All right. Let's get moving.

Good morning.

Today, if everything works out, we have some fun for you guys.

I hope it works out. We'll see.

What I am going to do today is a very major application of the frequency response and the frequency domain analysis of circuits. And this application area is called filters. The area of filters often times demands a full course or a couple of full courses all by itself. And filters are incredibly useful. They're used in virtually every electronic device in some form or another.

They're used in radio tuners. We will show you a demo of that today. They're also used in your cell phones. Every single cell phone has a set of filters. So, for example, how do you pick a conversation? You pick a conversation by picking a certain frequency and grabbing data from there.

They are also in wide area network wireless transmitters.

Do we have an access point here?

I don't see one, but you've seen wireless access points. Again, there they have filters in them. So, virtually every single electronic device contains a filter at some point or another.

And so, today we will look at this major, major application of frequency domain analysis. Before we get into that, I'd like to do a bit of review. The readings for today correspond to Chapter 14.4.2, 14.5 and 15.2 in the course notes. All right.

Let's start with the review. We looked at this circuit last Friday -- -- where I said that for our analysis, we are going to focus on this small, small region of the playground.

And what's special about this region of our playground is that I am going to focus on sinusoidal inputs.

And, second, I am going to focus on the steady state response. How does the response look like if I wait a long, long time?

And then we said that the full blown time domain analysis was hard. This was, remember, the agonizing approach? And then I taught you the impedance approach in the last lecture, which was blindingly simple. And, in that impedance approach, what we said we would do is -- I will apply the approach right now and in seconds derive the result for you. But the basic idea was we said what we are going to do is assume that we are going to apply inputs of the form Vi e to the j omega t.

Wherever you see a capital and a small, there is an implicate e to the j omega t next to it. I'm not showing you that.

And what I showed last time, and the class before that was once you find out the amplitude -- Once you find out the multiplier that multiplies e to the j omega t, it's a complex number, you have all the information you need.

And once you have this, you can find out the time domain response by simply taking the modulus of that, or the amplitude and the phase of that to get the angle.

And that gives you the time domain response.

So, our focus has been on these quantities.

The impedance method says what I am going to do is replace each of these by impedances. And then the corresponding impedance model looks like this.

Instead of R, I replace that with ZR.

And instead of the capacitor, I am going to replace that with ZC. And this is my Vc.

ZR is simply R and ZC was going to be one divided by sC where s was simply a shorthand notation for j omega.

Based on this, once I converted all my elements into impedances, I can go ahead and apply all the good-old linear analysis techniques.

I will discuss a bunch of them today.

As an example, I could analyze this using my simple voltage divider relationship.

Vc is simply ZC divided by ZC plus ZR times Vi.

And that, in turn, is, well, let's say I divide this by Vi so I can get the response relation, is ZC divided by ZC plus ZR. And ZC I know to be one by j omega C, plus R. And multiplying throughout by j omega C, I get one divided by one plus j omega CR.

It's incredibly simple. This is simply called the frequency response. And it's a transfer function representing the relationship between the output complex amplitude with the input. We can also plot this.

Notice that in our entire analysis we have not only assumed sinusoidal input, but we're also saying that let us look at this only in the steady state.

So, we will wait for time to be really, really large, and then look at the response. And so, therefore, we will plot the response not as a function of time, but rather we are going to plot the response as a function of omega. What we are going to say is I am going to input a sinusoid and my output is going to be some other sinusoid. And since I'm waiting for a long time to look at the output, time doesn't make sense anymore. Rather, my free variable is going to be my frequency, so I am going to change the frequency of the input that I apply.

And so, I am going to plot this as a function of omega.

This represents a completely complimentary view of circuits, the time domain view and then there is a frequency domain view. The frequency domain view says how did this circuit behave as I apply sinusoids of differing frequencies? I can plot that relationship in a graph like this, and this relationship is simply given by a parameter edge the transfer function, it's a function of omega. And I can also plot the absolute value of that. And let's take a look at what it looks like. So, I can look at functions like this and very quickly plot the response.

I am going to do a whole bunch of plots just by staring at circuits and staring at expressions like this.

And you will see a number of them today.

First of all, the way you plot these is look for the values where omega is very small and when omega is very large. When omega is very, very small this term goes away. And so, for very small values of omega the output is simply one.

Vc by Vi is simply one. This part goes away.

What happens when omega is very, very large?

When omega is really large, this part dominates, is much greater than one. If I ignore one in relation to this guy and take the absolute value of that then I simply get one divided by omega CR when omega is very large.

So, when omega is very large, I get a decay of the form one over omega CR. I know the value for small omega, and it looks like this for very large omega.

And, if you plot it out, this is how it's going to look like. Let's stare at this form for a little while longer. And let's plot some properties off it. First of all, you notice something else. When omega CR equals one then, in other words, when omega equals one by RC, notice that the output is given by one plus j.

And the absolute value of that is simply one divided the square root of two. So, in other words, when omega is one by RC -- When omega is one by CR then the output is one by square root two times its value when omega is very, very small. So, that is one little piece of information. If you look at the form of this, I would like you to stare at it for a few minutes and try to understand what this represents.

This says that for very low frequencies the response is virtually the same as the input in amplitude.

In other words, if I apply some very low frequency sinusoid of some amplitude then the output amplitude is going to be same as that amplitude.

And that's a one. Now, it also says when I apply a very high frequency, at very high frequencies it decays. So, this graph which says I am going to pass low frequencies without any attenuation, without hammering it, but I am going to clobber high frequencies and give you a very low amplitude signal at the output but pass through, almost without attenuation, the input at low frequencies. And so this is an example of what is called a low pass filter or LPF.

What this is saying is that this little circuit here acts like a low pass filter. It's a low pass filter because it passes low frequencies without attenuation but kills high frequencies. If I take some music, and you will do experiments with this in lab.

When is lab three? People are doing lab three right now, right? Lab three is going on right now and early next week as well. And, in lab three, you will play with looking at the response to music of different types of filters. If apply some music here, you will see that the output will pass low frequencies but really attenuate high frequencies.

You will hear a lot of the low sounding base and so on but attenuate a lot of the high frequencies.

All right. The other thing that I encourage you to do is Websim has built in pages for a large number of such circuits. You can go in there and play with the values of RC, or L for that matter, for a variety of circuits. And, if you click on frequency response, you actually get both the amplitude response and the phase as well. You can play with various values of RLC and see how the frequency response looks like for each of the circuits. As a next step, what I would like to do is just give you a sense of how impedances combine. This won't be very surprising given that they behave just like resistors, but it's good to go through it nonetheless. Suppose, just to build some insight, suppose I had two resistors in series.

All right. R1 and R2.

And this was my A and B terminals respectively.

And let's say the complex amplitude of the voltage was Vab across this. Then I could relate, let's say Iab was the current, I can relate these resistances.

Or, I could relate Vab and Iab as follows.

Simply Vab divided by Iab equals R1 plus R2.

I know that. And the same thing applies to R viewed as an impedance. It's still impedance R, and so this one still goes ahead and applies.

The second thing I can try is the circuit of this form.

A, B, and I have an R1 and an L in this case.

And what I can do is, in the impedance model, I can view this as an impedance of value j omega L.

And I can also combine them to get the impedance between A and B. Much as I got a resistance between A and B, I can get an impedance between A and B as Vab divided by Iab. And that will be given by ZR1 plus ZL, and that is simply R1 plus j omega L.

Similarly, I can do an even more complicated circuit.

So, resistance. And here I have a capacitor in series with the resistance, and then I apply inductor to it. This is A, B, Iab and plus, minus Vab.

And let me call this R1 and let me call this R2 and this is C and L. I can go about combining these in much the same manner that I combine my resistances in the series parallel simplifications. I can define an impedance Zab between the A and B terminals as ZR1 plus Z of this combination, impedance of this combination, which is simply impedance of C and that of R2 in parallel with each other.

I get Zc in parallel with ZR2. Notice that this notation simply says that look at the impedance of the capacitor in parallel with a resistor. And then, finally, I add to that the series impedance of the inductor ZL.

Exactly as you would have done for resistances, if all of these resistances you would have said R of this piece plus the R of the parallel combination plus the R of whatever was here. This time around we have impedances. And replacing this with the values, this is R1. I know for ZL it's j omega L. If I can arrange to have the

And so, for ZL, parallel ZR2 it is given by ZCZR2 divided by ZC plus ZR2, which is simply R1 here and j omega L. And let me just substitute the values here. I know that ZR2 is simply R2, ZC is one by j omega C, and then one by j omega C plus R2. And I can go ahead and simplify that further and get my impedance Zab.

Notice how simple analysis has become.

Using this technique, using the impedance method we've managed to convert our analysis from solving differential equations to going back to algebra.

A large part of what we do in circuits is see how we can get back to really simple algebra and try to be clever about how we do things. So, this is as far as analysis is concerned. In the next five minutes, I want to give you some insight into how you can build different kinds of impedances.

And I won't go into too much detail but give some insight into how you can get a sense for the kind of filters you want to design. Or, at the very least, given a filter, how can you very quickly get some insight into what kind of filter it is, how it performs, what its frequency response is and so on. And, this time around, this piece of intuition will be in honor of Umans.

And back to our Bend it Like Beckham series, I call this "Unleash it like Umans".

What experts in the field do is they don't go about sitting around writing differential equations, but rather use a lot of insight into how to solve these things.

And so in honor of Umans, I will label this unleash it like Umans. Let's get some insight into how the response of various elements look like.

Let's take, for example, I have some impedance Z.

Let's say this could be a resistor, it could be an inductor or it could be a capacitor.

Let's take a look at what the frequency response of just these elements look like. In other words, what are the frequency dependents of Z itself?

Let me just plot the impedance of each of these elements as a function of frequency. Let me just take the absolute value of their impedance. Notice that it's a complex number. For the inductor it's j omega L. And let me take the absolute value omega L in that case and plot it for you.

And use that to develop some insight.

Let's do a simple case first. If Z is a resistance of value R then no matter what the frequency my value is going to be R. If I have an inductor of value L then the impedance is going to look like j omega L, and so I am going to omega L for that.

And the dependence of that simply says that for low omega the impedance is very small. For omega zero the impedance is zero and it increases linearly with omega.

So, it's omega L for the inductor.

Impedance increases linerally as I increase the frequency.

What about for the capacitor? For the capacitor, the impedance is one divided by j omega C.

And so, therefore, I get the dependence being related to omega C. Which says that for very high frequencies impedance is very low, but for very low frequencies the impedance is very high and I get a behavior pattern that looks something like this.

It goes as one by omega C. As omega is very large, my impedance is very small. If omega is very small, my impedance goes towards that of an open circuit.

This is not surprising. You've known this before, right? That a capacitor behaves like an open circuit for DC. An inductor behaves like a short circuit for DC. Notice that zero frequency here corresponds to DC. The capacitor looks like an open circuit for DC, very high impedance.

The inductor looks like a short circuit for DC, very low impedance. And the opposite is true at very high frequencies. While R is a constant throughout. Let's use this to build some insight into how our circuits might look.

Let me do this example.

Let's say I have a Vi and I measure the response across the resistor.

So, I measure Vr divided by Vi and take the absolute value and take a look at how it's going to look like.

I want you to stare at this for me and help me with what the response is going to look like. Let's take incredibly high frequencies. At very high frequencies, this has a very high frequency, what do the capacitor look like to very high frequencies? Is it an open or is it a short?

A short circuit. At very high frequencies the capacitor looks like a short circuit.

Then Vi simply appears across the resistor, which means that at very high frequencies the output is very close to the input. At very low frequencies what happens? At very low frequencies the capacitor looks like an open circuit.

If this looks like an open circuit then very little voltage will drop across this resistor here because most of it is going to drop across the capacitor. What is going to happen is, for very low values, I am going to be looking at something out here. And, because of that, my response looks like this. And this is of a different form than the one you saw earlier. In this case, I pass high frequencies but attenuate low frequencies.

Not surprisingly, this is called a high pass filter.

You need to begin to be able to think about capacitors and inductors in terms of their high and low frequency properties.

And, if you develop that intuition, once you develop the intuition about capacitors and inductors and their frequency relationship, that will be a big step forward in 002. If you get that insight, you will go a long way in terms of knowing how to tackle problems and being able to quickly sketch responses.

Yes.

In the case of, if we get something like j omega L, what you can do is take the limit as omega goes to zero.

If it is omega L then notice that it is going to start linear. And, on the other hand, if when you get very high frequencies, for example, if you get one by something omega C then this is a hyperbolic relationship, so it is going to go ahead looking like this. So, you can take a look at a lot of these functions at their very low values and see how they look like at that point. All right.

The next one I would like to draw for you is something that looks like this.

Let's say, for example, I have an inductor L and a resistor R and I want to see what that looks like.

In this particular example, I have H, take the absolute value. So, what is this going to look like? I am going to look at the value across the resistor here. Here what I am going to find is that at very low frequencies this guy is a short circuit.

Since this guy is a short circuit, all the voltage drops across the resistor so it's going to look like this.

And, at very high frequencies, what I am going to find is that the inductor is going to appear like an open circuit.

And so, therefore, all the voltage is going to pretty much drop across the inductor.

It will be R divided by something plus omega L.

So, at high frequencies this guy is going to taper off to zero and is going to look like this.

And this is back to my low pass filter.

Just to go back to a question asked earlier, how do you know what this looks like?

I can very quickly write down the expression for H of j omega.

This is simply going to be R divided by R plus if this is VR.

VR is simply R divided by one by j omega C.

I multiply it out by j omega C in the numerator and the denominator. I'm going to find j omega C here and I am going to get one by j omega C here.

And what is going to happen with something like this is that as omega becomes very small then I am going to ignore this.

When omega becomes very small, I can ignore this with respect to one, and I get R j omega C. Given that, is what I've drawn here correct or wrong? This goes away with respect to one. I am left with R j omega C, right? For very low frequencies.

Given what I have drawn here, is that correct or is that wrong? Well, it's hard to say.

For very, very low frequencies it starts out being linear because it's an omega relationship, and then it goes up like this and then goes out there.

Let me go onto another example. Let me do another example here which is something like -- I need to make sure I don't make a mistake here. If I get R j omega C by R j omega C, you know what, this ends up being a first order system, and so is going to look like this. I blew it there.

Back to this system here. If I have an L and an R and I look at this equation to look at what happens across L, you can plot that again. And for very low frequencies it is going to be zero amplitude here and for very high frequencies this is going to be an open circuit, and so the response is going to look something like this.

That's going to end up being your high pass filter.

As another example, I would like to do a series RLC circuit -- -- and try to get you some sense of what that output looks like. Let's use our intuition and first write down what this looks like and then go and do some math and see if the math corresponds to what our intuition tells us. I want to plot Vr with respect to Vi. I want to plot it there.

For something like this, what happens at very low frequencies? We are just looking to get very, very crudely what this graph is going to look like.

Very, very crudely what this graph is going to look like.

Given that I am taking the voltage across VR, what happens at very low frequencies?

At incredibly low frequencies, the inductor looks like a short circuit, but the capacitor looks like open circuit.

An open circuit in series with a short circuit that ends up looking like an open circuit. And so, therefore, all my voltage falls across VR. Now, what happens at very high frequencies? At very high frequencies the capacitor looks like a short. But the inductor looks like an open circuit now for very high frequencies, correct?

Just remember, capacitor is short for high frequencies inductor open for high frequencies.

So, this ends up having a very high impedance.

At very high frequencies this guy has a very high impedance.

And, because of that, for a high value of frequency, I end up going in that manner. This behavior has the effect of the capacitor here. And for very high frequencies I get the effect of the inductor. And so this means that I have very low values for low frequencies, very low values for high frequencies. And, as the frequency increases, I do something like this.

I keep building up, then the inductor begins to play a role, and then I taper off again.

This kind of a filter where I kill low and high frequencies and pass intermediate frequencies is called a band pass filter, BPF. This means that it passes frequencies in some band. Let's get some more insight on this by writing down the equations.

So, Vr divided by Vi is simply R.

Using the impedance relation it is R divided by j omega L plus one divided by j omega C plus R. I am going to use this equation later, so let me stash it away on my stack and put a little notation there. I am going to multiply throughout by j omega C. And what I end up getting is j omega RC divided by one plus R j omega RC, and then here, I get j times j is minus one, so I get minus omega squared.

Let me rewrite it this way. I get minus omega squared.

So, j j is minus one, omega times omega is omega squared, and then I get an LC. That's what I end up getting.

And if I take the absolute value here, I end up getting, back to your complex algebra, the square root of this real value squared plus imaginary value squared.

So, one minus omega squared LC plus omega RC squared.

This is from, you can look it up in your complex algebra appendix in the course notes.

It's simply omega RC here, then square of the real value plus the square of the imaginary value, and take the square root of that. By staring at this, you can notice that you realize a really important property.

When omega equals LC. I'm sorry.

When omega equals one divided by LC, what happens?

Sorry, square root of LC. When omega is one divided by square root of LC then omega squared times LC becomes one.

When this is true then this becomes one, and one and one cancel out. And, not only that, when these cancel out, these two cancel out at that point, so I end up getting a one, which means that when omega equals omega nought equals one by square root of LC and I end up getting a value that is one. It's pretty amazing.

Which means that if I drive this at omega nought, if my sinusoid has a frequency omega nought where omega nought is one by square root of LC, if I'm sitting here and this is a black box on the right-hand side, and I drive this at a frequency omega nought equals one divided by square root of LC, what does this entire circuit look like to me?

I'm sitting there, the black box here.

I'm driving it at omega nought equals one by square root of LC at that frequency. What does that circuit look

like? Yes. It looks like a resistor. It's pretty amazing.

It means that even though I have an L and a C here, if I happen to drive this at omega nought then the circuit looks purely resistive and it seems to give me the same input appearing at the output. In other words, the effect of these two cancels out.

And that aspect is called driving the circuit at its resonance point. Resonance is when you're driving the circuit at omega nought equals one by a square root of LC.

I will very quickly sketch for you a couple of other ways of looking at circuits. Supposing I looked at this value here, Vlc, I looked at the value across the inductor and the capacitor, what will the frequency response look like? I am looking at the voltage across the inductor and the capacitor in series.

Let's see. Let's go back to our usual mantra. Think about Steve Umans when you do this. What would he do?

He would say ah-ha, at very low frequencies the capacitor is going to look like an open circuit.

In my voltage divider, I am measuring the voltage across an open circuit, so the entire Vi must drop across the inductor and capacitor.

Similarly, at very high frequencies the inductor looks like an open circuit now, so it looks like this.

At very high frequencies inductor is an open circuit.

And, again, I'm looking at the voltage divider across the near infinite resistance, impedance, so I get a high value here as well. Well, in the middle the value dips and I get something like this.

So, this thing is called a band stop filter.

Here I can nail any specific frequency, as long as the frequency falls in roughly that regime.

Yet another example.

The reason I'm working on so many examples is that to experts, a large part of what they do is look at a circuit and boom, give a rough form of how it looks like.

That can get you half the way there in most of what you're going to do. How did this look like?

If I take the voltage Vo versus Vi, let's take a look.

At very low frequencies, the inductor looks like a short circuit, correct? I am talking the voltage across a short circuit, so it looks like this.

At very high frequencies, I am taking a voltage across a parallel combination, but the capacitor is now a short circuit. So, that looks like a capacitor. This looks like an inductor out here and this is a capacitor holding sway here.

And so, somewhere in the middle it goes up and comes down like that. So, it's a band pass filter.

What is amazing is that you can take fairly complicated circuits, and just by doing a quick analysis of what happens at very low frequencies, what happens at very high frequencies, you can roughly sketch the response.

And then what you should do, in addition to that, is if it's a second order circuit, just assume that it's going to do something interesting at its resonance frequency, at omega nought equals one by square root of LC.

Something interesting is going to happen.

Check it out. And for circuits that are first order, RC or RL, the important number is the time constant RC. Usually, when you're driving it at one by RC, omega equals one by RC then what happens is that you often times end up getting a value that is one by square root two times the input value in the circuits we looked at here. Next, what I am going to do is talk about a major, major application of filters.

And that is an AM receiver. Let me do Radios 101 for 30 seconds. These guys have an antenna.

You take a ground here. You pick up a signal at your antenna. There is an implied ground as well. And what you do, as a first step, is you begin processing the signal now. What we place right there is a little filter that looks like this.

It is a inductor and a capacitor in parallel.

And this capacitor is really your tuner that you can tune to radio frequencies. And then what you have here is a bunch of other processing and end up with your speaker.

And the processing that happens here is you have a demodulator, you have an amplifier and a bunch of other things that let's not worry about them for now. What we do here is the antenna picks up a signal. So, in some sense, this part of the circuit here is your source.

I could replace it with its Thevenin equivalent as follows.

So, the front end of your radio looks like a Vi, R, L and a C. Where have you seen this before? Right there.

That's the front end of radios. Let me tell you why I need a band pass filter in a radio out here.

The way life works is as follows.

I have my frequency. Let me do this not in radians but in kilohertz for now, and let me plot your radio signal strength. In the Boston area, the signals go between 540 kilohertz and they go all the way to 1600 kilohertz. In some areas we have begun to use the 1700 extra band as well for some new stations.

This is the frequency range of interest.

If you look at your radio tuner, you will see 540 kilohertz all the way up to 1600 and you can tune your AM radio.

The way it works is that each station is given 10 kilohertz of spectrum here. And so, this is at 1000 kilohertz, 1010 kilohertz and so on.

And each station transmits its signal in plus or minus 5 kilohertz around that point. And this station transmits it here and this station transmits it here and so on.

This is 1030. This guy is WBZ News Radio 1030, for those of you who listen to it.

What happens is that at 10 kilohertz, each station gets 10 kilohertz, and so WBZ transmits in the 10 kilohertz around 1030.

Notice that each of these signals transmitted by radio stations happen within small bands.

Now, you will learn a lot more about modulation and how do you get a signal to go in a small band and all that stuff.

You will learn about that in For now, don't worry about how I did all of this.

How do you listen to that station?

The way you listen to that station is you put a low pass filter here. You put a low pass filter that does the following. Let's say I want to hear WBZ If I pass this entire signal through that filter.

And if I arrange to have the omega nought of my filter at omega nought at 1030 then this is the response of my filter.

And I am going to pick out this guy and cut out everything else.

I am just going to get this.

Let's listen to the station for some time.

So, you can see I can tune to the station WBUL.