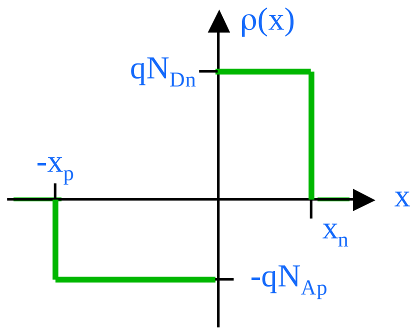
Lecture 5 - p-n Junction Injection and Flow - Outline

- **Review**
  - **Depletion approximation for an abrupt p-n junction**
  - **Depletion charge storage and depletion capacitance** **(Rec. Fri.)**
    - $q_{DP}(v_{AB}) = – AqN_{Ap}x_p = – A[2\varepsilon q(\phi_b\text{-}v_{AB})\{N_{Ap}N_{Dn}/(N_{Ap}+N_{Dn})\}]^{1/2}$
    - $C_{dp}(V_{AB}) \equiv \partial q_{DP}/\partial v_{AB}|_{V_{AB}} = A[\varepsilon q\{N_{Ap}N_{Dn}/(N_{Ap}+N_{Dn})\}/2(\phi_b\text{-}V_{AB})]^{1/2}$
- **Biased p-n Diodes**
  - **Depletion regions change** **(Lecture 4)**
  - **Currents flow: two components**
    - – flow issues in quasi-neutral regions **(Today)**
    - – boundary conditions on p' and n' at $-x_p$ and x_n **(Lecture 6)**
- **Minority carrier flow in quasi-neutral regions**
  - **The importance of minority carrier diffusion**
  - **Boundary conditions**
  - **Minority carrier profiles and currents in QNRs**
    - – Short base situations
    - – Long base situations
    - – Intermediate situations

The Depletion Approximation: *an informed first estimate of $\rho(x)$*

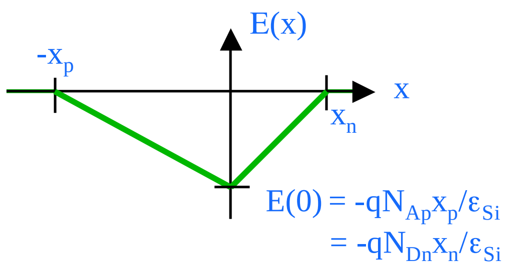
Assume full depletion for $-x_p < x < x_n$, where x_p and x_n are two unknowns yet to be determined. This leads to:

$$\rho(x) = \begin{cases} 0 & \text{for} \quad x < -x_p \\ -qN_{Ap} & \text{for} \quad -x_p < x < 0 \\ qN_{Dn} & \text{for} \quad 0 < x < x_n \\ 0 & \text{for} \quad x_n < x \end{cases}$$



Integrating the charge once gives the electric field

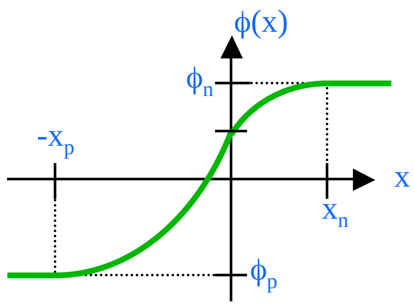
$$E(x) = \begin{cases} 0 & \text{for} \quad x < -x_p \\ -\dfrac{qN_{Ap}}{\varepsilon_{Si}}\left(x + x_p\right) & \text{for} \quad -x_p < x < 0 \\ \dfrac{qN_{Dn}}{\varepsilon_{Si}}\left(x - x_n\right) & \text{for} \quad 0 < x < x_n \\ 0 & \text{for} \quad x_n < x \end{cases}$$



The Depletion Approximation, cont.:

Integrating again gives the electrostatic potential:

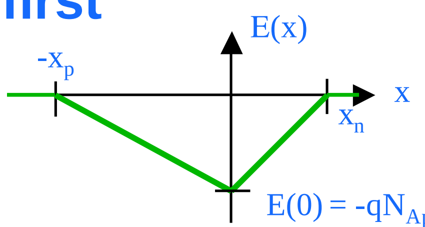
$$\phi(x) = \begin{cases} \phi_p & \text{for} \quad x < -x_p \\ \phi_p + \dfrac{qN_{Ap}}{2\varepsilon_{Si}}\left(x + x_p\right)^2 & \text{for} \quad -x_p < x < 0 \\ \phi_n - \dfrac{qN_{Dn}}{2\varepsilon_{Si}}\left(x - x_n\right)^2 & \text{for} \quad 0 < x < x_n \\ \phi_n & \text{for} \quad x_n < x \end{cases}$$



$\phi(0) = \phi_p + qN_{Ap}x_p^2/2\varepsilon_{Si}$
$= \phi_n - qN_{Dn}x_n^2/2\varepsilon_{Si}$

Insisting E(x) is continuous at x = 0 yields our first equation relating our unknowns, x_n and x_p:

$$\boxed{N_{Ap}x_p = N_{Dn}x_n} \qquad \mathbf{1}$$



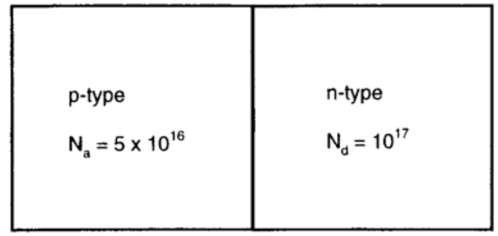
$E(0) = -qN_{Ap}x_p/\varepsilon_{Si}$
$= -qN_{Dn}x_n/\varepsilon_{Si}$

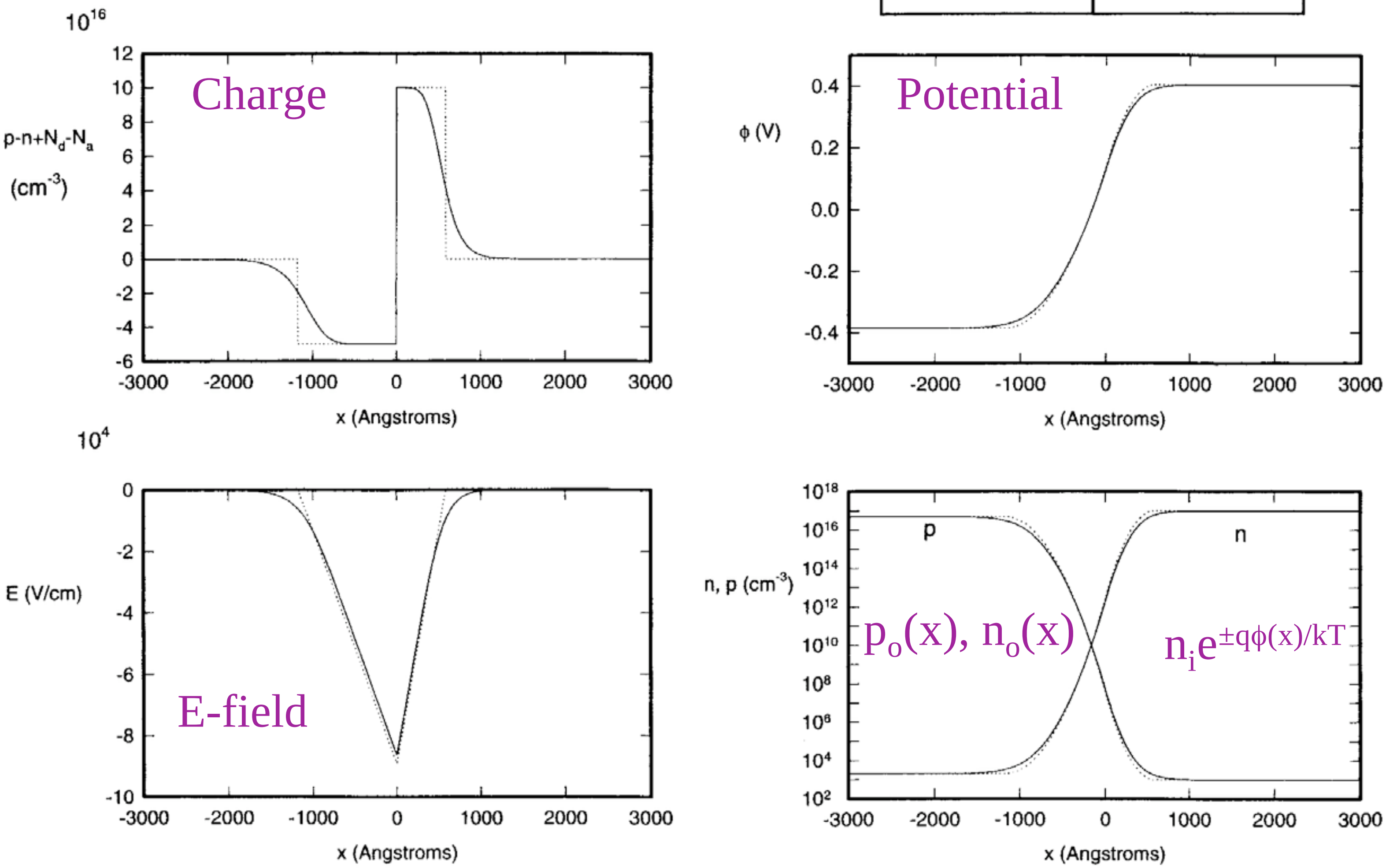
Requiring that the potential be continuous at x = 0 gives us our second relationship between x_n and x_p:

$$\boxed{\phi_p + \frac{qN_{Ap}}{2\varepsilon_{Si}}x_p^2 = \phi_n - \frac{qN_{Dn}}{2\varepsilon_{Si}}x_n^2} \qquad \mathbf{2}$$

Comparing the depletion approximation with a full solution:

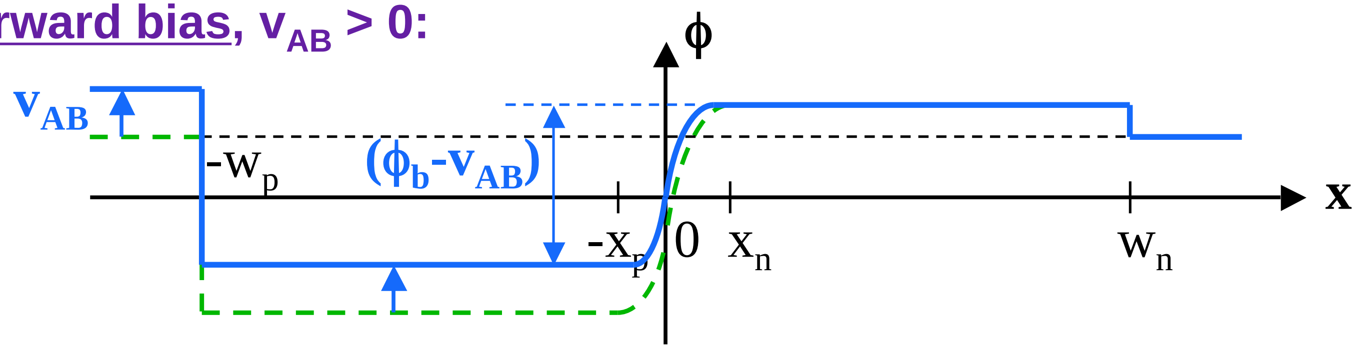
Example: An unbiased abrupt p-n junction with $N_{Ap} = 10^{17}\ cm^{-3}$, $N_{Dn} = 5 \times 10^{16}\ cm^{-3}$



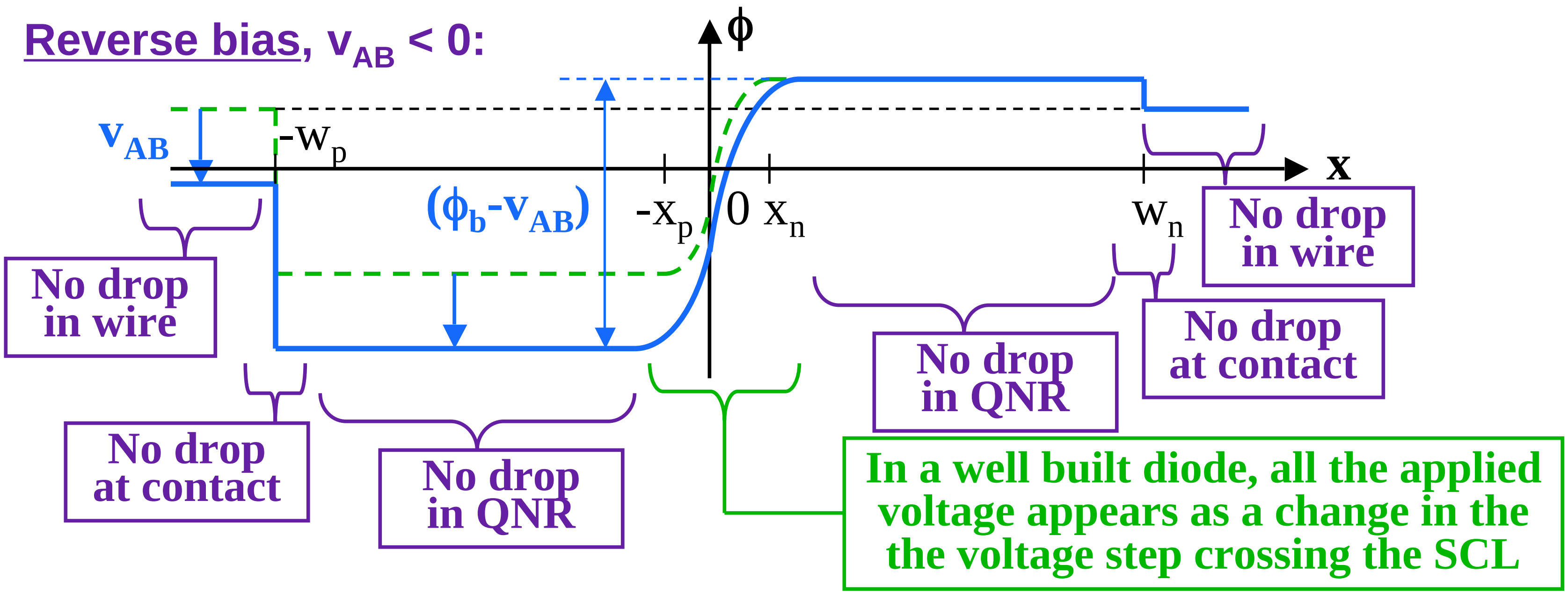


<u>Depletion approximation</u>: Applied bias

<u>Forward bias</u>, $v_{AB} > 0$:



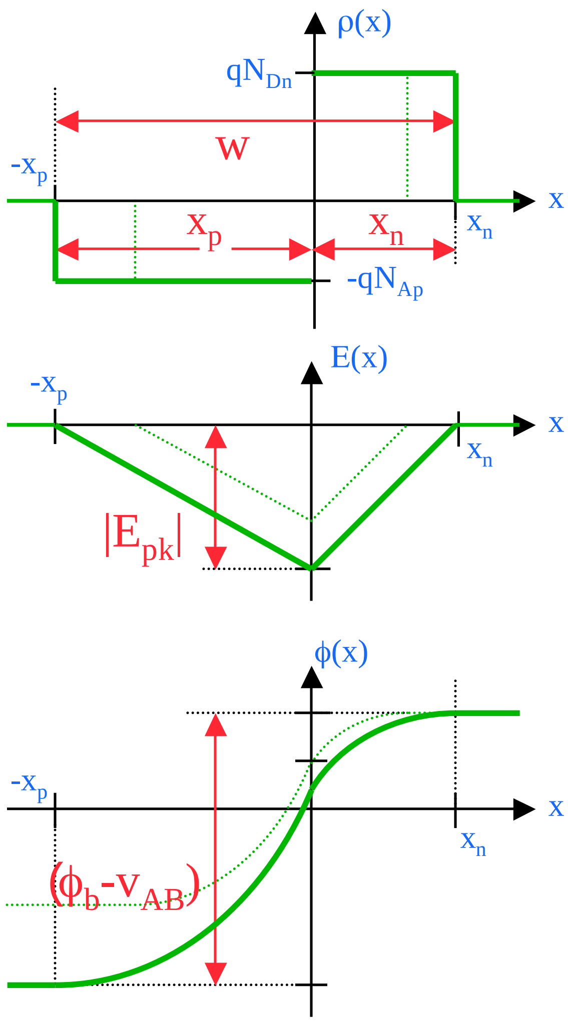
<u>Reverse bias</u>, $v_{AB} < 0$:



<u>Note</u>: With applied bias we are no longer in thermal equilibrium so it is no longer true that $n(x) = n_i\, e^{q\phi(x)/kT}$ and $p(x) = n_i\, e^{-q\phi(x)/kT}$.

<u>The Depletion Approximation</u>: Applied bias, cont.

Adding v_{AB} to our earlier sketches: *assume reverse bias, $v_{AB} < 0$*



$$w = \sqrt{\frac{2\varepsilon_{Si}\left(\phi_b - v_{AB}\right)}{q}\frac{\left(N_{Ap} + N_{Dn}\right)}{N_{Ap}N_{Dn}}}$$

$$x_p = \frac{N_{Dn}w}{\left(N_{Ap} + N_{Dn}\right)}, \quad x_n = \frac{N_{Ap}w}{\left(N_{Ap} + N_{Dn}\right)}$$

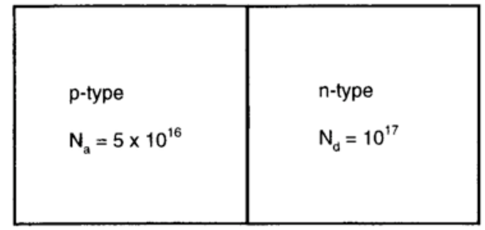
$$\left|E_{pk}\right| = \sqrt{\frac{2q\left(\phi_b - v_{AB}\right)}{\varepsilon_{Si}}\frac{N_{Ap}N_{Dn}}{\left(N_{Ap} + N_{Dn}\right)}}$$

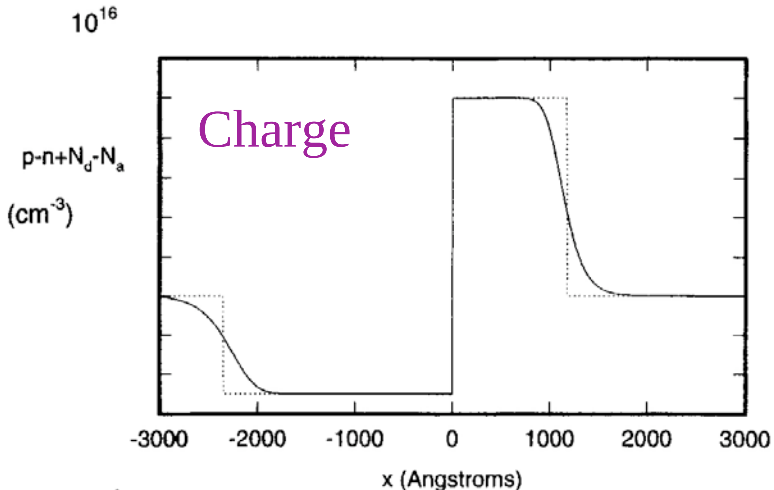
$$\Delta\phi = \phi_b - v_{AB}$$

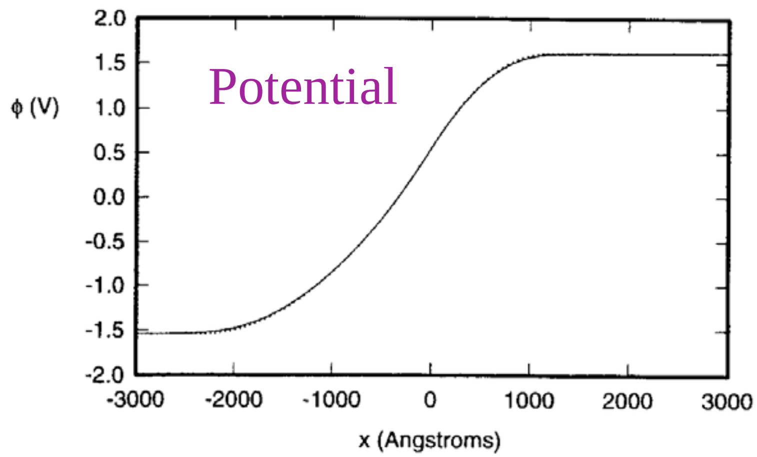
and $$\phi_b = \frac{kT}{q}\ln\frac{N_{Dn}N_{Ap}}{n_i^2}$$

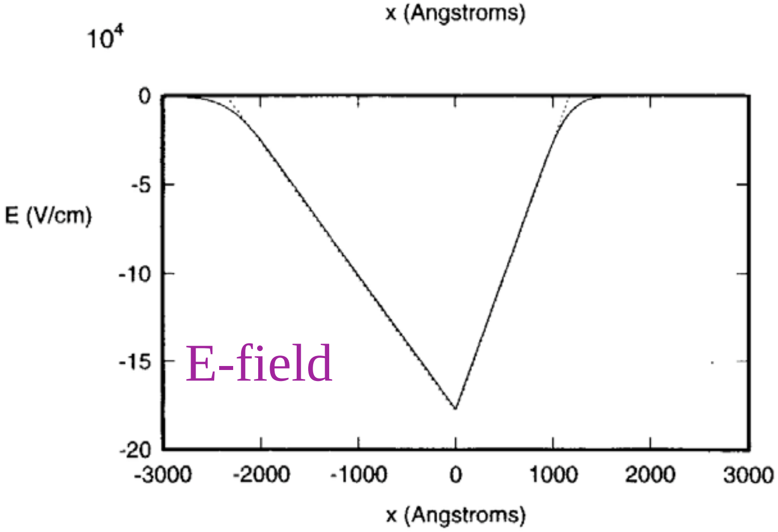
The Depletion Approximation: comparison cont.

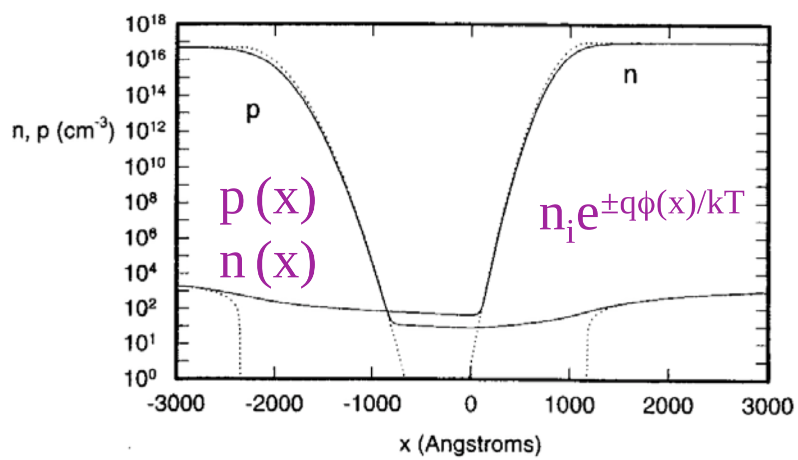
Example: Same sample, reverse biased -2.4 V





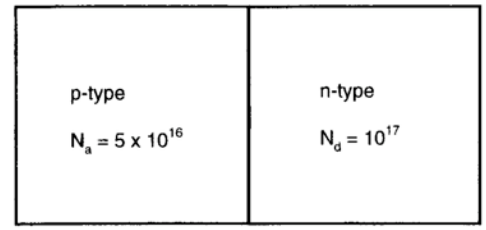


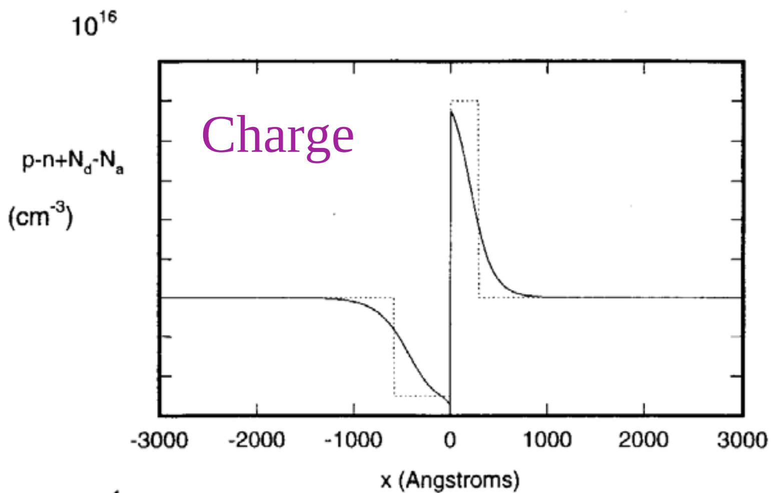


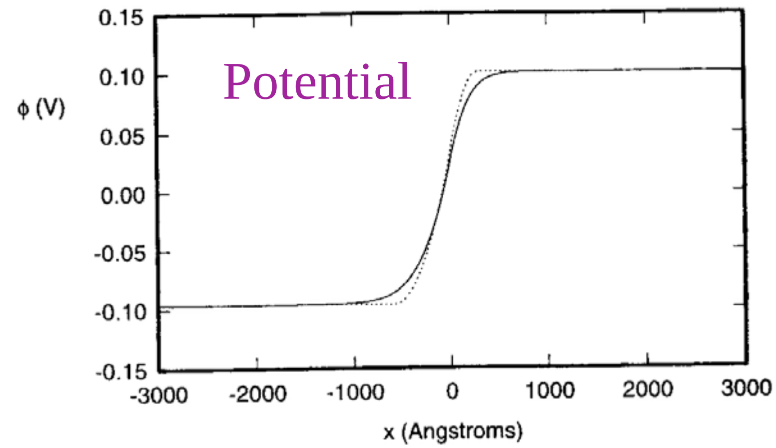


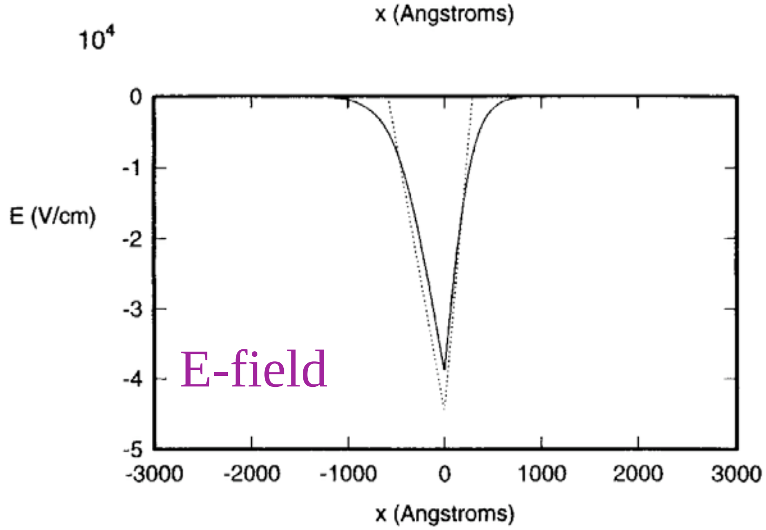
<u>The Depletion Approximation</u>: comparison cont.

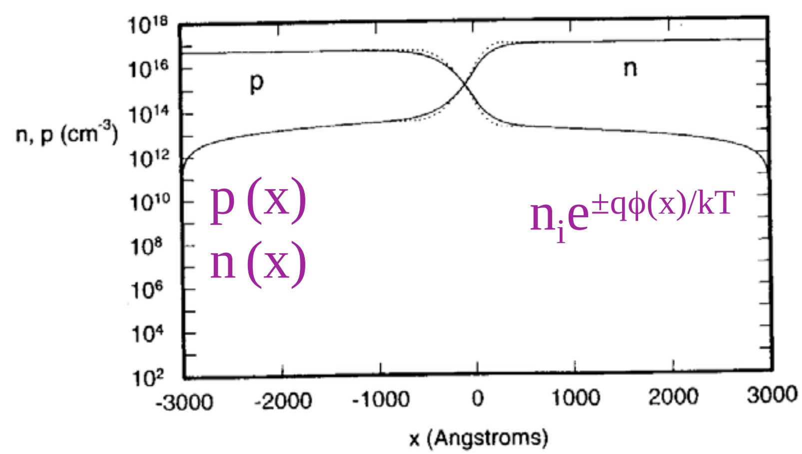
<u>Example</u>: Same sample, forward biased 0.6 V











The value of the depletion approximation

The plots look good, but equally important is that

1. **It gives an excellent model for making hand calculations <u>and</u> gives us good values for quantities we care about:**
   - **Depletion region width**
   - **Peak electric field**
   - **Potential step**
2. **It gives us the proper dependences of these quantities on the doping levels (relative and absolute) and the bias voltage.**

Apply bias; what happens?

Two things happen

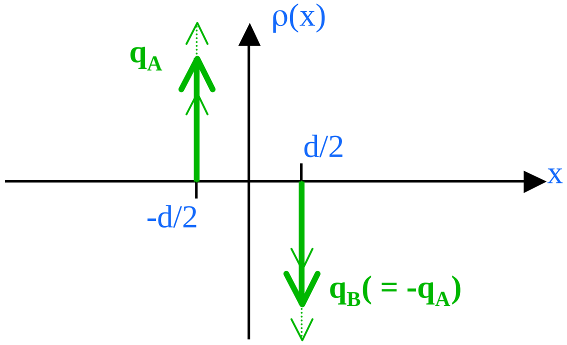
1. The depletion width changes

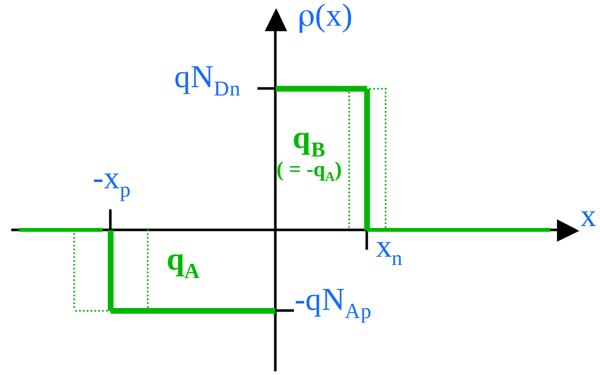
- **(ϕ_b - v_{AB}) replaces ϕ_b in the Depletion Approximation Eqs.**

2. Currents flow

- **This is the main topic of today's lecture**

Depletion capacitance: Comparing depletion charge stores with a parallel plate capacitor





Parallel plate capacitor

$$q_{A,PP} = A\frac{\varepsilon}{d}v_{AB}$$

$$C_{pp}(V_{AB}) \equiv \left.\frac{\partial q_{A,PP}}{\partial v_{AB}}\right|_{v_{AB}=V_{AB}}$$

$$= \frac{A\varepsilon}{d}$$

Depletion region charge store

$$q_{A,DP}(v_{AB}) = -AqN_{Ap}x_p(v_{AB})$$

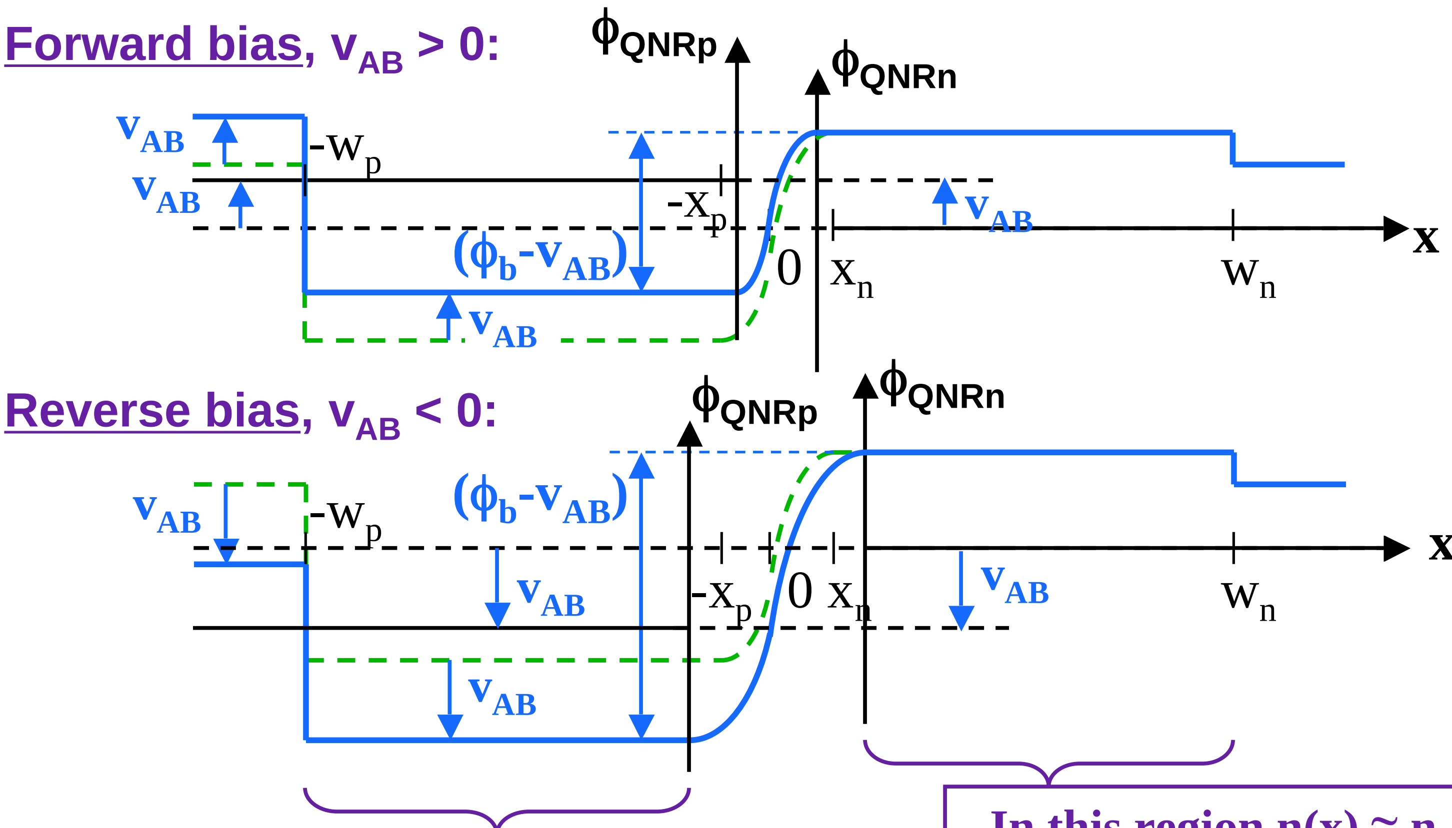
$$= -A\sqrt{2q\varepsilon_{Si}\left[\phi_b - v_{AB}\right]\frac{N_{Ap}N_{Dn}}{\left[N_{Ap}+N_{Dn}\right]}}$$

$$C_{dp}(V_{AB}) \equiv \left.\frac{\partial q_{A,DP}}{\partial v_{AB}}\right|_{v_{AB}=V_{AB}}$$

$$= A\sqrt{\frac{q\varepsilon_{Si}}{2\left[\phi_b - V_{AB}\right]}\frac{N_{Ap}N_{Dn}}{\left[N_{Ap}+N_{Dn}\right]}} = \frac{A\,\varepsilon_{Si}}{w(V_{AB})}$$

Many similarities; important differences.

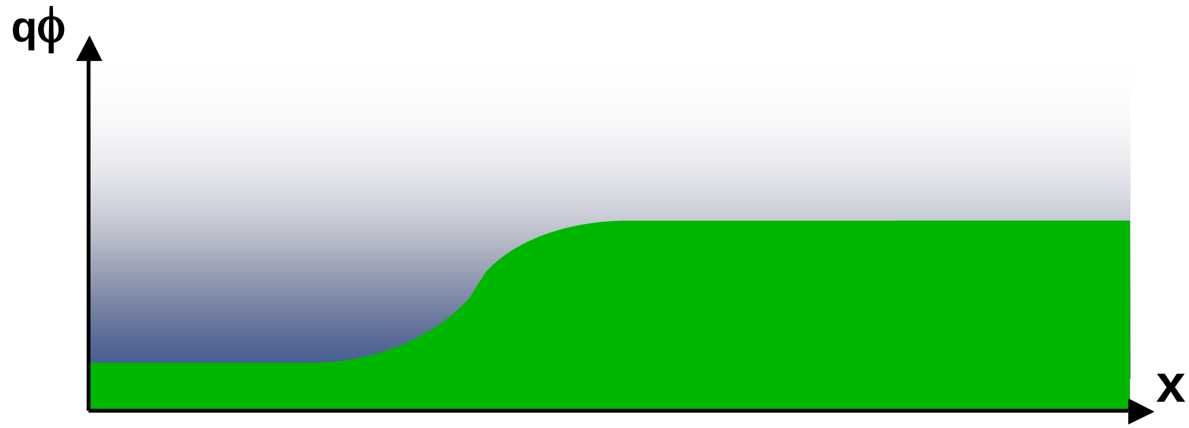
Bias applied, cont.: With $v_{AB} \neq 0$, it is not true that $n(x) = n_i\, e^{q\phi(x)/kT}$ and $p(x) = n_i\, e^{-q\phi(x)/kT}$ because we are no longer in TE. However, outside of the depletion region things are in quasi-equilibrium, and we can define local electrostatic potentials for which the equilibrium relationships hold for the majority carriers, assuming LLI.



In this region $p(x) \approx n_i\, e^{-q\phi_{QNRp}/kT}$

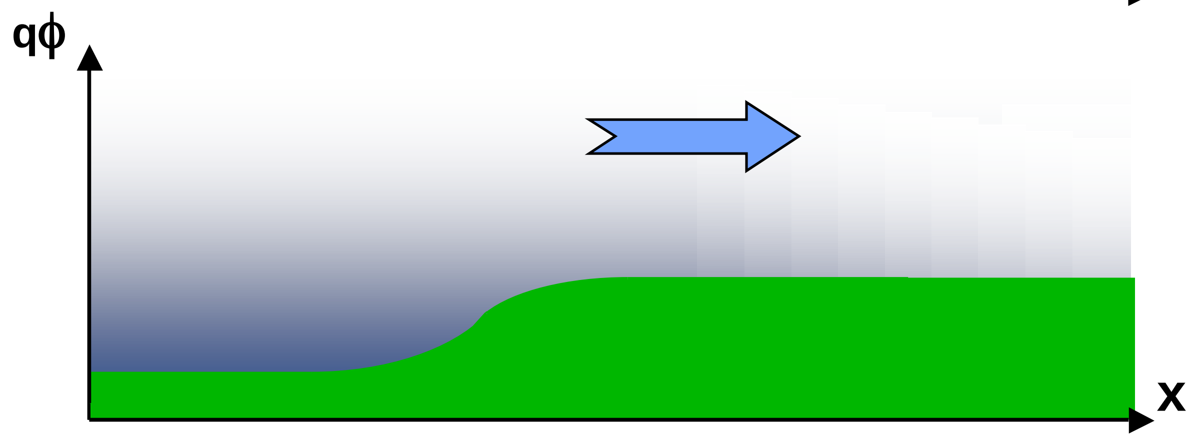
In this region $n(x) \approx n_i\, e^{q\phi_{QNRn}/kT}$

Current Flow



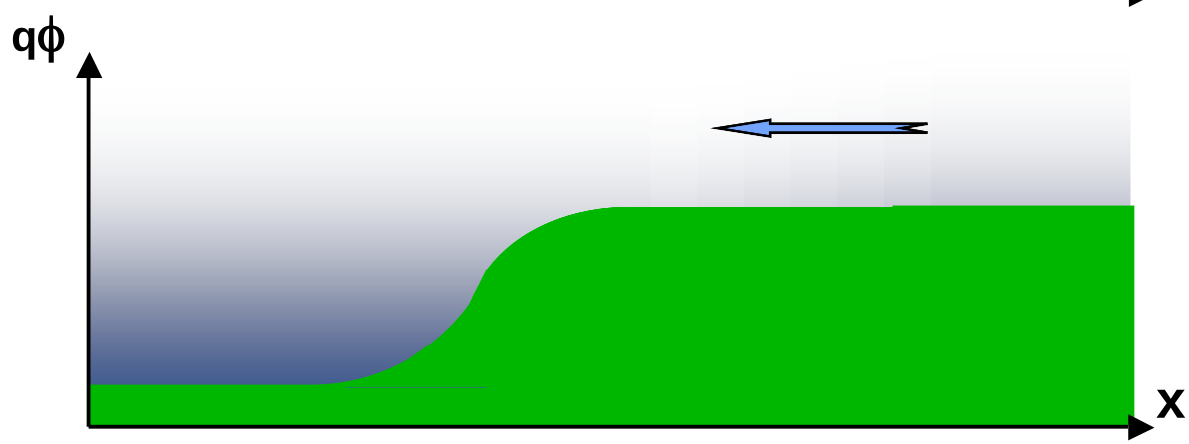
Unbiased junction

Population in equilibrium with barrier



Forward bias on junction

Barrier lowered so carriers to left can cross over it.



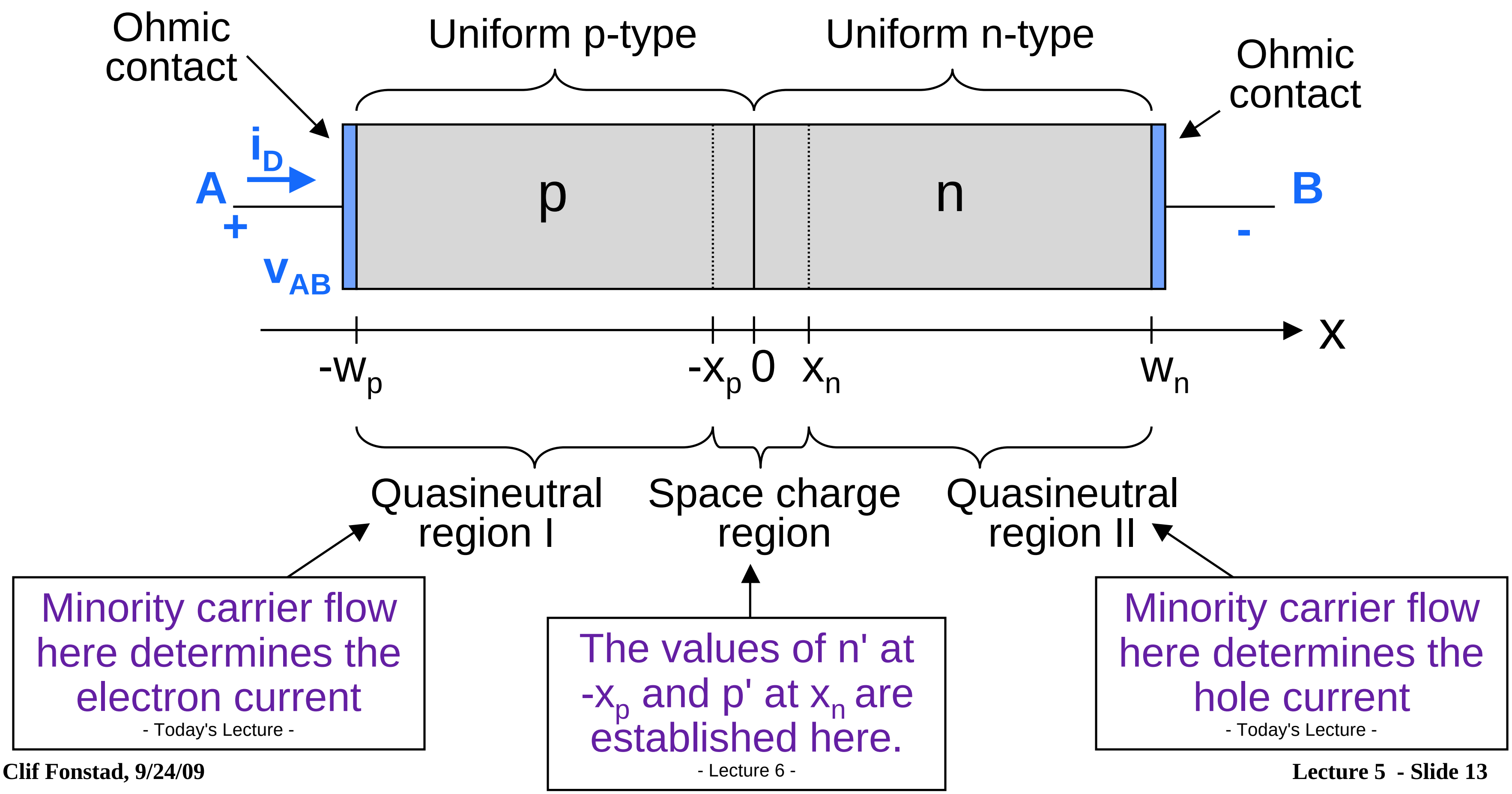
Reverse bias on junction

Barrier raised so the few carriers on top spill back down it.

Current flow: finding the relationship between i_D and v_{AB}

There are two pieces to the problem:

- Minority carrier flow in the QNRs is what limits the current.
- Carrier equilibrium across the SCR determines $n'(-x_p)$ and $p'(x_n)$, the boundary conditions of the QNR minority carrier flow problems.



Solving the five equations: special cases we can handle

1. Uniform doping, thermal equilibrium ($n_o p_o$ product, n_o, p_o):

$$\frac{\partial}{\partial x}=0,\quad \frac{\partial}{\partial t}=0,\quad g_L(x,t)=0,\quad J_e=J_h=0$$

Lecture 1

2. Uniform doping and E-field (drift conduction, Ohms law):

$$\frac{\partial}{\partial x}=0,\quad \frac{\partial}{\partial t}=0,\quad g_L(x,t)=0,\quad E_x \text{ constant}$$

Lecture 1

3. Uniform doping and uniform low level optical injection (τ_{min}):

$$\frac{\partial}{\partial x}=0,\quad g_L(t),\quad n' << p_o$$

Lecture 2

3'. Uniform doping, optical injection, <u>and</u> E-field (photoconductivity):

$$\frac{\partial}{\partial x}=0,\quad E_x \text{ constant},\quad g_L(t)$$

Lecture 2

4. Non-uniform doping in thermal equilibrium (junctions, interfaces)

$$\frac{\partial}{\partial t}=0,\quad g_L(x,t)=0,\quad J_e=J_h=0$$

Lectures 3,4

5. Uniform doping, non-uniform LL injection (QNR diffusion)

$$\frac{\partial N_d}{\partial x}=\frac{\partial N_a}{\partial x}=0,\quad n'\approx p',\quad n' << p_o,\quad J_e\approx qD_e\frac{\partial n'}{\partial x},\quad \frac{\partial}{\partial t}\approx 0$$

TODAY

Lecture 5

QNR Flow: Uniform doping, non-uniform LL injection

What we have:

Five things we care about (i.e. want to know):

Hole and electron concentrations: $p(x,t)$ and $n(x,t)$

Hole and electron currents: $J_{hx}(x,t)$ and $J_{ex}(x,t)$

Electric field: $E_x(x,t)$

And, five equations relating them:

Hole continuity: $$\frac{\partial p(x,t)}{\partial t} + \frac{1}{q}\frac{\partial J_h(x,t)}{\partial x} = G - R \approx G_{ext}(x,t) - \left[n(x,t)p(x,t) - n_i^2\right] r(t)$$

Electron continuity: $$\frac{\partial n(x,t)}{\partial t} - \frac{1}{q}\frac{\partial J_e(x,t)}{\partial x} = G - R \approx G_{ext}(x,t) - \left[n(x,t)p(x,t) - n_i^2\right] r(t)$$

Hole current density: $$J_h(x,t) = q\mu_h p(x,t)E(x,t) - qD_h \frac{\partial p(x,t)}{\partial x}$$

Electron current density: $$J_e(x,t) = q\mu_e n(x,t)E(x,t) + qD_e \frac{\partial n(x,t)}{\partial x}$$

Charge conservation: $$\rho(x,t) = \frac{\partial\left[\varepsilon(x)E_x(x,t)\right]}{\partial x} \approx q\left[p(x,t) - n(x,t) + N_d(x) - N_a(x)\right]$$

We can get approximate analytical solutions if 5 conditions are met!

QNR Flow, cont.: Uniform doping, non-uniform LL injection

Five unknowns, five equations, five flow problem conditions:

1. Uniform doping

$$\frac{dn_o}{dx} = \frac{dp_o}{dx} = 0 \quad \Rightarrow \quad \frac{\partial n}{\partial x} = \frac{\partial n'}{\partial x}, \quad \frac{\partial p}{\partial x} = \frac{\partial p'}{\partial x}$$

$$p_o - n_o + N_d - N_a = 0 \quad \Rightarrow \quad \rho = q\left(p - n + N_d - N_a\right) = q\left(p' - n'\right)$$

2. Low level injection
(in p-type, for example)

$$n' << p_o \quad \Rightarrow \quad \left(np - n_i^2\right) r \approx n' p_o r = \frac{n'}{\tau_e}$$

3. Quasineutrality holds

$$n' \approx p', \quad \frac{\partial n'}{\partial x} \approx \frac{\partial p'}{\partial x}$$

4. Minority carrier drift is negligible
(continuing to assume p-type)

$$J_e(x,t) \approx qD_e \frac{\partial n'(x,t)}{\partial x}$$

Note: It is also always true that

$$\frac{\partial n}{\partial t} = \frac{\partial n'}{\partial t}, \quad \frac{\partial p}{\partial t} = \frac{\partial p'}{\partial t}$$

QNR Flow, cont.: Uniform doping, non-uniform LL injection

With these first four conditions our five equations become:
(assuming for purposes of discussion that we have a p-type sample)

$$1,2: \quad \frac{\partial p'(x,t)}{\partial t} + \frac{1}{q}\frac{\partial J_h(x,t)}{\partial x} = \frac{\partial n'(x,t)}{\partial t} - \frac{1}{q}\frac{\partial J_e(x,t)}{\partial x} = g_L(x,t) - \frac{n'(x,t)}{\tau_e}$$

$$3: \quad J_e(x,t) \approx +qD_e \frac{\partial n'(x,t)}{\partial x}$$

$$4: \quad J_h(x,t) = q\mu_h p(x,t)E(x,t) + qD_h \frac{\partial p'(x,t)}{\partial x}$$

$$5: \quad \frac{\partial E(x,t)}{\partial x} = \frac{q}{\varepsilon}\left[p'(x,t) - n'(x,t)\right]$$

In preparation for continuing to our fifth condition, we note that combining Equations 1 and 3 yields one equation in n'(x,t):

$$\boxed{\frac{\partial n'(x,t)}{\partial t} - D_e \frac{\partial^2 n'(x,t)}{\partial x^2} = g_L(x,t) - \frac{n'(x,t)}{\tau_e}}$$

✓ **The time dependent diffusion equation**

QNR Flow, cont.: Uniform doping, non-uniform LL injection

The time dependent diffusion equation, which is repeated below, is in general still very difficult to solve

$$\frac{\partial n'(x,t)}{\partial t} - D_e \frac{\partial^2 n'(x,t)}{\partial x^2} = g_L(x,t) - \frac{n'(x,t)}{\tau_e}$$

but things get much easier if we impose a fifth constraint:

5. Quasi-static excitation $\quad g_L(x,t)$ such that all $\frac{\partial}{\partial t} \approx 0$

With this constraint the above equation becomes a second order linear differential equation:

$$-D_e \frac{d^2 n'(x)}{dx^2} = g_L(x) - \frac{n'(x)}{\tau_e}$$

which in turn becomes, after rearranging the terms :

$$\boxed{\frac{d^2 n'(x)}{dx^2} - \frac{n'(x)}{D_e \tau_e} = -\frac{1}{D_e} g_L(x)}$$

✓ The steady state diffusion equation

QNR Flow, cont.: Solving the steady state diffusion equation

The steady state diffusion equation in p-type material is:

$$\frac{d^2 n'(x)}{dx^2} - \frac{n'(x)}{L_e^2} = -\frac{1}{D_e} g_L(x)$$

and for n-type material it is:

$$\frac{d^2 p'(x)}{dx^2} - \frac{p'(x)}{L_h^2} = -\frac{1}{D_h} g_L(x)$$

In writing these expressions we have introduced L_e and L_h, the minority carrier diffusion lengths for holes and electrons, as:

$$L_e \equiv \sqrt{D_e \tau_e} \qquad L_h \equiv \sqrt{D_h \tau_h}$$

We'll see that the minority carrier diffusion length tells us how far the average minority carrier diffuses before it recombines.

In a basic p-n diode, we have $g_L = 0$ which means we only need the homogenous solutions, i.e. expressions that satisfy:

n-side: $$\frac{d^2 p'(x)}{dx^2} - \frac{p'(x)}{L_h^2} = 0$$

p-side: $$\frac{d^2 n'(x)}{dx^2} - \frac{n'(x)}{L_e^2} = 0$$

QNR Flow, cont.: Solving the steady state diffusion equation

For convenience, we focus on the n-side to start with and find p'(x) for $x_n \leq x \leq w_n$. p'(x) satisfies

$$\frac{d^2 p'(x)}{dx^2} = \frac{p'(x)}{L_h^2}$$

subject to the boundary conditions:

$$p'(w_n) = 0 \quad \text{and} \quad p'(x_n) = \text{something we'll find next time}$$

The general solution to this static diffusion equation is:

$$p'(x) = Ae^{-x/L_h} + Be^{+x/L_h}$$

where A and B are constants that satisfy the boundary conditions. Solving for them and putting them into this equation yields the final general result:

$$p'(x) = \frac{p'(x_n)e^{(w_n - x_n)/L_h}}{e^{(w_n - x_n)/L_h} - e^{-(w_n - x_n)/L_h}} e^{-(x - x_n)/L_h} - \frac{p'(x_n)e^{-(w_n - x_n)/L_h}}{e^{(w_n - x_n)/L_h} - e^{-(w_n - x_n)/L_h}} e^{+(x - x_n)/L_h}$$

$$\text{for} \quad x_n \leq x \leq w_n$$

QNR Flow, cont.: Solving the steady state diffusion equation

We seldom care about this general result. Instead, we find that most diodes fall into one of two cases:

Case I - Long-base diode: $w_n >> L_h$

Case II - Short-base diode: $L_h >> w_n$

Case I: When $w_n >> L_h$, which is the situation in an LED, for example, the solution is

$$p'(x) \approx p'(x_n) e^{-(x - x_n)/L_h} \quad \text{for} \quad x_n \leq x \leq w_n$$

This profile decays from $p'(x_n)$ to 0 exponentially as $e^{-x//L_h}$.

The corresponding hole current for $x_n \leq x \leq w_n$ in Case I is

$$J_h(x) \approx -qD_h \frac{dp'(x)}{dx} = \frac{qD_h}{L_h} p'(x_n) e^{-(x - x_n)/L_h} \quad \text{for} \quad x_n \leq x \leq w_n$$

The current decays to zero also, indicating that all of the excess minority carriers have recombined before getting to the contact.

QNR Flow, cont.: Solving the steady state diffusion equation

Case II: When $L_h >> w_n$, which is the situation in integrated Si diodes, for example, the differential equation simplifies to:

$$\frac{d^2 p'(x)}{dx^2} = \frac{p'(x)}{L_h^2} \approx 0$$

We see immediately that p'(x) is linear: $p'(x) = A x + B$

Fitting the boundary conditions we find:

$$p'(x) \approx p'(x_n)\left[1-\left(\frac{x-x_n}{w_n-x_n}\right)\right] \quad \text{for} \quad x_n \le x \le w_n$$

This profile is a straight line, decreasing from $p'(x_n)$ at x_n to 0 at w_n.

In Case II the current is constant for $x_n \le x \le w_n$:

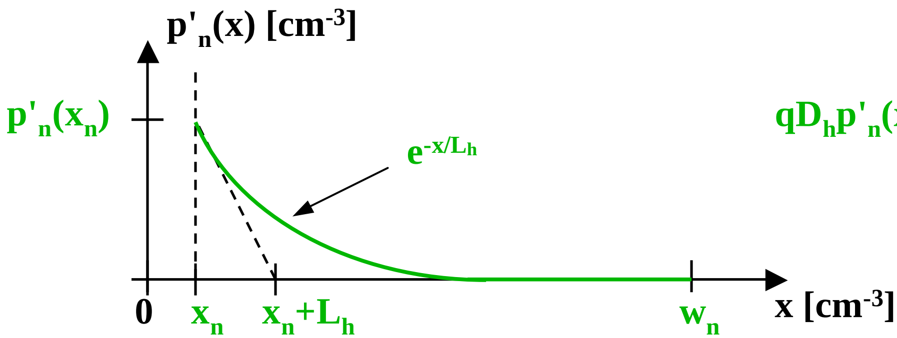
$$J_h(x) \approx -qD_h \frac{dp'(x)}{dx} = \frac{qD_h}{w_n - x_n} p'(x_n) \quad \text{for} \quad x_n \le x \le w_n$$

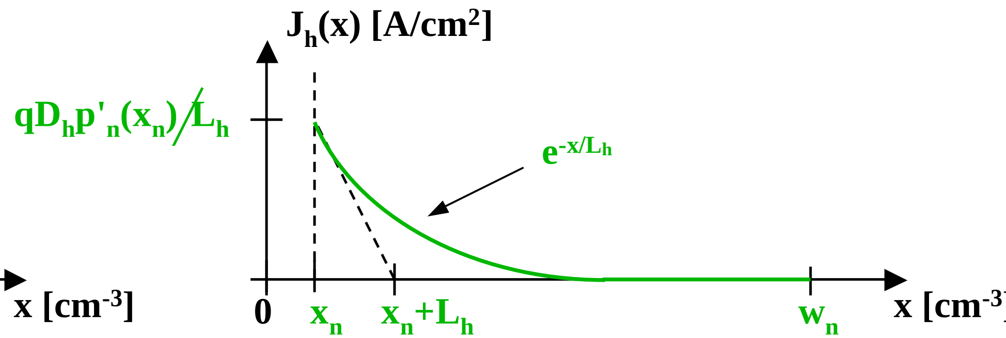
The constant current indicates that no carriers recombine before reaching the contact.

QNR Flow, cont.: Uniform doping, non-uniform LL injection

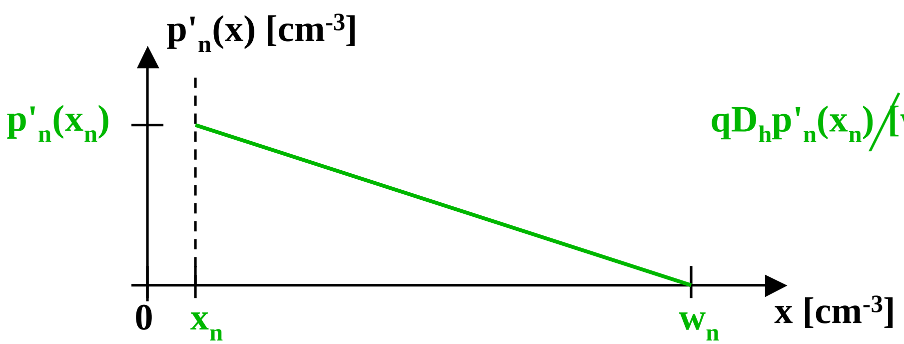
Sketching and comparing the limiting cases: $w_n >> L_h$, $w_n << L_h$

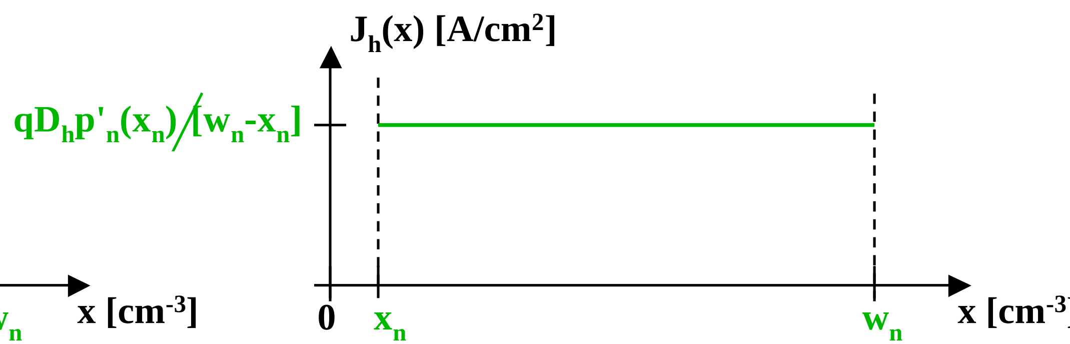
Case I - Long base: $w_n >> L_n$ (the situation in LEDs)





Case II - Short base: $w_n << L_n$ (the situation in most Si diodes and transistors)





QNR Flow, cont.: Uniform doping, non-uniform LL injection

The four other unknowns

⇒ **Solving the steady state diffusion equation gives n'.**

⇒ **Knowing n'..... we can easily get p', J_e, J_h, and E_x:**

First find J_e: $$J_e(x) \approx qD_e \frac{dn'(t)}{dx}$$

Then find J_h: $$J_h(x) = J_{Tot} - J_e(x)$$

Next find E_x: $$E_x(x) \approx \frac{1}{q\mu_h p_o}\left[J_h(x) - \frac{D_h}{D_e} J_e(x)\right]$$

Then find p': $$p'(x) \approx n'(x) + \frac{\varepsilon}{q}\frac{dE_x(x)}{dx}$$

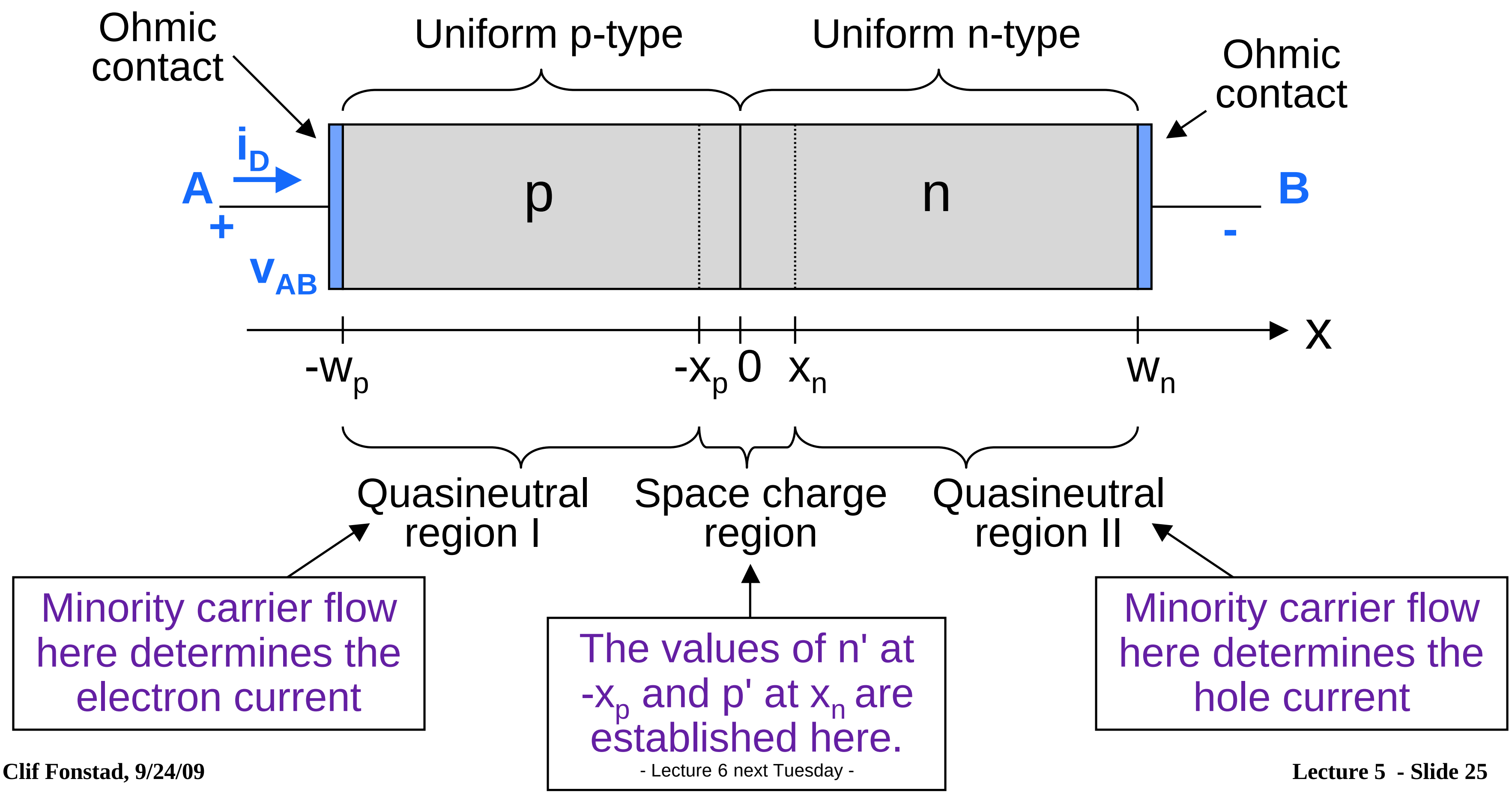
Finally, go back and check that all of the five conditions are met by the solution.

✓ **Once we solve the diffusion equation to get the minority excess, n', we know everything.**

Current flow: finding the relationship between i_D and v_{AB}

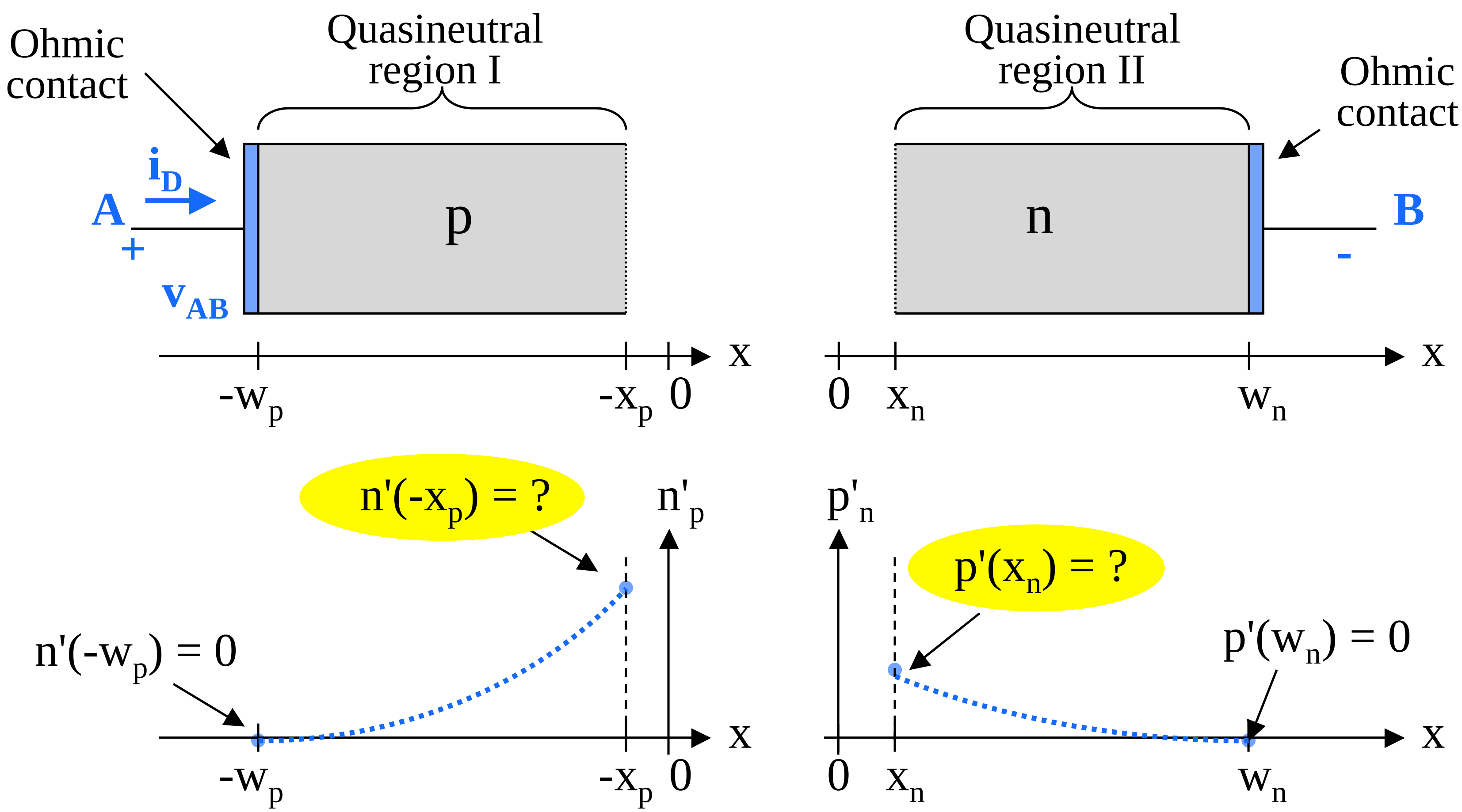
There are two pieces to the problem:

- Minority carrier flow in the QNRs is what limits the current.
- Carrier equilibrium across the SCR determines $n'(-x_p)$ and $p'(x_n)$, the boundary conditions of the QNR minority carrier flow problems.



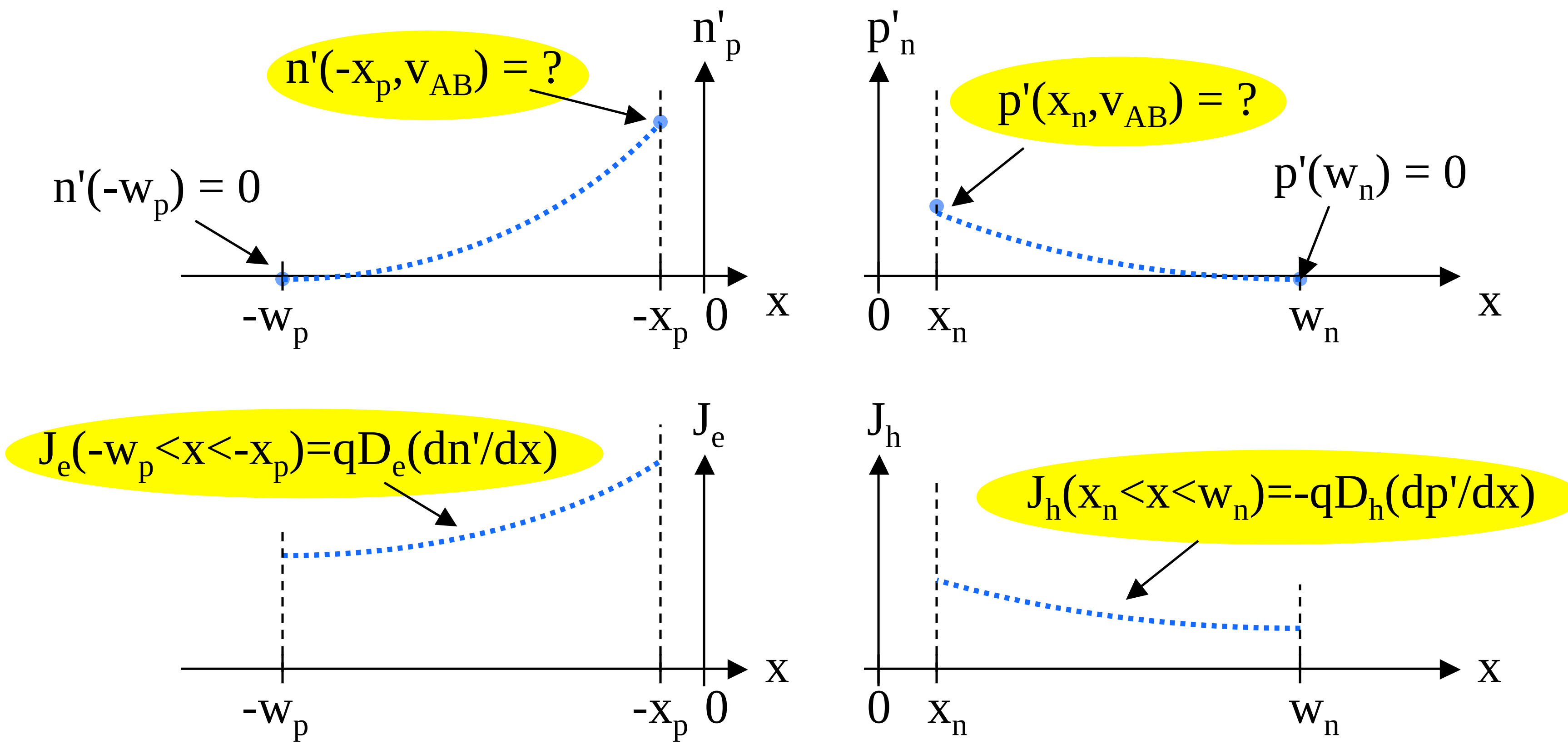
The p-n Junction Diode: *the game plan for getting $i_D(v_{AB})$*

We have two QNR's and a flow problem in each:



If we knew $n'(-x_p)$ and $p'(x_n)$, we could solve the flow problems and we could get $n'(x)$ for $-w_p<x<-x_p$, and $p'(x)$ for $x_n<x<w_n$ …

….and knowing $n'(x)$ for $-w_p<x<-x_p$, and $p'(x)$ for $x_n<x<w_n$, we can find $J_e(x)$ for $-w_p<x<-x_p$, and $J_h(x)$ for $x_n<x<w_n$.



Having $J_e(x)$ for $-w_p<x<-x_p$, and $J_h(x)$ for $x_n<x<w_n$, we can get i_D because we will argue that $i_D(v_{AB}) = A[J_e(-x_p,v_{AB})+J_h(x_n,v_{AB})]$…

…but first we need to know $n'(-x_p,v_{AB})$ and $p'(x_n,v_{AB})$.

We will do this in Lecture 6.

Lecture 5 - p-n Junction Injection and Flow - Summary

- **Biased p-n Diodes**
  - **Depletion regions change** (Lecture 4)
  - **Currents flow: two components**
    - flow issues in quasi-neutral regions
    - boundary conditions on p' and n' at $-x_p$ and x_n
- **Minority carrier flow in quasi-neutral regions**
  - **The importance of minority carrier diffusion**
    - minority carrier drift is negligible
  - **Boundary conditions**
  - **Minority carrier profiles and currents in QNRs**
    - Short base situations
    - Long base situations
- **Carrier populations across the depletion region** (Lecture 6)
  - **Potential barriers and carrier populations**
  - **Relating minority populations at $-x_p$ and x_n to v_{AB}**
  - **Excess minority carriers at $-x_p$ and x_n**

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