6.003: Signals and Systems

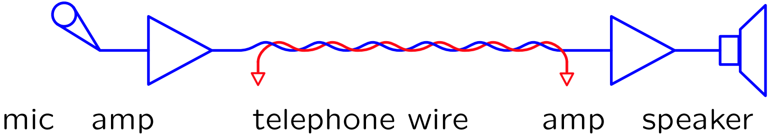
Modulation

December 1, 2011

Modulation

Applications of signals and systems in communication systems.

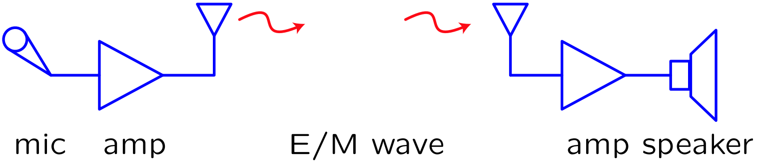
Example: Transmit voice via telephone wires (copper)



Works well: basis of local land-based telephones.

Wireless Communication

In cellular communication systems, signals are transmitted via electromagnetic (E/M) waves.



For efficient transmission and reception, antenna length should be on the order of the wavelength.

Telephone-quality speech contains frequencies from 200 to 3000 Hz.

How long should the antenna be?

Check Yourself

For efficient transmission and reception, the antenna length should be on the order of the wavelength.

Telephone-quality speech contains frequencies between 200 Hz and 3000 Hz.

How long should the antenna be?

1. < 1 mm
2. $\sim$ cm
3. $\sim$ m
4. $\sim$ km
5. > 100 km

Check Yourself

Wavelength is $\lambda = c/f$ so the lowest frequencies (200 Hz) produce the longest wavelengths

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8\,\text{m/s}}{200\,\text{Hz}} = 1.5 \times 10^6\,\text{m} = 1500\,\text{km}\,.$$

and the highest frequencies (3000 Hz) produce the shortest wavelengths

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8\,\text{m/s}}{3000\,\text{Hz}} = 10^5\,\text{m} = 100\,\text{km}\,.$$

On the order of hundreds of miles!

Check Yourself

For efficient transmission and reception, the antenna length should be on the order of the wavelength.

Telephone-quality speech contains frequencies between 200 Hz and 3000 Hz.

How long should the antenna be? 5

1. < 1 mm
2. $\sim$ cm
3. $\sim$ m
4. $\sim$ km
5. > 100 km

Check Yourself

What frequency E/M wave is well matched to an antenna with a length of 10 cm (about 4 inches)?

1. < 100 kHz
2. 1 MHz
3. 10 MHz
4. 100 MHz
5. > 1 GHz

Check Yourself

A wavelength of $10\,\text{cm}$ corresponds to a frequency of

$$f = \frac{c}{\lambda} \sim \frac{3 \times 10^8\,\text{m/s}}{10\,\text{cm}} \approx 3\,\text{GHz}\,.$$

Modern cell phones use frequencies near 2 GHz.

Check Yourself

What frequency E/M wave is well matched to an antenna with a length of 10 cm (about 4 inches)? 5

1. < 100 kHz
2. 1 MHz
3. 10 MHz
4. 100 MHz
5. > 1 GHz

Wireless Communication

Speech is not well matched to the wireless medium.

Many applications require the use of signals that are not well matched to the required media.

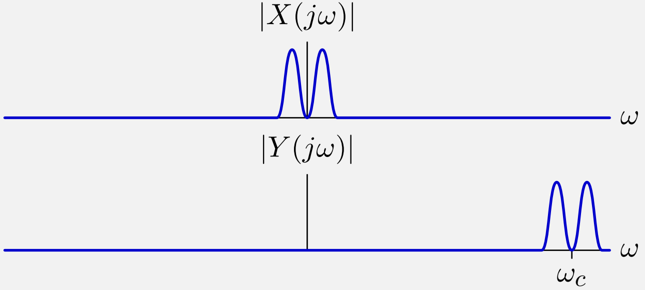
signal	applications
audio	telephone, radio, phonograph, CD, cell phone, MP3
video	television, cinema, HDTV, DVD
internet	coax, twisted pair, cable TV, DSL, optical fiber, E/M

We can often modify the signals to obtain a better match.

Today we will introduce simple matching strategies based on **modulation**.

Check Yourself

Construct a signal Y that codes the audio frequency information in X using frequency components near 2 GHz.



Determine an expression for Y in terms of X.

1. $y(t) = x(t)\,e^{j\omega_c t}$
2. $y(t) = x(t) * e^{j\omega_c t}$
3. $y(t) = x(t)\cos(\omega_c t)$
4. $y(t) = x(t) * \cos(\omega_c t)$
5. none of the above

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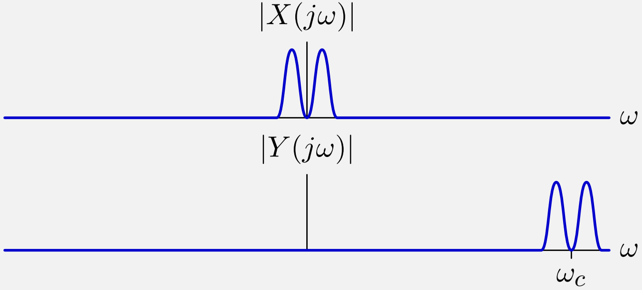

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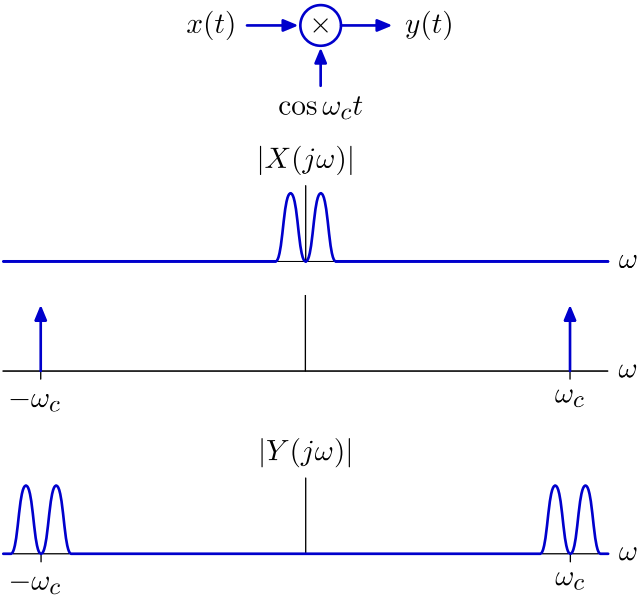
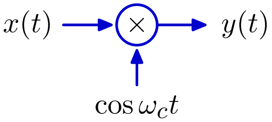
Amplitude Modulation

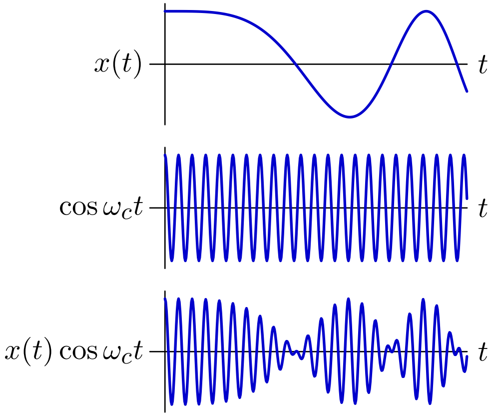
Multiplying a signal by a sinusoidal **carrier** signal is called amplitude modulation (AM). AM shifts the frequency components of X by $\pm\omega_c$.



Amplitude Modulation

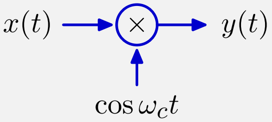
Multiplying a signal by a sinusoidal **carrier** signal is called amplitude modulation. The signal "modulates" the amplitude of the carrier.





Amplitude Modulation

How could you recover $x(t)$ from $y(t)$?



Synchronous Demodulation

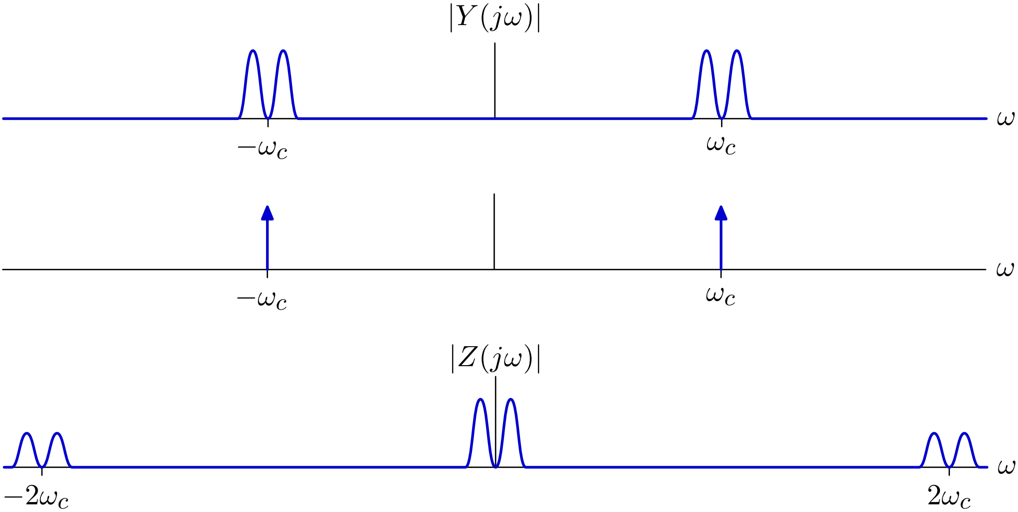
X can be recovered by multiplying by the carrier and then low-pass filtering. This process is called **synchronous demodulation**.

$$y(t) = x(t)\cos\omega_c t$$

$$z(t) = y(t)\cos\omega_c t = x(t) \times \cos\omega_c t \times \cos\omega_c t = x(t)\left(\frac{1}{2} + \frac{1}{2}\cos(2\omega_c t)\right)$$

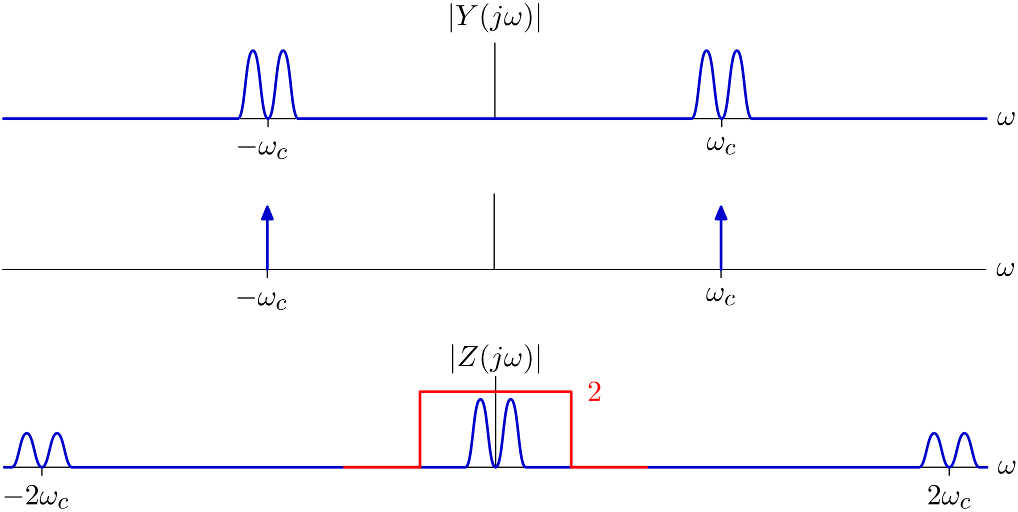
Synchronous Demodulation

Synchronous demodulation: convolution in frequency.



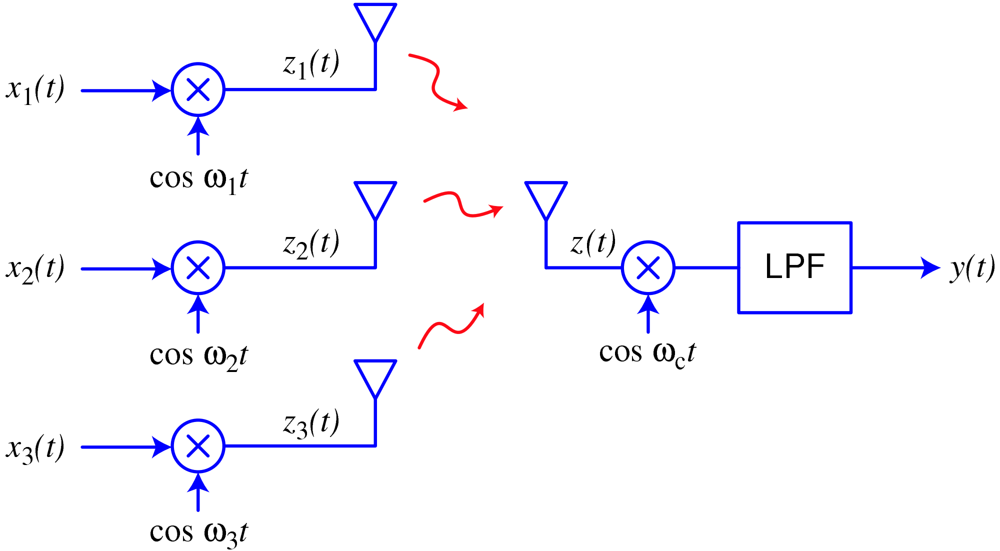
Synchronous Demodulation

We can recover X by low-pass filtering.



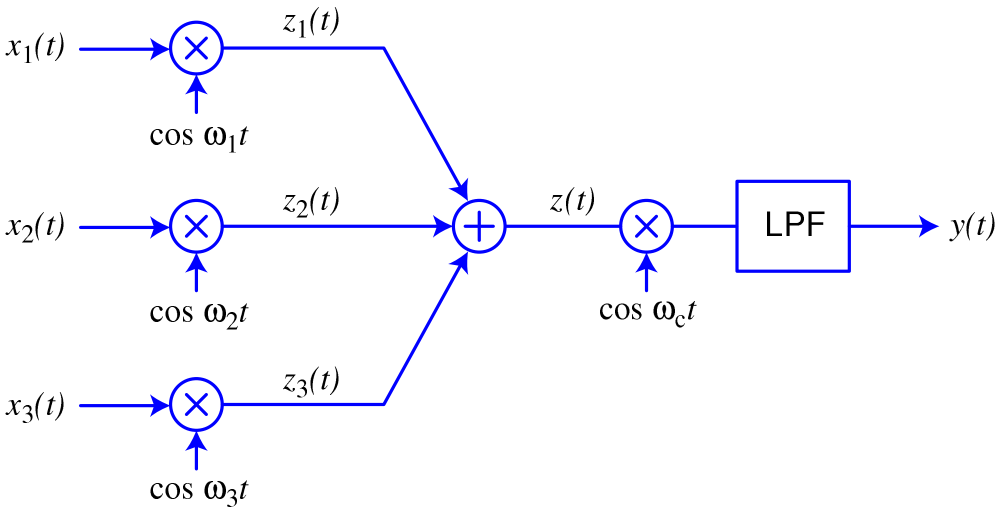
Frequency-Division Multiplexing

Multiple transmitters can co-exist, as long as the frequencies that they transmit do not overlap.



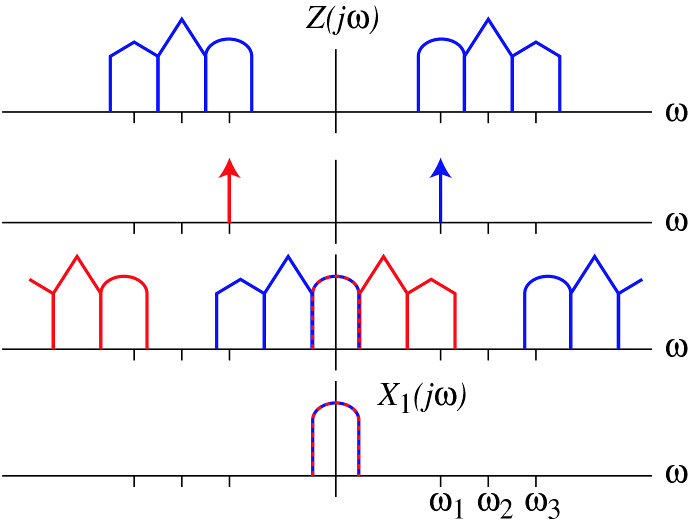
Frequency-Division Multiplexing

Multiple transmitters simply sum (to first order).



Frequency-Division Multiplexing

The receiver can select the transmitter of interest by choosing the corresponding demodulation frequency.



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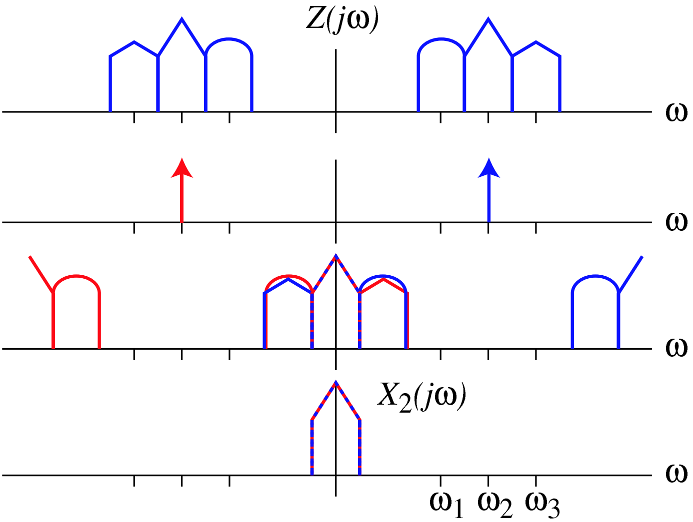

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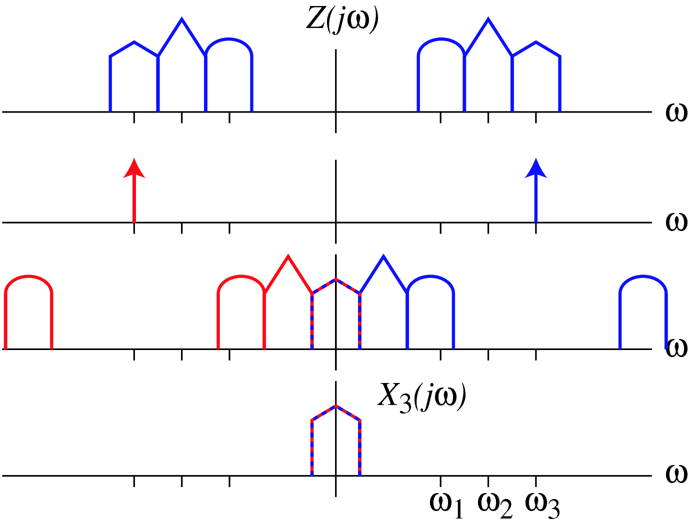


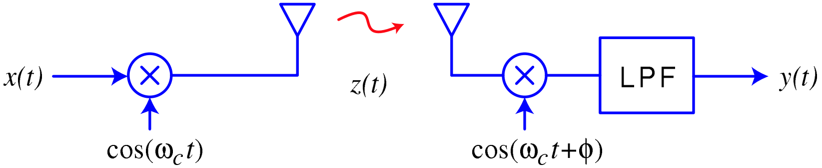
Broadcast Radio

"Broadcast" radio was championed by David Sarnoff, who previously worked at Marconi Wireless Telegraphy Company (point-to-point).

- envisioned "radio music boxes"
- analogous to newspaper, but at speed of light
- receiver must be cheap (as with newsprint)
- transmitter can be expensive (as with printing press)

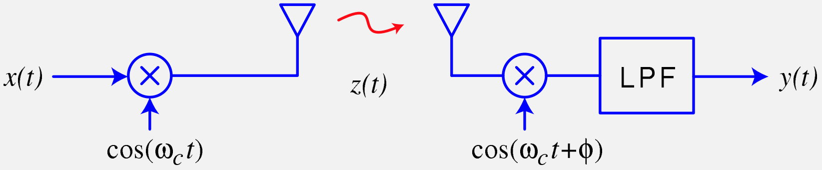
Inexpensive Radio Receiver

The problem with making an inexpensive radio receiver is that you must know the carrier signal exactly!



Check Yourself

The problem with making an inexpensive radio receiver is that you must know the carrier signal exactly!



What happens if there is a phase shift ϕ between the signal used to modulate and that used to demodulate?

Check Yourself

$$y(t) = x(t) \times \cos(\omega_c t) \times \cos(\omega_c t + \phi)$$
$$= x(t) \times \left(\frac{1}{2}\cos\phi + \frac{1}{2}\cos(2\omega_c t + \phi)\right)$$

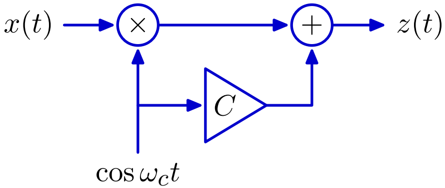
Passing $y(t)$ through a low pass filter yields $\frac{1}{2}x(t)\cos\phi$.

If $\phi = \pi/2$, the output is zero!

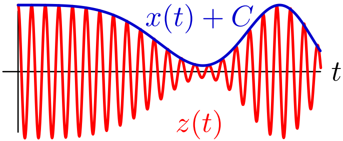
If ϕ changes with time, then the signal "fades."

AM with Carrier

One way to synchronize the sender and receiver is to send the carrier along with the message.



$$z(t) = x(t)\cos\omega_c t + C\cos\omega_c t = (x(t) + C)\cos\omega_c t$$

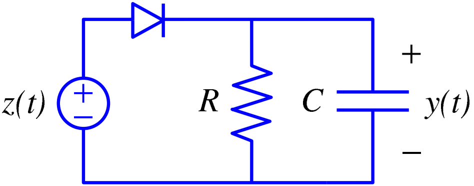


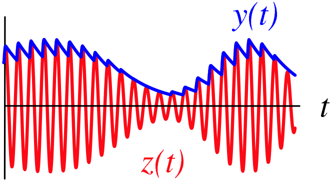
Adding carrier is equivalent to shifting the DC value of $x(t)$.

If we shift the DC value sufficiently, the message is easy to decode: it is just the envelope (minus the DC shift).

Inexpensive Radio Receiver

If the carrier frequency is much greater than the highest frequency in the message, AM with carrier can be demodulated with a peak detector.





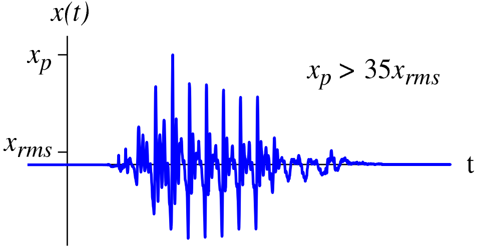
In AM radio, the highest frequency in the message is 5 kHz and the carrier frequency is between 500 kHz and 1500 kHz.

This circuit is simple and inexpensive.

But there is a problem.

Inexpensive Radio Receiver

AM with carrier requires more power to transmit the carrier than to transmit the message!



Speech sounds have high crest factors (peak value divided by rms value). The DC offset C must be larger than x_p for simple envelope detection to work.

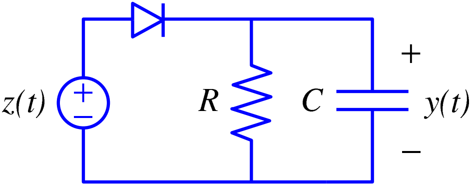
The power needed to transmit the carrier can be $35^2 \approx 1000\times$ that needed to transmit the message.

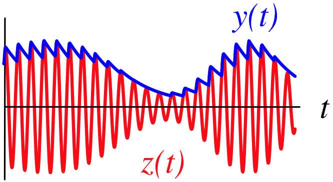
Okay for broadcast radio (WBZ: 50 kwatts).

Not for point-to-point (cell phone batteries wouldn't last long!).

Inexpensive Radio Receiver

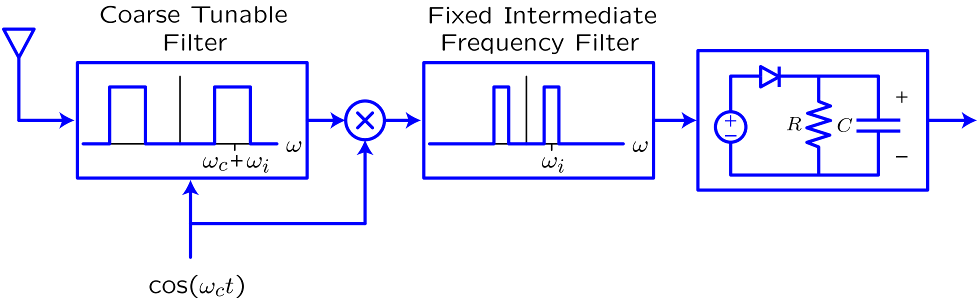
Envelope detection also cannot separate multiple senders.





Superheterodyne Receiver

Edwin Howard Armstrong invented the superheterodyne receiver, which made broadcast AM practical.



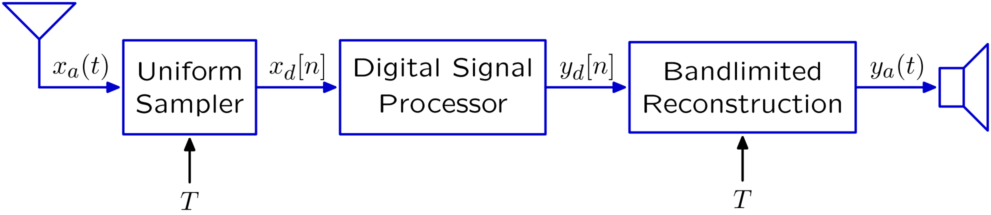
Edwin Howard Armstrong also invented and patented the "regenerative" (positive feedback) circuit for amplifying radio signals (while he was a junior at Columbia University). He also invented wide-band FM.

Digital Radio

Could we implement a radio with digital electronics?

Commercial AM radio

- 106 channels
- each channel is allocated 10 kHz bandwidth
- center frequencies from 540 to 1600 kHz



Check Yourself

Determine T to decode commercial AM radio.

Commercial AM radio

- 106 channels
- each channel is allocated 10 kHz bandwidth
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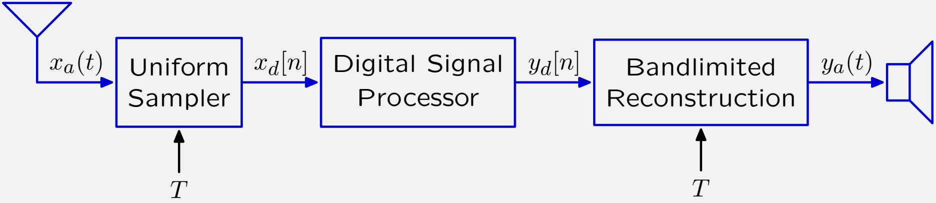

The maximum value of T is approximately

1. 0.3 fs
2. 0.3 ns
3. 0.3μs
4. 0.3 ms
5. none of these

Check Yourself

Determine T to decode commercial AM radio. 3.

Commercial AM radio

- 106 channels
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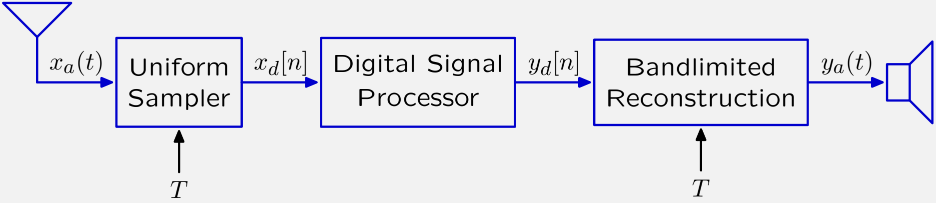

The maximum value of T is approximately

1. 0.3 fs
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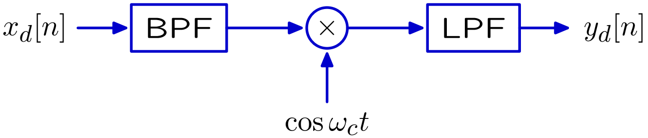
5. none of these

Digital Radio

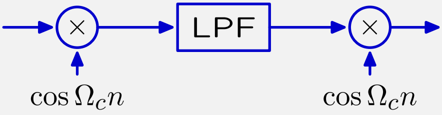
The digital electronics must implement a bandpass filter, multiplication by $\cos\omega_c t$, and a lowpass filter.



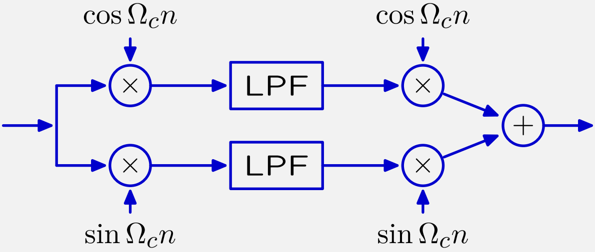
Check Yourself

Which of following systems implement a bandpass filter?

System 1



System 2



System 3

Check Yourself

$$h[n] = h_{LPF}[n]\ \cos\Omega_c n$$

$$\begin{aligned}
y[n] &= x[n] * \left(h_{LPF}[n]\ \cos\Omega_c n\right) \\
&= \sum_k x[k] h_{LPF}[n-k]\ \cos\Omega_c(n-k) \\
&= \sum_k x[k] h_{LPF}[n-k]\ (\cos\Omega_c n \cos\Omega_c k + \sin\Omega_c n \sin\Omega_c k) \\
&= \left(\sum_k x[k] \cos\Omega_c k\, h_{LPF}[n-k]\right) \cos\Omega_c n \\
&\quad + \left(\sum_k x[k] \sin\Omega_c k\, h_{LPF}[n-k]\right) \sin\Omega_c n \\
&= \Big((x[n]\cos\Omega_c n) * h_{LPF}[n]\Big)\ \cos\Omega_c n \\
&\quad + \Big((x[n]\sin\Omega_c n) * h_{LPF}[n]\Big)\ \sin\Omega_c n
\end{aligned}$$

Check Yourself

Which of following systems implement a bandpass filter?

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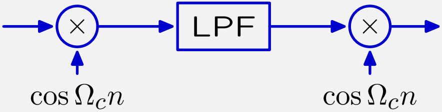
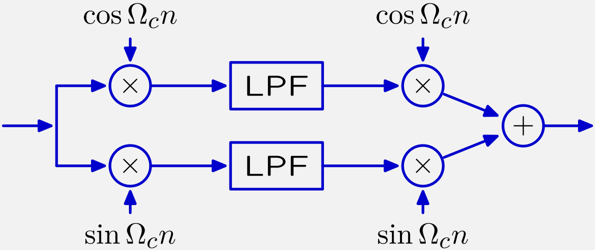


System 2



System 3

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Fall 2011