# Lecture 5 - PN Junction and MOS Electrostatics (II)

## pn Junction in Thermal Equilibrium

September 22, 2005

**Contents**:

1. Introduction to pn junction
2. Electrostatics of pn junction in thermal equilibrium
3. The depletion approximation
4. Contact potentials

**Reading assignment:**

Howe and Sodini, Ch. 3, §§3.3-3.4

# Key questions

- What happens if the doping distribution in a semiconductor abruptly changes from n-type to p-type?
- Is there a simple description of the electrostatics of a pn junction?

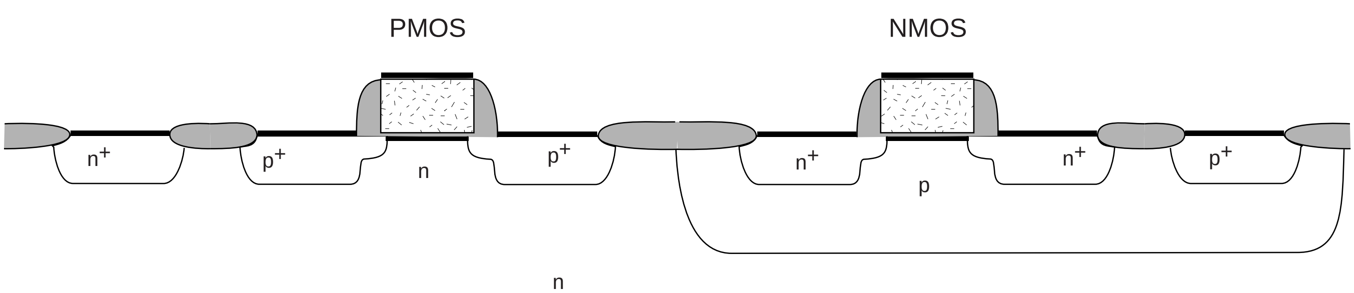
# 1. Introduction to pn junction

- pn junction: p-region and n-region in intimate contact

- Why is the p-n junction worth studying?

It is present in virtually every semiconductor device!

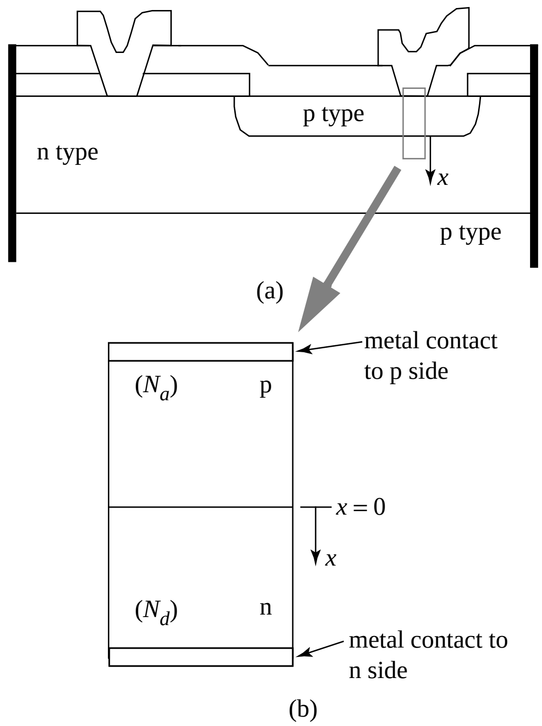
Example: CMOS cross section



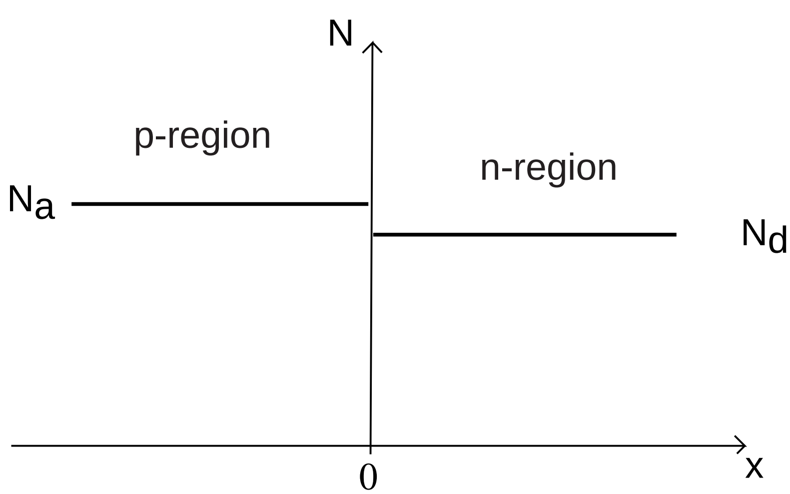

Understanding p-n junction is essential to understanding transistor operation.

## 2. Electrostatics of p-n junction in equilibrium

Focus on intrinsic region:

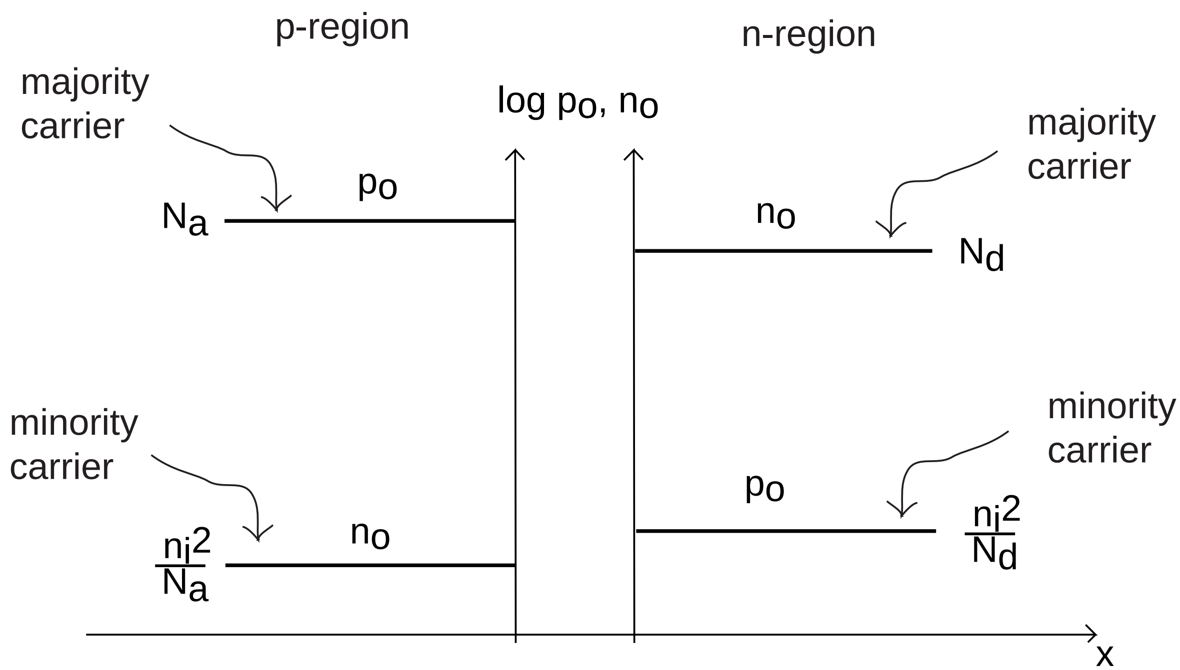



Doping distribution of <u>abrupt</u> p-n junction:



What is the carrier concentration distribution in thermal equilibrium?

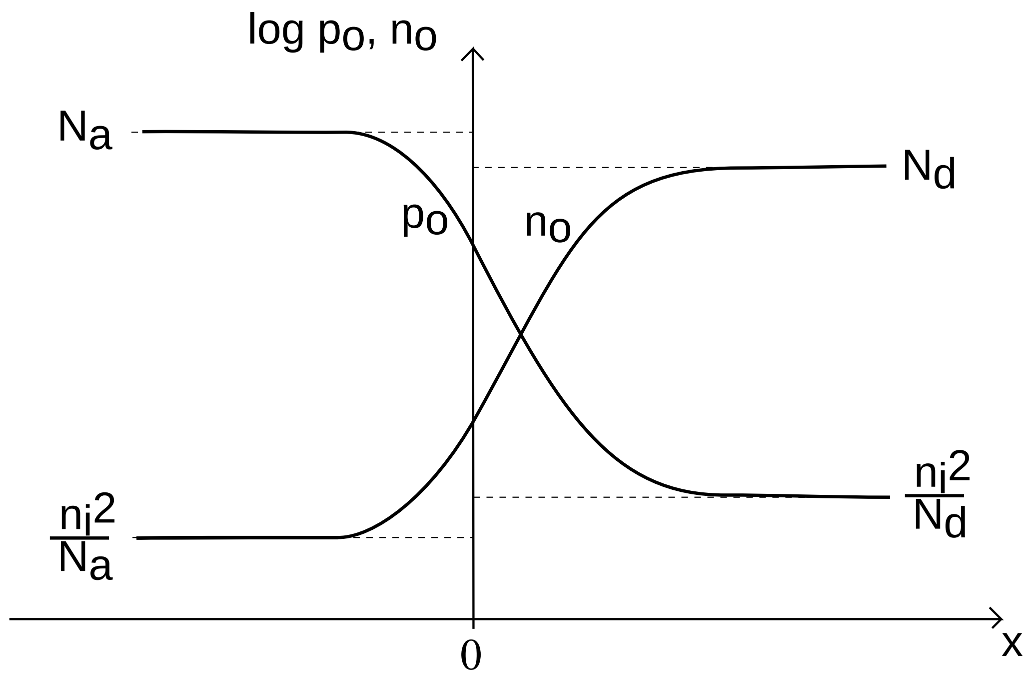
First think of two sides separately:




Now bring them together. What happens?

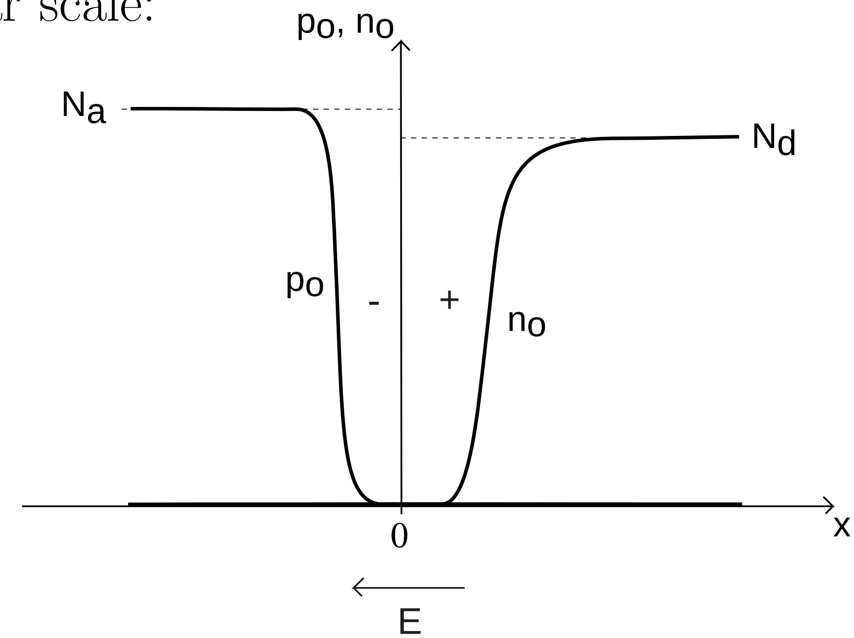
Diffusion of electrons and holes from majority carrier side to minority carrier side until drift balances diffusion.

Resulting carrier profile in thermal equilibrium:

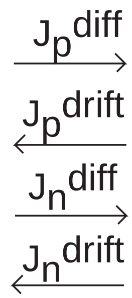



- Far away from metallurgical junction: nothing happens
  - two *quasi-neutral regions*
- Around metallurgical junction: carrier drift must cancel diffusion
  - *space-charge region*

In a linear scale:




Thermal equilibrium: balance between drift and diffusion




Can divide semiconductor in three regions:

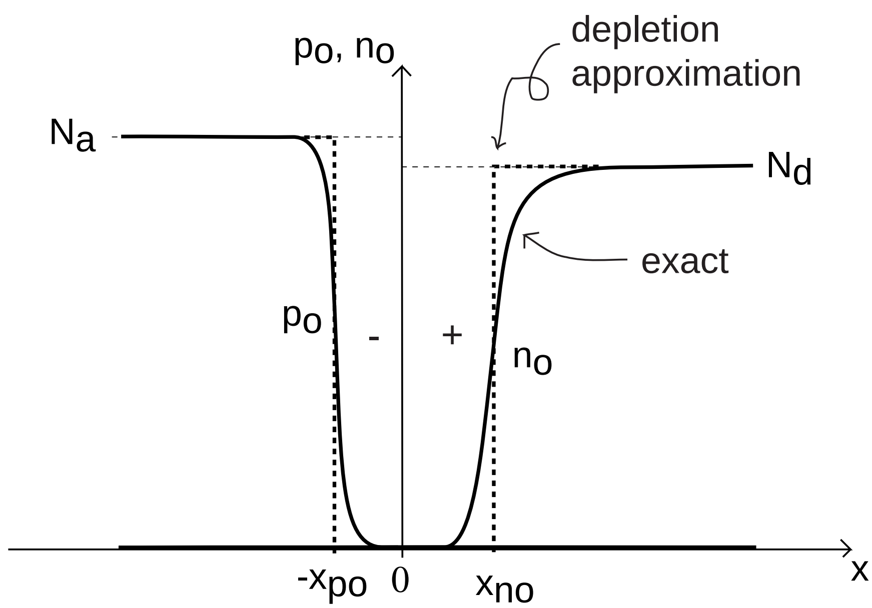
- two quasi-neutral n- and p-regions (QNR's)
- one space charge region (SCR)

Now, want to know $n_o(x)$, $p_o(x)$, $\rho(x)$, $E(x)$, and $\phi(x)$.

Solve electrostatics using simple, powerful approximation.

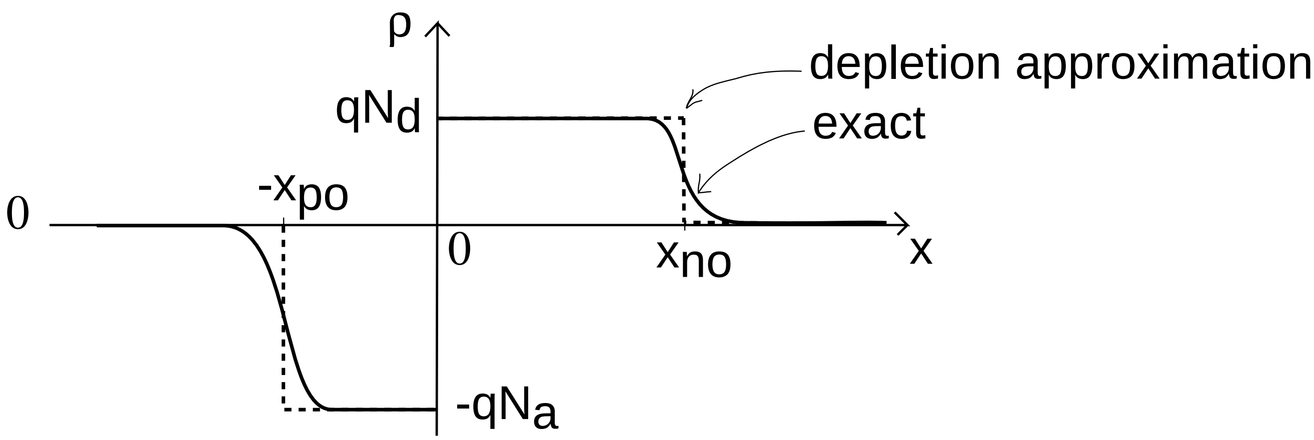
## 3. The depletion approximation

- Assume QNR's perfectly charge neutral
- assume SCR depleted of carriers (*depletion region*)
- transition between SCR and QNR's sharp (must calculate where to place $-x_{po}$ and $x_{no}$)




- $x < -x_{po}$ $\qquad p_o(x) = N_a,\ n_o(x) = \frac{n_i^2}{N_a}$
- $-x_{po} < x < 0$ $\qquad p_o(x),\ n_o(x) \ll N_a$
- $0 < x < x_{no}$ $\qquad n_o(x),\ p_o(x) \ll N_d$
- $x_{no} < x$ $\qquad n_o(x) = N_d,\ p_o(x) = \frac{n_i^2}{N_d}$

- **Space charge density**

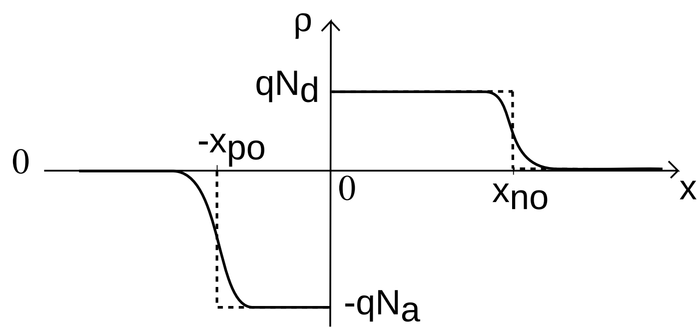


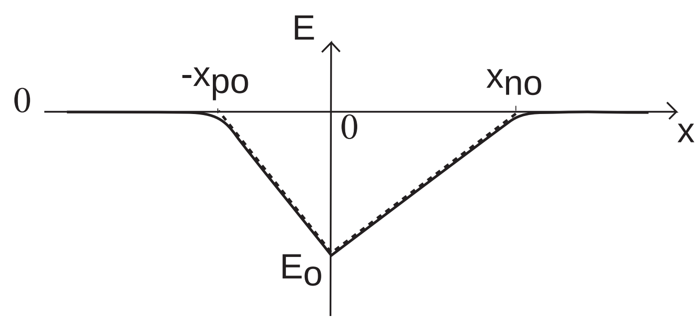

$$
\begin{aligned}
\rho(x) &= 0 & x < -x_{po} \\
&= -qN_a & -x_{po} < x < 0 \\
&= qN_d & 0 < x < x_{no} \\
&= 0 & x_{no} < x
\end{aligned}
$$

• Electric field

Integrate Gauss' equation:

$$E(x_2) - E(x_1) = \frac{1}{\epsilon_s} \int_{x_1}^{x_2} \rho(x)dx$$




• $x < -x_{po}$ $\qquad E(x) = 0$

• $-x_{po} < x < 0$ $\qquad E(x) - E(-x_{po}) = \frac{1}{\epsilon_s} \int_{-x_{po}}^{x} -qN_a dx$

$$= \frac{-qN_a}{\epsilon_s} x\Big|_{-x_{po}}^{x} = \frac{-qN_a}{\epsilon_s}(x + x_{po})$$

• $0 < x < x_{no}$ $\qquad E(x) = \frac{qN_d}{\epsilon_s}(x - x_{no})$

• $x_{no} < x$ $\qquad E(x) = 0$

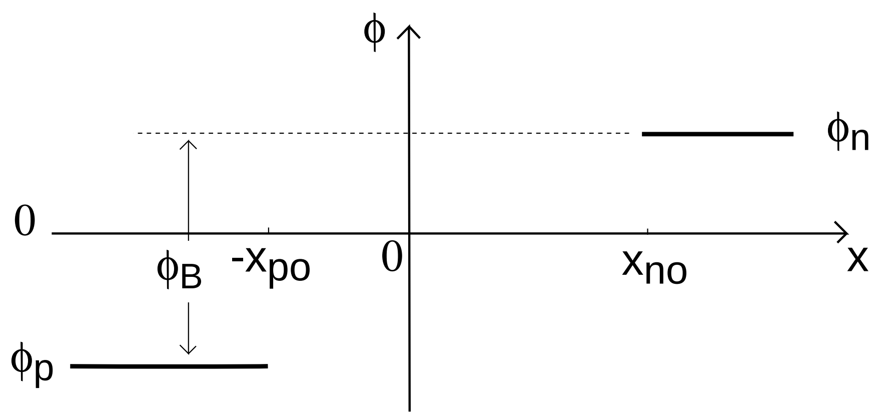
- ELECTROSTATIC POTENTIAL
  (with $\phi = 0$ @ $n_o = p_o = n_i$):

$$\phi = \frac{kT}{q} \ln \frac{n_o}{n_i} \qquad \phi = -\frac{kT}{q} \ln \frac{p_o}{n_i}$$

In QNR's, $n_o$, $p_o$ known $\Rightarrow$ can determine $\phi$:

in p-QNR: $p_o = N_a \;\Rightarrow\; \phi_p = -\frac{kT}{q} \ln \frac{N_a}{n_i}$

in n-QNR: $n_o = N_d \;\Rightarrow\; \phi_n = \frac{kT}{q} \ln \frac{N_d}{n_i}$



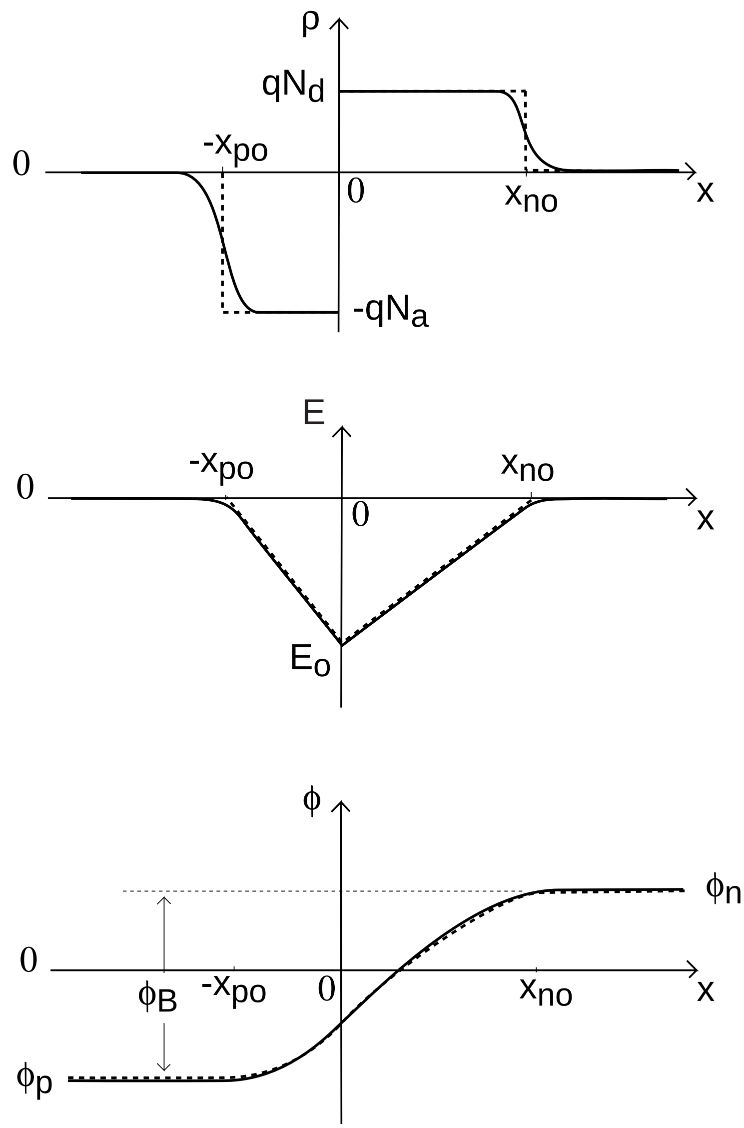

Built-in potential:

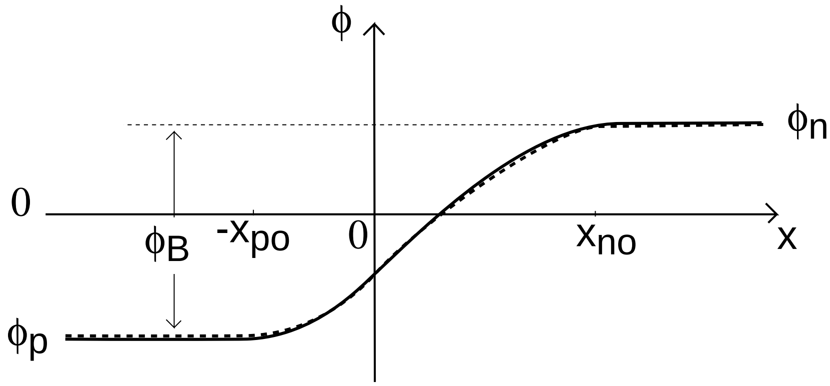
$$\phi_B = \phi_n - \phi_p = \frac{kT}{q} \ln \frac{N_a N_d}{n_i^2}$$

General expression: did not use depletion approximation.

To get $\phi(x)$ in between, integrate $E(x)$:

$$\phi(x_2) - \phi(x_1) = -\int_{x_1}^{x_2} E(x)dx$$






- $x < -x_{po}$ $\qquad \phi(x) = \phi_p$

- $-x_{po} < x < 0$ $\qquad \phi(x) - \phi(-x_{po})$

$$= -\int_{-x_{po}}^{x} -\frac{qN_a}{\epsilon_s}(x + x_{po})dx$$
$$= \frac{qN_a}{2\epsilon_s}(x + x_{po})^2$$

$$\phi(x) = \phi_p + \frac{qN_a}{2\epsilon_s}(x + x_{po})^2$$

- $0 < x < x_{no}$ $\qquad \phi(x) = \phi_n - \frac{qN_d}{2\epsilon_s}(x - x_{no})^2$

- $x_{no} < x$ $\qquad \phi(x) = \phi_n$

Almost done...

Still don't know $x_{no}$ and $x_{po}$ $\Rightarrow$ need two more equations

1. Require overall charge neutrality:

$$qN_a x_{po} = qN_d x_{no}$$

2. Require $\phi(x)$ continuous at $x = 0$:

$$\phi_p + \frac{qN_a}{2\epsilon_s} x_{po}^2 = \phi_n - \frac{qN_d}{2\epsilon_s} x_{no}^2$$

Two equations with two unknowns. Solution:

$$x_{no} = \sqrt{\frac{2\epsilon_s \phi_B N_a}{q(N_a + N_d)N_d}} \qquad x_{po} = \sqrt{\frac{2\epsilon_s \phi_B N_d}{q(N_a + N_d)N_a}}$$

Now problem completely solved.

Other results:

Total width of space charge region:

$$x_{do} = x_{no} + x_{po} = \sqrt{\frac{2\epsilon_s \phi_B (N_a + N_d)}{q N_a N_d}}$$

Field at metallurgical junction:

$$|E_o| = \sqrt{\frac{2q\phi_B N_a N_d}{\epsilon_s (N_a + N_d)}}$$

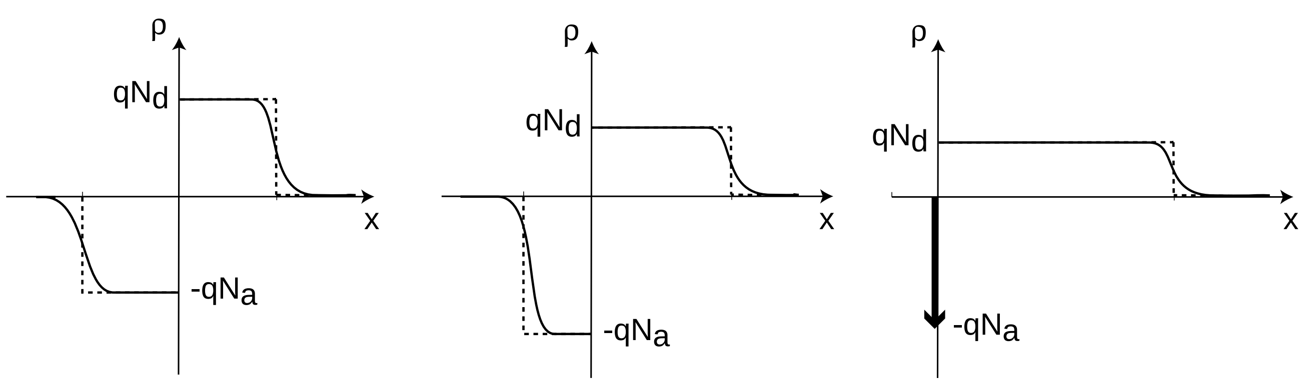
Three cases:

- Symmetric junction: $N_a = N_d \;\Rightarrow\; x_{po} = x_{no}$
- Asymmetric junction: $N_a > N_d \;\Rightarrow\; x_{po} < x_{no}$
- Strongly asymmetric junction:
  *i.e.* p$^+$n junction: $N_a \gg N_d$

$$x_{po} \ll x_{no} \simeq x_{do} \simeq \sqrt{\frac{2\epsilon_s \phi_B}{qN_d}} \propto \frac{1}{\sqrt{N_d}}$$

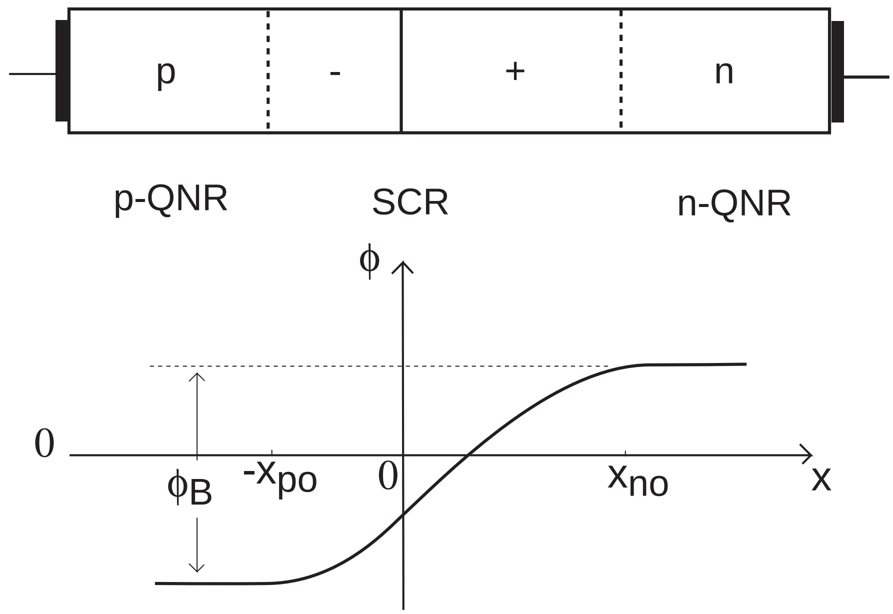
$$|E_o| \simeq \sqrt{\frac{2q\phi_B N_d}{\epsilon_s}} \propto \sqrt{N_d}$$

The lowly-doped side controls the electrostatics of the pn junction.



# 4. Contact potentials

Potential distribution in thermal equilibrium so far:



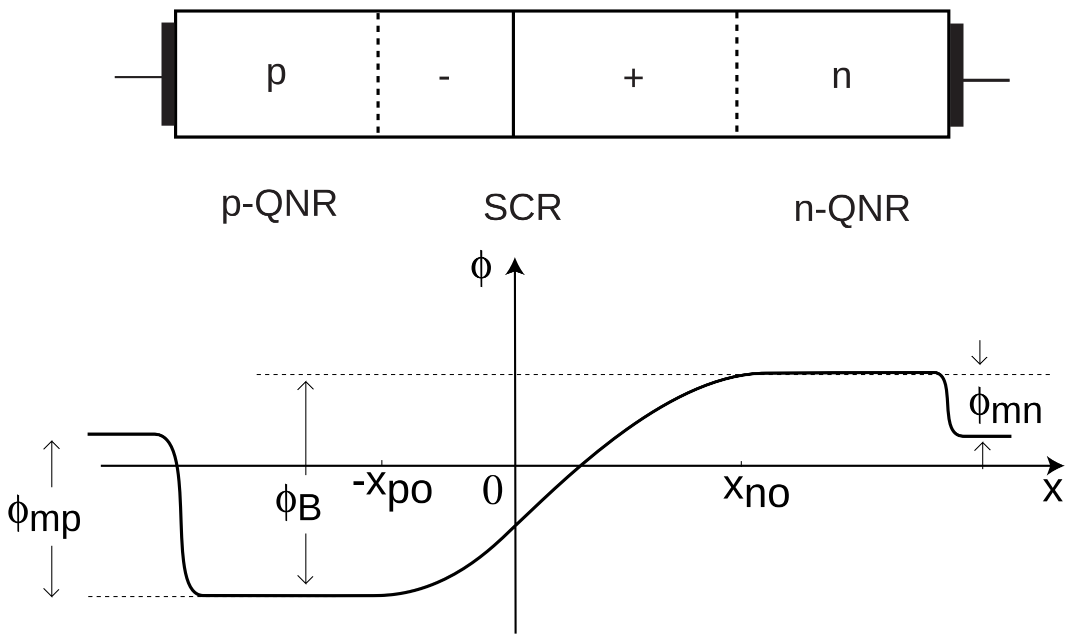

Question 1: *If I apply a voltmeter across diode, do I measure $\phi_B$?*

☐ yes ☐ no ☐ it depends

Question 2: *If I short diode terminals, does current flow on outside circuit?*

☐ yes ☐ no ☐ sometimes

We are missing *contact potential* at metal-semiconductor contacts:




Metal-semiconductor contacts: junctions of dissimilar materials
$\Rightarrow$ built-in potentials: $\phi_{mn}$, $\phi_{mp}$

Potential difference across structure must be zero
$\Rightarrow$ cannot measure $\phi_B$!

$$\phi_B = \phi_{mn} + \phi_{mp}$$

# Key conclusions

- Electrostatics of pn junction in equilibrium:
  - a *space-charge region*
  - surrounded by two *quasi-neutral regions*

  ⇒ built-in potential across p-n junction

- To first order, carrier concentrations in space-charge region are much smaller than doping level

  ⇒ *depletion approximation.*

- Contact potential at metal-semiconductor junctions:
  ⇒ from contact to contact, there is no potential build-up across pn junction