DENNIS FREEMAN: Hello. Welcome to the first lecture in the last topic of this class. So we'll spend this lecture and the next lecture talking about modulation.

Modulation, like sampling, is an excellent illustration of the power of thinking about signals in terms of their Fourier transforms. We'll see, just like we saw in sampling, that a problem that was potentially very complicated to understand, sampling, was very easy when you thought about it in the Fourier domain. And precisely the same thing happens with modulation, which is precisely the reason we're talking about it now.

So I'll talk about modulation in the context of a communication system. That's just for convenience. In fact, modulation is used in lots of other places. In fact, next time, in the next lecture, I'll show you an application of modulation from my research group, where we used modulation to improve the resolution of an optical microscope-- nothing whatever to do with communications. But it's still-- the optical microscope-- the enhancement to resolution that we achieve is directly based on the principles that I'll start to describe today.

So I'll talk about modulation today in the context of communications. Probably the most convenient, easiest to think about communication system to all of us, being humans, is speech. We use speech for communication all the time. It's very easy to think about how speech works as a communication medium. Somebody talks, somebody listens.

One of the first ways of thinking about it as a technological feat was the telephone-- the idea being that you convert the sound that I'm emitting when I'm speaking by a microphone into an electrical representation that gets shot down a wire. Then at the other end, you take the electrical signal that's coming down the wire and turn it back into sound. That's the principle of a telephone-- works very well.

Modulation comes up when we start to think about how would we generalize this notion for wireless communication. In particular, if we do cell phone communication, cell phones transmit the signal that is picked up from the microphone. That signal gets converted into electromagnetic signals. That's the basis by which cell phones communicate with the cell tower.

The towers may communicate with other towers via lot of different kinds of technologies. I'm ignoring those for now. But the communication between the phone and the tower is via electromagnetic waves.

And there's an interesting thing that happens when you try to recode the signal that would have been perfectly happy running down the copper wire, when you try to recode that signal into an electromagnetic wave. And that has to do with basic physics of electromagnetic waves, which I'm sure you all know. And so I'll just remind you of one simple idea.

So for efficient transmission from an electrical representation to an electromagnetic wave, first off, that transduction is mediated by something we call an antenna. Antennas will take an electrical signal and convert it into an electromagnetic wave, and vice versa. But the efficiency with which an antenna works has to do, among other things, with its size.

It's very difficult-- by which I mean it takes lots of power-- to transform a signal from an electrical representation to an E&M representation if the antenna is smaller-- is significantly smaller-- than the wavelength of interest. So it's not that you can't do it. It's that it takes lots of power. So if you don't want to burn a lot of power doing that transformation, then you need to make an antenna that's roughly the size of the wavelength that you're trying to transmit as an electromagnetic wave.

So if we were thinking about this kind of a scheme for the communication of voice-- we've talked about voice many times-- Telephone-quality voice is usually defined to be frequencies between about 200 hertz and about 3 kilohertz. If I had a signal that was composed of those kinds of frequencies, how big should I make the antenna to get efficient coupling between the electrical representation in the cell phone and the electromagnetic wave that the cell phone wants to launch to get to the cell tower? So look at your neighbor. That involves turning your head.

[LAUGHTER]

Say hello and figure out how long should the antenna be.

[INTERPOSING VOICES]

DENNIS FREEMAN: Does everybody have an answer? Raise your hand if you don't have an answer. How do you like that for a switch of-- Raise your hand if you don't have an answer. You all have answers? OK, so how big should the antenna be-- 1, 2, 3, 4, or 5?

OK, very good. So the answer is really big. So you think about the relationship between wavelength and distance, so you can think about the relationship is given by speed. The speed of a wave is the wave length-- how long it goes per cycle-- times the number of cycles per second, f. So you can solve that expression to find out the wavelength in terms of the speed, which for an electromagnetic wave is the speed of light, 3 times 10 to the eighth meters per second.

The hardest one to launch is the lowest frequency. That'll take the biggest antenna. So the lowest frequency in telephony-- in telephone-quality speech-- is 200 hertz. So we get about 1,500 kilometers-- kind of big, kind of useless.

If you were thinking about trying to make cellular communication and your antenna actually had to be 1,500 kilometers, that just isn't going to work. So what do we do? We obviously don't do this. So the answer is that you would need an antenna hundreds of miles in length.

So what frequency should you be using if you wanted to build a phone that kind of fit in your hand-- 10 centimeters or so? What would be the frequency-- what would be the interesting range of frequencies that you would want to use for such a device?

And the answer is-- Of course. So the answer is, you go to the bottom one, and it's bigger than a gigahertz. And it's just running the same expression in the other direction. So you think about, the frequency that you would like would be the speed of light divided by the wavelength.

If you wanted the wavelength to be 10 centimeters, then you would end up with a frequency on the order of gigahertz. And it shouldn't come as a surprise to you then that that's what we use in cellular communication. So a modern cell phone uses 2.1 gigahertz.

So the point is that when we're thinking about how we would like to use the electromagnetic spectrum for a communications task, that spectrum is not necessarily well-matched to the communications problem of interest. You might think that that cellular example is an exception. In fact, that's the rule.

If you try to have a signal of interest transmitted over a medium or stored on some sort of a medium, it is generally the case that there is a matching problem, that the characteristics of the medium don't match the characteristics of the message. And so part of communications engineering is trying to come up with a coding scheme that matches the characteristics of the message to the characteristics of the medium. And so what I'm going to do today is talk about some matching schemes based on modulation.

So we just saw that if we wanted to do cellular communication of voice, voice might have a spectrum represented by this magnitude function, X, where the bandwidth is on the order of a few kilohertz. But we might want to transmit a signal that has the same information. However, we would like the frequencies to be up around 2 gigahertz.

Which of these coding schemes, Y as a function of X, achieves this transformation? Take the stuff that you have in a low frequency range and shift it to a high frequency range. What would you do?

The obvious answer is to stare with a blank face in it. It'll definitely come to you, right? You don't want to talk to somebody else. That would give it away.

[INTERPOSING VOICES]

DENNIS FREEMAN: So what's the relationship between X of t and Y of t? Is it relation 1, 2, 3, 4, or none of the above? OK, it's about 80% correct.

So how do I think about it? So let's see, I want to figure out a relationship between the Fourier transform of Y and X. If I wanted to figure out the Fourier transform of Y, I would integrate Y of t e to the minus j omega t dt. That looks kind of right.

And Y would be X of the t-- let's try the first one-- X of t e to the j omega c t e to the minus j omega t dt. Well, that's almost the Fourier transform of X. All I've done is shifted omega. So in fact, that's the same as X of j omega minus omega c. And that's what I want to do.

So the idea is that, if you multiply by this complex exponential, the effect of that multiplication in time is to shift frequencies. Can somebody say that to me-- say the same transformation but in slightly different words? Guess that was kind of big. Yes.

AUDIENCE: Couldn't you [INAUDIBLE] multiplication by the exponential in time.

DENNIS FREEMAN: [INAUDIBLE] by the exponential in time--

AUDIENCE: Yeah equal to--

DENNIS FREEMAN: --should correspond to--

AUDIENCE: Which gives you a frequency [INAUDIBLE].

DENNIS FREEMAN: So generally, a multiplication in time corresponds to--

AUDIENCE: [INAUDIBLE]

DENNIS FREEMAN: --convolution. So equivalently, instead of saying it shifted in frequency, you could say--

AUDIENCE: [INAUDIBLE]

DENNIS FREEMAN: --it got convolved--

AUDIENCE: With delta.

DENNIS FREEMAN: --with the delta function. So a different way of saying the same thing would be that you think about convolving with the delta function in frequency-- same thing. Now, we don't actually do this. When we're doing the cell phone thing, we don't actually multiply it by e to the j omega c t. Anyone know why?

This is kind of the simplest way that you could imagine. I've taken frequency content centered near 0 and turning it into frequency content centered near omega c. But that's not what we really do. Why don't we really do that?

AUDIENCE: [INAUDIBLE]

DENNIS FREEMAN: Exactly. We don't really do it, because the signals aren't real. So how do you know the signal is not real?

AUDIENCE: Because magnitudes [INAUDIBLE].

DENNIS FREEMAN: If the signal had been real, the Fourier transform would have been-- If the signal had been real the Fourier transform would have been-- X of j omega. If the signal had been real, the Fourier transform would have been--

AUDIENCE: Symmetry.

DENNIS FREEMAN: --some kind of symmetry. How do I see symmetry in that expression?

AUDIENCE: Conjugate.

AUDIENCE: Conjugate.

DENNIS FREEMAN: Conjugate symmetry. How do I see conjugate symmetry in that expression? Well, this is cos omega t plus j sine omega t. So if this is a real signal, then it gives rise to something here that is symmetric. The cosine terms are all symmetric about the origin.

And it has an imaginary part that is antisymmetric about the origin, because you add together a bunch of cosines. You can't get anything that's anything other than symmetric about the origin. You add together a bunch of sines. You can't get anything except something that is antisymmetric about the origin.

So you know this thing has to be-- so a real signal would have had conjugate symmetry. The real part would have been symmetric. The imaginary part would be antisymmetric. And you can see that this is not conjugate symmetric. Everybody knows what I'm talking about?

AUDIENCE: [INAUDIBLE]

DENNIS FREEMAN: Conjugate symmetry would mean that the real part of the signal is symmetric about the origin, which means that if this is supposed to represent a real signal, then there should have been a reflection over here to make it symmetric about the origin.

AUDIENCE: Just was it symmetric before?

DENNIS FREEMAN: It was symmetric, but I shifted it by a complex number. I shifted it by cos omega c t plus j sine omega c t. So by that multiplication, I generated a complex number.

AUDIENCE: [INAUDIBLE].

DENNIS FREEMAN: So this signal was complex valued and not conjugate symmetric. So the point is trying to get you to remember the kinds of things that we're supposed to know about Fourier transforms. So by shifting with a complex exponential, we wreck the realness of the original signal. The real original signal would have been conjugate symmetric in frequency. But the wrecking of it gave rise to a signal that was no longer conjugate symmetric in frequency.

So we don't really modulate this way. But we do the obvious extension. What we would do is modulate with a cosine wave. So now, if instead of multiplying by a complex exponential, I multiply by the cosine omega c t, by Euler's expression, I can think about that as being the sum of two components-- one at plus omega c and one at minus omega c. And now, when I do the convolution, I get a signal that is conjugate symmetric.

So when I convolve this with this one, this one gives me a copy of this here. And when I convolve this with this one, this one gives me a copy of this one down here. So now, the resulting signal, which I know by construction-- if this was real and this was real, then that's real-- I know by construction that this signal must have been real. But I can also see it in the transform, because I can see now that there's a symmetry that is consistent with it being conjugate symmetric. Yes. Somebody had a question?

So that's what we mean by modulation. This is modulation. Modulation just is a fancy word that means multiply. So what we're going to do is multiply the signal by a carrier. The carrier is going to be a signal that carries the message through the medium.

The carrier is chosen so that it goes efficiently through the medium. And then the carrier carries the message through the medium. So we think about this as modulation.

And we want to be familiar with going back and forth between time and frequency. You can also think about the result of modulation in time. So if this were my message, which is intended to be represented as a low frequency, and this is my carrier, which is intended to be represented as a higher frequency, then when I modulate it, I get a modulated signal, by which I mean-- this is called amplitude modulation-- the amplitude of the carrier is modulated by the message. That's all we mean.

So you can see that this transformation, which has the property of moving the information from a low frequency that's hard to transmit to a high frequency that is easy to transmit, that has the effect of doing a very particular pattern to the time-domain waveform.

Now, it would be completely useless as a communication scheme if it weren't easy to invert. So imagine that I have this signal. And what I'd like to do is recover x. What should I do to recover x, the original message?

So the idea is, I have an original message available in my cell phone. It gets modulated so that it can be launched into electromagnetic waves. The electromagnetic waves go to a receiver thousands of miles away. And now, the idea is to reconstruct my original signal x. What would I do-- what kind of a system would I use to recover x from y?

AUDIENCE: [INAUDIBLE].

DENNIS FREEMAN: I'm sorry.

AUDIENCE: Couldn't you divide out the cosine?

DENNIS FREEMAN: You could divide out the cosine. So you could take x of t cosine omega c t times something-- what do I want to say-- a of t designed so that this times this is 1. That's kind of ugly. Anybody see anything ugly about that? Yeah.

AUDIENCE: [INAUDIBLE] shift it back the other way?

DENNIS FREEMAN: You could also shift it back the other way. That's right. So before we do that, why is this ugly?

AUDIENCE: The zeros.

DENNIS FREEMAN: The zeros. So if we wanted to take a signal that looks like a cosine wave and multiply it by some signal that generates 1, that's not too hard to do here. You would do that with something like this. But it becomes very hard to do here. So you would end up making some signal that does some awful-- So it would periodically be a mass.

But you can do what you said. An alternative would be to multiply it by another cosine, which in the frequency domain is easy to think about. It would just shift it back-- so convolve with a pair of impulses to move something that was at DC out to some high frequency, convolve again to take the thing that was at high frequency and bring it back to DC. And you can think about that in either frequency or time.

It's easy to think about it in time. If you think about it in time, here we've got the product of two cosines. But the product of two cosines is just 1/2 plus half the cosine of double. Well, that's good.

Why is that good? Well, if you multiply x of t by this, this is a super high frequency. If omega c was a high frequency, 2 omega c is even higher. So what you could do is remove x times 1/2 cos 2 omega c t with a low pass filter, since omega c is such a high frequency. And that would just leave you with half the message, which would be easy then to reconstruct, because what you would do is just put it through a low pass filter and then multiply by 2.

You can similarly think about the same thing in frequency. If I took y and convolved it-- if I multiply in time by another cosine wave, that second cosine wave is a pair of impulses-- one at minus omega c and one at omega c. And now, when I convolve the y signal, this one shifts these two up, and this one shifts these two down. And two of them land on top of each other.

But each of these was only of height 1/2. So by Euler's expression, cosine of something was 1/2 e to the whatever plus 1/2 e to the whatever. So I got 1/2's on each of those amplitudes. So the result is then that I have to multiply the low frequency part by a factor of two to undo the 1/2's.

So this kind of a scheme is especially nice, because you can scramble together multiple messages and still get them separated at the destination. If you imagine having three similar transmitters that use their own omega c-- so the first one uses omega 1, omega 2, omega 3-- so if each one of the transmitters had their own frequency, and if the frequencies were far enough apart, and if the frequencies were all big compared to the message frequency, then you could combine them all and select out the one of interest by tuning the receiver, by choosing the demodulation frequency.

So now, if the receiver chose omega c equals omega 1, you would decode message 1. If omega c were omega 2, you'd decode message 2. And that's because the medium works approximately linearly.

So if you launch multiple waves into the air-- I don't want to get too much into electromagnetic theory-- so the presence of the antennas distort it from linearity. But once the antennas are all there, then it's perfectly linear system. And the thing that gets into the air as a result of a sum is the sum of the individual parts.

So the idea then is illustrated here. If I had three different messages represented by different style houses, and each one of the messages was at a different frequency-- omega 1, omega 2, omega 3-- by tuning the omega c of the receiver, if I put omega c at omega 1, the convolution of this one would suck this one up to here. And by this one would lower that one down there. You get overlap of the lowest frequency pair.

And so if you built a low pass filter of exactly the right width, you would decode message 1. Where, if you just changed the frequency of the demodulator-- if you make the demodulation frequency now be omega 2-- now, the effect of shifting that different amount means that the low pass filter recovers message 2, rather the message 1. So that's the idea.

So that's the idea that we use in commercial AM radio. And that was, in fact, a revolutionary idea that enabled people to think about for the first time a communication system that did a lot of things that were very different from previous communication systems. In particular, it went at the speed of light. Even more importantly, or at least as important, is the fact that it was a broadcast system.

So broadcast was an idea that was championed by David Sarnoff. Sarnoff was a visionary. He was the person who was very excited about the idea of broadcast, which is a little ironic, because he got his start with Marconi. Anybody ever hear of Marconi? Good, good, you're supposed to have heard of Marconi.

So Sarnoff got his start-- this is Sarnoff; this is Marconi-- Sarnoff got his start with Marconi. Marconi made his mint with wireless telegraphy. Anybody ever hear of telegraphy? Of course not.

So telegraphy, that's telegraph. Shake your heads yes. It's ancient, I realize. So Marconi made his fortune with wireless telegraphy.

Telegraphy was what we call point to point. The idea in telegraphy was precisely the same as the idea of the US post office, except it was at the speed of light, or not quite the speed of light. So the idea in the post office is you take a sheet of paper and do something to it that makes it magically appear at somebody else's place. So point A communicated to point B.

Telegraphy was precisely the same. You take your sheet of paper to the telegraph office. And somebody who's very skilled with their hands, or specifically with their finger, would do something that caused that piece of paper to be regenerated hundreds of miles away. Then that piece of paper got delivered. So message went from point A to point B.

So telegraphy came long before Marconi. And it was a revolution in how you do communications. But it was point to point-- one person sent one message to one person.

Sarnoff got his start in newspaper business. He was a Russian immigrant, impoverished, and had a newspaper route as a kid. And he was ambitious. Newspapers are broadcast.

The idea in point to point and broadcast are very different. In broadcast, you're allowed to spend a fortune on the transmitter-- the printing press-- but not on the thing that the individuals get. The individuals get newsprint. So the paper and ink have to be cheap. The printing press doesn't. So that was Sarnoff's background.

So he was interested as a kid in broadcast, in newspaper. But then he got his reputation working for Marconi in wireless telegraphy. Marconi, the inventor of radio, thought of a way of doing point-to-point telegraphy wirelessly via radio-- radio telegraphy. And he sold it to ships.

And Sarnoff made his reputation, because he was the guy operating the radio telegraphy system at that Marconi company when the Titanic sank. Everybody know about the Titanic, right? Big ship sank. So Sarnoff was known as an amazing telegraphy operator. And he stayed at the station for 72 consecutive hours getting emergency messages from the Titanic, telling everybody everything that he could, trying to tell them the situation, whose family was in good shape, whose family was not in good shape.

It had an enormous impact-- big enough that Congress made a law saying that every ship had to have wireless telegraphy. Now, that made Marconi extremely rich. And it indirectly made Sarnoff extremely rich too.

Sarnoff then got very interested in extending the idea of radio, which then was point to point to broadcast. So the idea was to somehow-- he called it a radio music box. Somehow, it was supposed to be like a newspaper but at the speed of light. So the idea was to somehow make mass consumption of radio. At the time, radio was per ship, point to point. So you have the land-based station talking to ship A, or the land-based station talking to ship B, or ship A talking to ship B, but it was all point to point.

Sarnoff's idea was, let's make a newspaper out of this. The key to doing that was making a cheap receiver. It's like newsprint. The printer can cost a mint. The transmitter for radio broadcast music is allowed to cost a mint, but the receivers are not allowed to cost a mint. That's like the newsprint.

So the trick was to make an inexpensive receiver. The problem with making an expensive receiver is that the scheme that we just talked about, where you decode the signal by multiplying by cos omega c t-- omega c chosen to be the frequency that you want to listen to--

the problem with that-- that's called synchronous demodulation-- the problem with that is that you've got to be exactly synchronized. If you want to listen to the message at omega 2, omega c must equal omega 2, not omega 2 plus 3.

So if you want to listen to a particular frequency, to a particular message, you had to have the frequency chosen to match the carrier of the message you wanted to listen to. Today, that's not so hard. The way we would make frequencies today is with crystal.

Crystals are great because the frequencies are determined by the distances in crystal lattices, which are determined by nanomechanical processes very precisely. And in fact, we can make crystals with no problem with frequency resolutions of 10 to the minus seventh, even 10 to the minus eighth. So the errors are very small compared to the frequencies that we're trying to generate.

Even that wouldn't be good enough. So back then-- so Sarnoff was working back in the 1800s-- back then, they couldn't possibly do 10 to the minus eighth precision. They were doing something more like 10 to the minus second precision. They didn't have a technology based on crystals.

So it would have been impossible to match even to within a factor of 10 to the minus third. But even if they could have matched to within 10 to the minus eighth, which we could today, even that wouldn't work, because not only does the frequency have to match, the phase has to match. So if you're multiplying by cos omega c t and you want omega c to be omega 2, it better actually be the cosine of omega 2 and not the sine of omega 2. Sine and cos differ by a phase.

So the question is, what's the effect of phase when you're trying to demodulate a signal? So look at your neighbor. What would happen if you tried to demodulate by precisely the right frequency but you were slipped by phase?

[INTERPOSING VOICES]

AUDIENCE: [INAUDIBLE] over here and be much smaller at the end.

[INTERPOSING VOICES]

AUDIENCE: And then it's to the point [INAUDIBLE]. [INAUDIBLE]

[INTERPOSING VOICES]

AUDIENCE: And then if you [INAUDIBLE]

[INTERPOSING VOICES]

DENNIS FREEMAN: So what would happen if there is a shift between this carrier that was modulating the signal and the carrier that is demodulating the signal? What would happen if there's a phase shift of phi between those two? What's the effect of phi not being 0?

Ideally, phi would be 0. Ideally, phi would be 0. What would happen if phi were not 0? You did all that talking, and you don't have-- Yes.

AUDIENCE: So the low pass filter of the two will not work. It would be-- It will be lots more than what you want it to be, like 1/2 the original.

DENNIS FREEMAN: So that's exactly right. So the effect of phi-- if you think about-- so now, you have cosine of some omega c t and cos omega c t plus phi. So you have cos A cos B. so that gives you the cos of the difference and the cos of the sum. The cos of the difference was supposed to be-- the difference was supposed to be 0. The cos of the difference would be the cos of zero, which would be 1, and that's where the 1/2 came from.

But now, the difference isn't 0 anymore. So instead of getting 1/2 cos of 0, we get 1/2 cos of phi, which means that if phi is a constant, you just get the wrong amplitude. But if phi is a slowly varying signal, which it would be, even if you had a frequency reliability of one part in 10 to the minus seventh-- one way of thinking about that would be that there's a slowly varying phase-- the effect of the slowly varying phase would be to make the amplitude vary with time. So we call that fading. And it would be a very distracting thing to have happen.

So this kind of a technology would even be difficult today, when we can match frequencies very well. It was completely out of the question back in the 1800s, when they couldn't match frequencies that well. So the trick was to not only send the message but also send the carrier.

So ideally, when we first talked about modulation, there is no C path. All you do is you take the signal, and you modulate it by a carrier, and you send that out on the antenna. Now, instead, add on a little bit of the carrier.

That way what's in the air is the carrier and the message. So now when you receive it, somehow if you can receive the carrier, you can use the carrier to tell you information about not only the frequency but also the phase of the carrier. And you can use that then to demodulate the message. That's the idea.

Notice that adding in a little bit C of the carrier is precisely the same as adding a constant to the message before you modulate-- mathematically identical. And that gives an easy way of thinking about the effect of this carrier. If you think about the message added to C and if C is big enough, you can make the message positive only.

You remember in the previous illustration, every time the message went through 0, which might happen here, the modulating message went through 0 also, which means that sign changes were affected by a 180-degree phase shifts of the carrier, which was kind of a subtle thing. So now, the message appears entirely as the positive envelope of the carrier. Well, that's nice, because that makes it very easy to decode in a way that has no dependence whatever on the carrier frequency.

If the carrier frequency is big enough-- if the frequency of the carrier is sufficiently larger than the maximum frequency of the message, there's a trivial way to decode such a system with a nonlinear circuit of this type. What's intended here is that you take the message that's received from the antenna, reconstruct y, which is intended to be the output message, such that if z, the signal on the antenna, exceeds the current value of the message, the diode comes on. And that makes the blue line, the decoded signal, rapidly go back up to the red line-- the thing that's coming off the antenna.

But if the antenna signal shrinks below the blue line, let the blue line discharge, because there is an RC decay constant. So there's a fast attack through the diode, so that the blue quickly goes to the peak value of the red and slowly decays back toward 0. The result is that if the difference in frequency between the carrier frequency and the message frequencies is sufficiently large, you can effectively separate the blue from the red with a very simple circuit. And that's the way they do it-- or that's the way they did it in the early 1900s.

But there's still a problem with that. The problem is that the messages-- audio of the type that I'm speaking, speech-- is characterized by an enormous peak-to-average ratio. The strongest pressures that are generated by speech are enormously more powerful than the average pressure that's generated in speech. You can see that in this diagram by these peaks.

There are several things that generate peaks. Peaks are generated at about 60 or 70 hertz by my vocal chords. But they're also generated by my lips in plosives. When I do plosive, there's a sudden jump in the instantaneous frequency that's not there on average.

And for normal speech, that ratio can be as high as 35 to 1. 35 to 1, not a big deal. The problem is that power goes like the square of voltage. So 35 to 1 becomes 1,000 to 1. It takes 1,000 times more energy to code the peaks than it does to code the average.

And the problem with that is that in this coding scheme that we talked about, you have to add a constant that is big enough so that the signal never goes negative. So the constant that you add has to go in proportion to the peak value. So you end up transmitting almost all of your power By the ratio of 1,000 to 1, that's the amount of power that gets used to transmit the carrier compared to the message.

Well, that's a terrible scheme, if what we were trying to do is point to point. Imagine your cell phone, if you had to transmit enough power to, in the worst case, do the peaks, you would on average be transmitting power at 1,000 times the rate that you would necessarily have to. So that's OK for broadcast.

So for example, WBZ broadcast radio, WBZ uses a 50-kilowatt transmitter. 50 kilowatts is the amount of power that would otherwise be sufficient to generate 500 100-watt light bulbs. That's a fair amount of power. Imagine the heat that comes off 500 100-watt light bulbs. That's how much power is being radiated by the antenna for WBZ.

That power is not necessary to transmit the average message. It's necessary to transmit the peak message. You can imagine how long your cell phone battery would last if you were transmitting 50 kilowatts. That doesn't work.

So that's how the broadcast radio takes advantage of broadcast. It makes no sense to use this coding scheme for a point-to-point system. It's fine if what you're trying to do is have one transmitter, WBZ, that services a million listeners. That's fine.

The problem with this scheme for decoding is it still doesn't separate different channels. And the way to fix that was developed by Edwin Armstrong. So Sarnoff who was kind of the visionary. He had the idea for broadcast radio. He's the entrepreneurial type, who thought of how to do this.

Armstrong was the technical genius. He knew how to do it. So Armstrong's idea here, which we call superheterodyne, was let's make the signal look like it always comes from omega i regardless of what channel it comes from. So omega i, the intermediate frequency, will always take whatever frequency you're interested in and turn it into omega i and will do that by just modulating.

And the cleverness had to do with a lot of technical details. He worked out a scheme where this modulation was very easy. The cutoff-- the sidebands on the bandpass filters didn't have to be very steep, which made them easy to implement. The sharp band pass filters were all at that one intermediate frequency.

So he had to generate one very sharp band pass filter, but that same sharp bandpass filter could then be used for all the different channels. So the idea then was use a coarse tunable filter to map the frequency of interest to omega i, put that through a very sharp filter, of which there is exactly one in each receiver, and then use this decoding scheme to demodulate the carrier, wi.

That's how they did it. We would never do it that way. That's part of the theme of the course. We are interested in schemes that let us map continuous time to discrete time, that sort of thing.

So one way we might do it is implement a radio digitally. So the idea would be, what if you took the antenna, put it through a sampler, turned the radio signal, which contains a gazillion number of bands-- for commercial radio, there's 100 channels in the frequency band 500 to 600 kilohertz-- but just take the whole signal off of the antenna, turn it into a bunch of bits, run that through some digital logic that does, by magic, picks out the one that you're interested in, generates a new stream of bits, yd, from which you can do bandlimited reconstruction.

So this is the last two lectures-- do you do this, how you do this. Now, all we do is we put a particular algorithm in there. And we've got a radio. That's the idea.

So the key to being able to do that is whether or not you can build that sampler. So what would be the required sampling time in order to make a digital radio? And since I'm running out of time, I'll just tell you that the important thing it's sampling-- it's what we did last time-- the answer is you need T, so that the sampling frequency is at least twice the maximum frequency of the thing that you're trying to code.

So the biggest frequency here is 1,600 kilohertz. You need to sample that with omega sampling more than twice that frequency-- so bigger than 2 pi 1,600 kilohertz. And if you work that out, that leaves sampling time of about 1/3 of a microsecond. And the point is that that's easy to do these days. That's the kind of part that you get from DigiKey for $2. So that's easy.

So the only thing that you need to do is worry about, well, then how much computation is there? And that also turns out to be easy. The principal thing you need to do is make a bandpass filter. The question is, how would you make a bandpass filter?

And here are three possible systems for making a bandpass filter. Should I take my digitized antenna signal modulate low pass modulate, or modulate low pass modulate, modulate low pass modulate, cosine, sine, cosine, sine, or put it through a filter that looks just like that unit sample response, except that it's multiplied by cos omega c t n?

Some number of those work. And I'll leave it for you to figure out which of those work. Good to see you. Have a good day.