6.006 Recitation

Build 2008.12

PS1 Solutions

- Posted on homework page
- Password-protected
    - Please write down username/password

Coming Up Next...

- More hashing!
- Rabin-Karp (String Matching)
  - vs the ~~dumb~~ naive algorithm
  - Rolling Hashes
  - Black Magic: why it works

Hashing without tables

- Fancy names: fingerprint, message digest
- Idea (hashing repeated):
  - given an object, compute a summary that's easier to work with
- Very versatile concept! Don't forget it!!

Hashing human beings

Hashing human beings

- Want something easy to handle

Hashing human beings

- Want something easy to handle
  - fingerprints (doh)
  - DNA samples
  - iris scans
  - face picture

Naive String Matching

- Want to find pattern in text
- Slide pattern over text one by one character
  - If pattern matches overlapping characters of text, report match

Rabin-Karp

- Want to find pattern in text
- Slide pattern over text one by one character
    - If hash(pattern) matches hash(overlapping characters of text)
        - If pattern matches overlapping characters of text report match

Making Rabin-Karp fast

- Good hash function
    - If many false positives, then many useless full-string comparisons
- Fast hash update when “sliding” pattern across text
    - If we rehash every time, might as well use naive string comparison

Introducing Rolling Hashes

- Data Structure (just like hash table)
  - start with empty list
  - append(val): appends val at the end of list
  - skip(): removes the first list element
  - hash(): computes a hash of the list

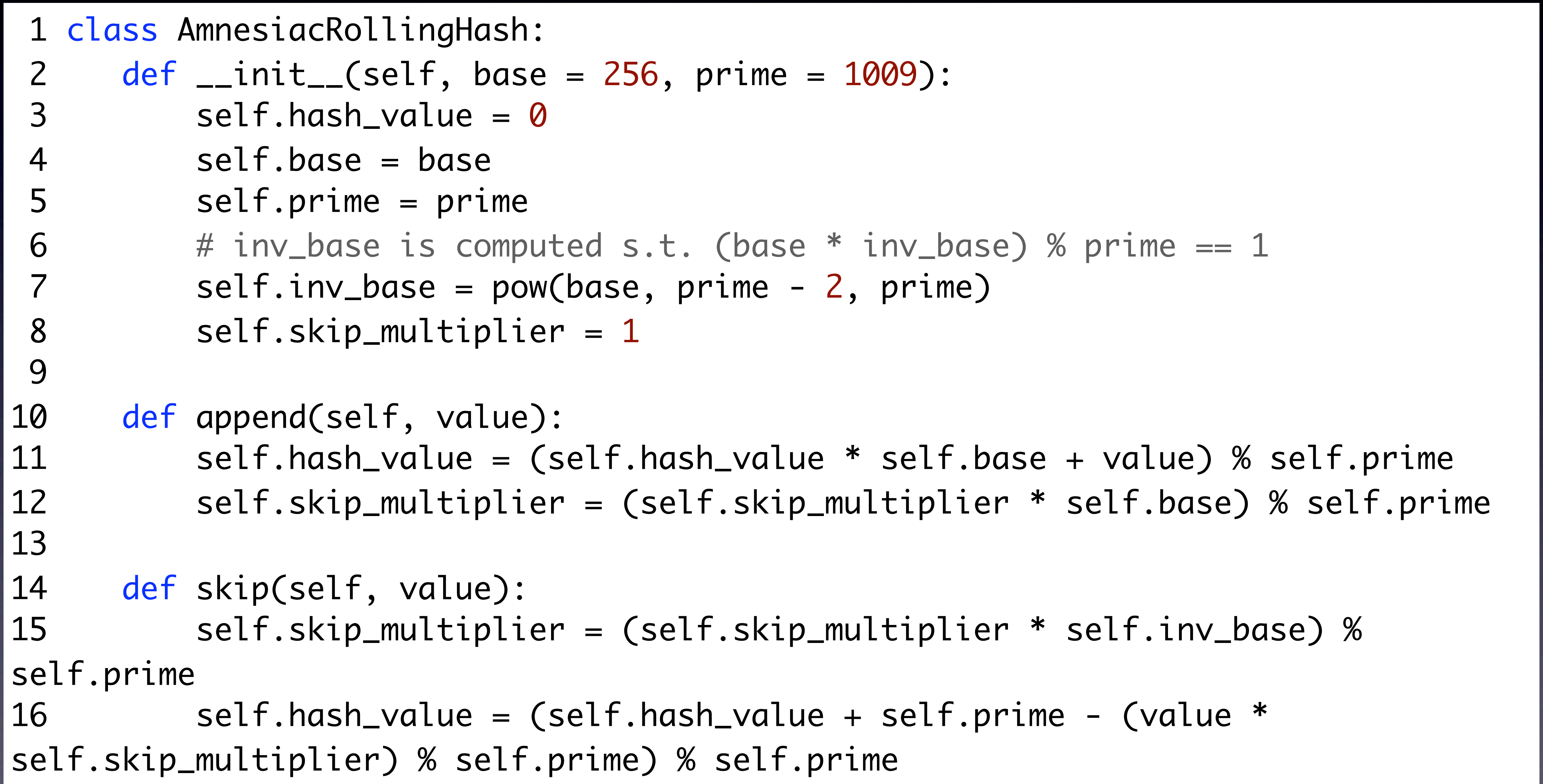
But we have strings

- Characters are numbers (ASCII, Unicode)
  - 'A' = 65, 'B' = 66
- Then strings are lists of numbers
  - "Boom! Headshot" = [66, 111, 111, 109, 33, 32, 72, 101, 97, 100, 115, 104, 111, 116]
- So we can work with lists of numbers

Building Rolling Hashes

- Key Idea: use division method for hashing
  - “concatenate” list items into big number
  - hash value: big number mod prime
  - reason: skip() is doable (not true for most other hashing methods)

Goal: Getting to This



```
class AmnesiacRollingHash:
    def __init__(self, base = 256, prime = 1009):
        self.hash_value = 0
        self.base = base
        self.prime = prime
        # inv_base is computed s.t. (base * inv_base) % prime == 1
        self.inv_base = pow(base, prime - 2, prime)
        self.skip_multiplier = 1

    def append(self, value):
        self.hash_value = (self.hash_value * self.base + value) % self.prime
        self.skip_multiplier = (self.skip_multiplier * self.base) % self.prime

    def skip(self, value):
        self.skip_multiplier = (self.skip_multiplier * self.inv_base) %
self.prime
        self.hash_value = (self.hash_value + self.prime - (value *
self.skip_multiplier) % self.prime) % self.prime
```

Hashing Intuition

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- Base 100, modulo 23
- Hash [61, **8**, 19, **91**, 37]

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- Hash [61, **8**, 19, **91**, 37]
    - (61**08**19**91**37 mod 23) = 12
- Hash [a_3, a_2, a_1, a_0]

Hashing Intuition

- Base 100, modulo 23
- Hash [61, **8**, 19, **91**, 37]
  - (61**08**19**91**37 mod 23) = 12
- Hash [a_3, a_2, a_1, a_0]
  - $(a_3 \cdot 100^3 + a_2 \cdot 100^2 + a_1 \cdot 100^1 + a_0 \cdot 100^0) \bmod 23$

Sliding Intuition

- Base 100, mod 23
- List: [3, 14, 15, 92, 65, 35, 89, 79, 31]
- [3, 14, 15, 92, 65] to [14, 15, 92, 65, 35]
    - get from 11 to 6
- [14, 15, 92, 65, 35] to [15, 92, 65, 35, 89]
    - get from 6 to 5

Simple Rolling Hashes

- formulas for updating the hash value on append and skip

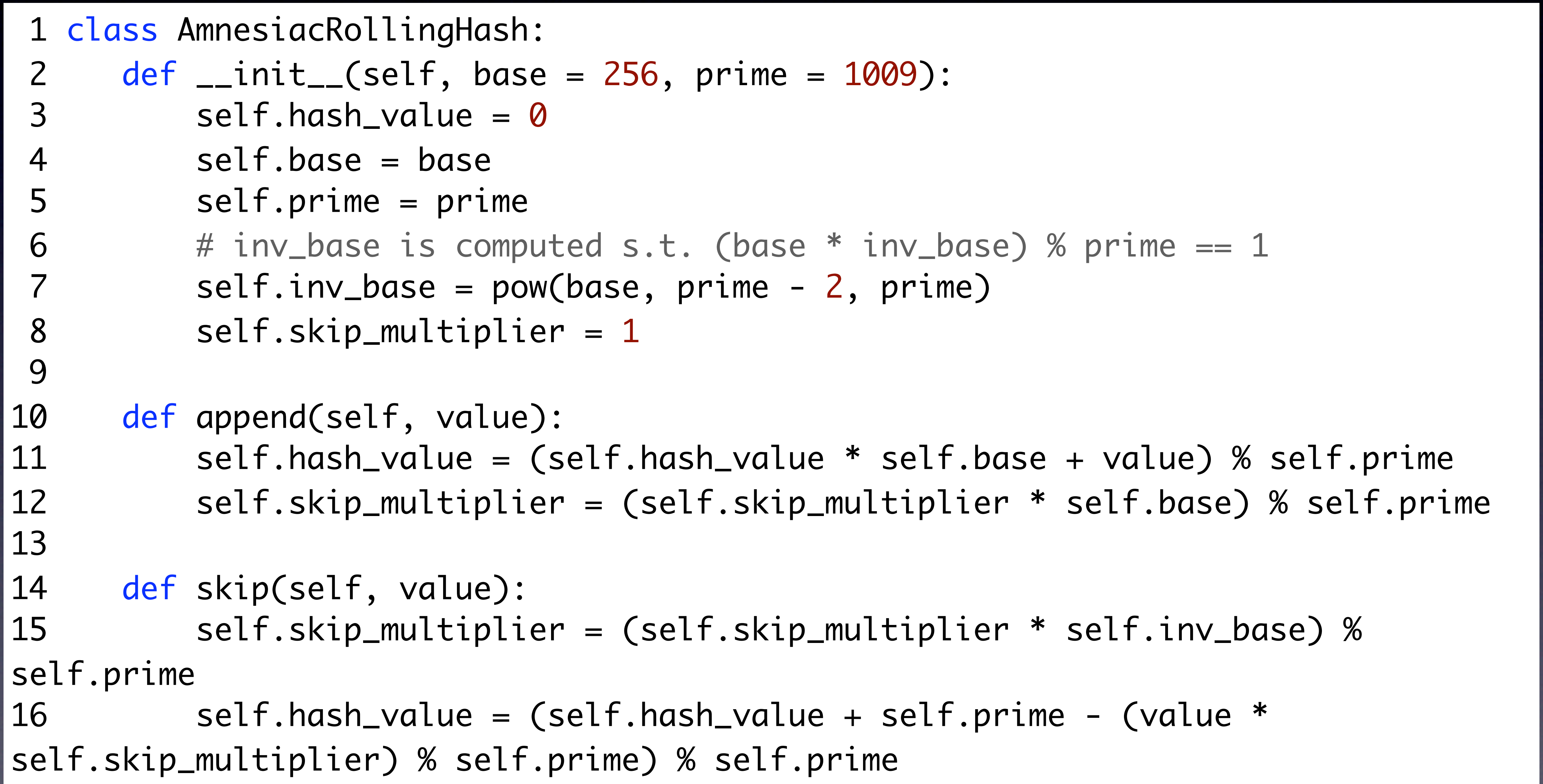
Fast Rolling Hashes

- need to avoid exponentiation in skip
- cache the result (base ** length mod p)
    - append: multiply by base
    - skip: divide by base
        - can’t divide, use multiplicative inverse

Python design

- Step 1: Amnesiac Hash -- forgets list items
  - need to remind skip() what's the front element of the list
- Step 2: Easy Hash -- keeps track of items
  - builds upon Amnesiac Hash
  - keeps track of list items

Python: Amnesiac Hash



```python
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    def __init__(self, base = 256, prime = 1009):
        self.hash_value = 0
        self.base = base
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        # inv_base is computed s.t. (base * inv_base) % prime == 1
        self.inv_base = pow(base, prime - 2, prime)
        self.skip_multiplier = 1

    def append(self, value):
        self.hash_value = (self.hash_value * self.base + value) % self.prime
        self.skip_multiplier = (self.skip_multiplier * self.base) % self.prime

    def skip(self, value):
        self.skip_multiplier = (self.skip_multiplier * self.inv_base) % self.prime
        self.hash_value = (self.hash_value + self.prime - (value * self.skip_multiplier) % self.prime) % self.prime
```
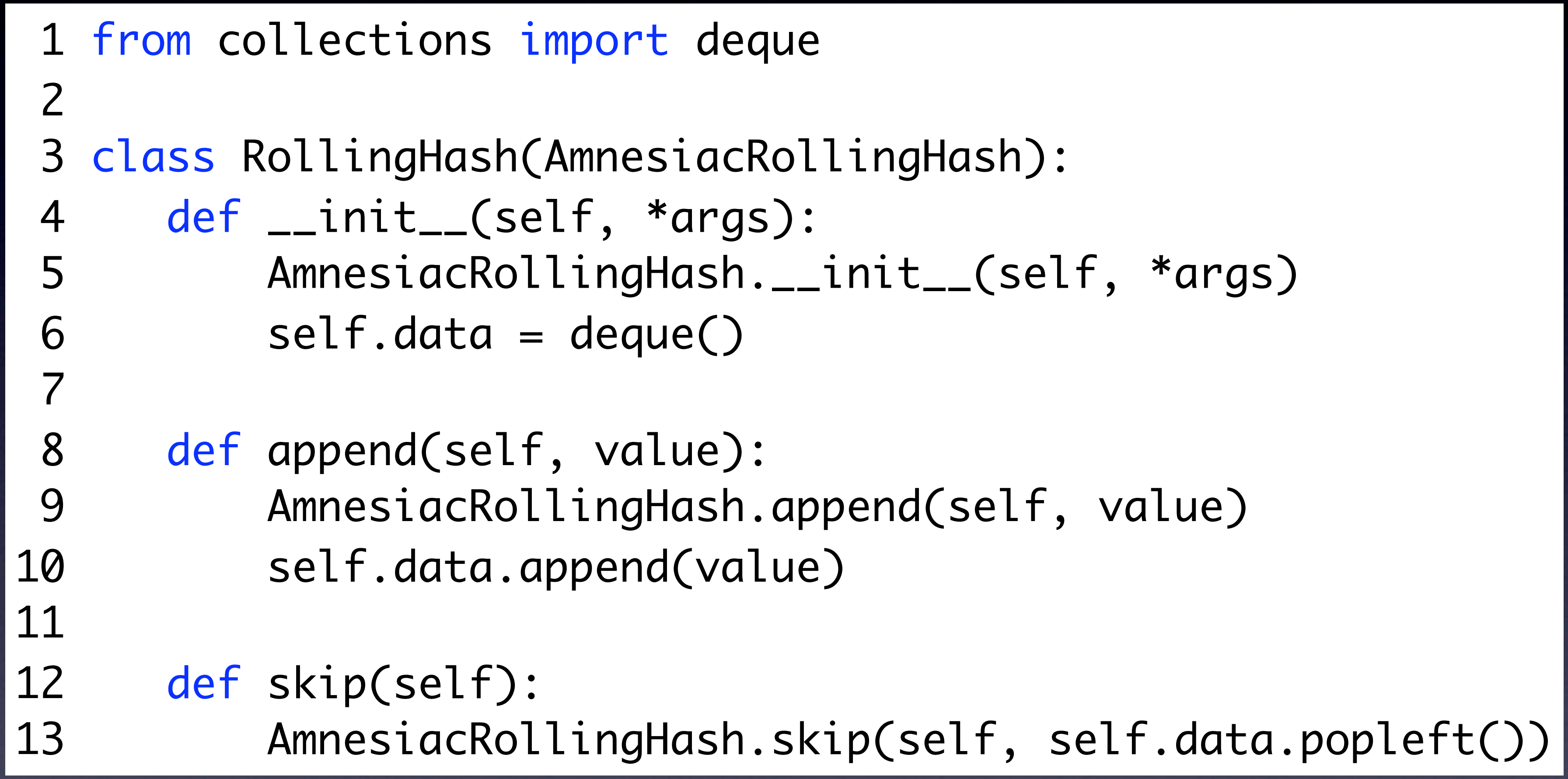

Python: Easy Hash



```python
from collections import deque

class RollingHash(AmnesiacRollingHash):
    def __init__(self, *args):
        AmnesiacRollingHash.__init__(self, *args)
        self.data = deque()

    def append(self, value):
        AmnesiacRollingHash.append(self, value)
        self.data.append(value)

    def skip(self):
        AmnesiacRollingHash.skip(self, self.data.popleft())
```