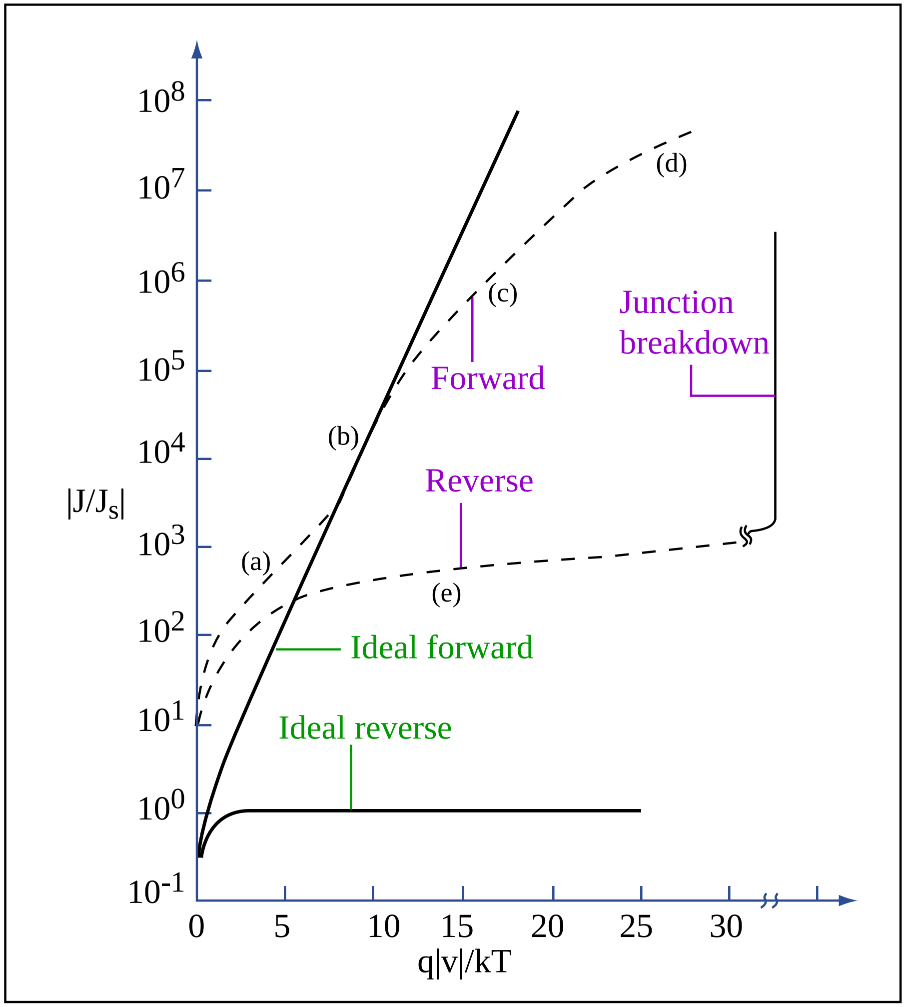


Figure by MIT OpenCourseWare.

Ref: Figure 18 in S. M. Sze, "Physics of Semi-conductor Devices" 1st. Ed (Wiley, 1969)

Asymmetrically doped junctions: *an important special case*

Depletion region impacts/issues

A p+-n junction ($N_{Ap} >> N_{Dn}$): $N_{Ap}N_{Dn}/\left(N_{Ap}+N_{Dn}\right) \approx N_{Dn}$

$$x_n >> x_p, \quad w \approx x_n \approx \sqrt{\frac{2\varepsilon_{Si}\left(\phi_b - v_{AB}\right)}{qN_{Dn}}}, \quad \left|E_{pk}\right| \approx \sqrt{\frac{2q\left(\phi_b - v_{AB}\right)N_{Dn}}{\varepsilon_{Si}}}$$

An n+-p junction ($N_{Dn} >> N_{Ap}$): $N_{Ap}N_{Dn}/\left(N_{Ap}+N_{Dn}\right) \approx N_{Ap}$

$$x_p >> x_n, \quad w \approx x_p \approx \sqrt{\frac{2\varepsilon_{Si}\left(\phi_b - v_{AB}\right)}{qN_{Ap}}}, \quad \left|E_{pk}\right| \approx \sqrt{\frac{2q\left(\phi_b - v_{AB}\right)N_{Ap}}{\varepsilon_{Si}}}$$

Note that in both cases the depletion region is predominately on the lightly doped side, and it is the doping level of the more lightly doped junction that matters (i.e., dominates).

Note also that as the doping level increases the depletion width decreases and the peak E-field increases. [This is also true in symmetrical diodes.]

Two very important and useful observations!!

Asymmetrically doped junctions: *an important special case*

Current flow impact/issues

A p+-n junction ($N_{Ap} >> N_{Dn}$):

$$i_D = Aqn_i^2\left[\frac{D_h}{N_{Dn}w_{n,eff}} + \cancel{\frac{D_e}{N_{Ap}w_{p,eff}}}\right]\left[e^{qv_{AB}/kT} - 1\right] \approx \underbrace{Aqn_i^2\frac{D_h}{N_{Dn}w_{n,eff}}}_{\text{Hole injection into n-side}}\left[e^{qv_{AB}/kT} - 1\right]$$

An n+-p junction ($N_{Dn} >> N_{Ap}$):

$$i_D = Aqn_i^2\left[\cancel{\frac{D_h}{N_{Dn}w_{n,eff}}} + \frac{D_e}{N_{Ap}w_{p,eff}}\right]\left[e^{qv_{AB}/kT} - 1\right] \approx \underbrace{Aqn_i^2\frac{D_e}{N_{Ap}w_{p,eff}}}_{\text{Electron injection into p-side}}\left[e^{qv_{AB}/kT} - 1\right]$$

Note that in both cases the minority carrier injection is predominately into the lightly doped side.

Note also that it is the doping level of the more lightly doped junction that determines the magnitude of the current, and as the doping level on the lightly doped side decreases, the magnitude of the current increases.

Two very important and useful observations!!

Lecture 6 - p-n Junctions: I-V Relationship - Summary

- **I-V relationship for an abrupt p-n junction**

  Focus is on minority carrier diffusion on either side of SCL

  Voltage across SCL sets excess populations $-x_p$ and x_n:

  $$n'(-x_p) = n_{no}e^{-q[\phi_b - v_{AB}]/kT} - n_{po} = n_{po}(e^{qv_{AB}/kT}-1) = (n_i^2/N_{Ap})(e^{qv_{AB}/kT}-1)$$

  $$p'(x_n) = p_{po}e^{-q[\phi_b - v_{AB}]/kT} - p_{no} = p_{no}(e^{qv_{AB}/kT}-1) = (n_i^2/N_{Dn})(e^{qv_{AB}/kT}-1)$$

  Flow problems in QNR regions give minority currents:

  $$J_e(-w_p<x<-x_p) = q(D_e/L_e)[\cosh(w_p-x)/\sinh(w_p-x_p)](n_i^2/N_{Ap})(e^{qv_{AB}/kT}-1)$$

  $$J_h(x_n<x<w_n) = q(D_h/L_h)[\cosh(w_n-x)/\sinh(w_n-x_n)](n_i^2/N_{Dn})(e^{qv_{AB}/kT}-1)$$

  Total current is found from continuity across SCL:

  $$i_D(v_{AB}) = A\,[J_e(-x_p) + J_h(x_n)] = I_S\,(e^{qv_{AB}/kT}-1), \text{ with}$$

  $$I_S \equiv A\,q\,n_i^2\,[\underbrace{(D_h/N_{Dn}\,w_n^*)}_{\text{(hole component)}} + \underbrace{(D_e/N_{Ap}\,w_p^*)}_{\text{(electron component)}}]$$

  Note: w_p^* and w_n^* are the effective widths of the p- and n-sides

  If $L_e >> w_p$, then $w_p^* \approx (w_p - x_p)$, and if $L_e << w_p$, then $w_p^* \approx L_e$

  If $L_h >> w_n$, then $w_n^* \approx (w_n - x_n)$, and if $L_h << w_n$, then $w_n^* \approx L_h$

- **Features and limitations of the model**

  Exponential dependence enters via boundary conditions

  Injection is predominantly into more lightly doped side

  Saturation current, I_S, goes down as doping levels go up

  Limits: 1. SCL g-r may dominate at low current levels

  2. Series resistance may reduce junction voltage at high currents

  3. Junction may breakdown (conduct) at large reverse bias

6.012 Microelectronic Devices and Circuits
Fall 2009