Lecture: Sampling and Standard Error

Announcements

- Relevant reading: Chapter 17
- No lecture Wednesday of next week!

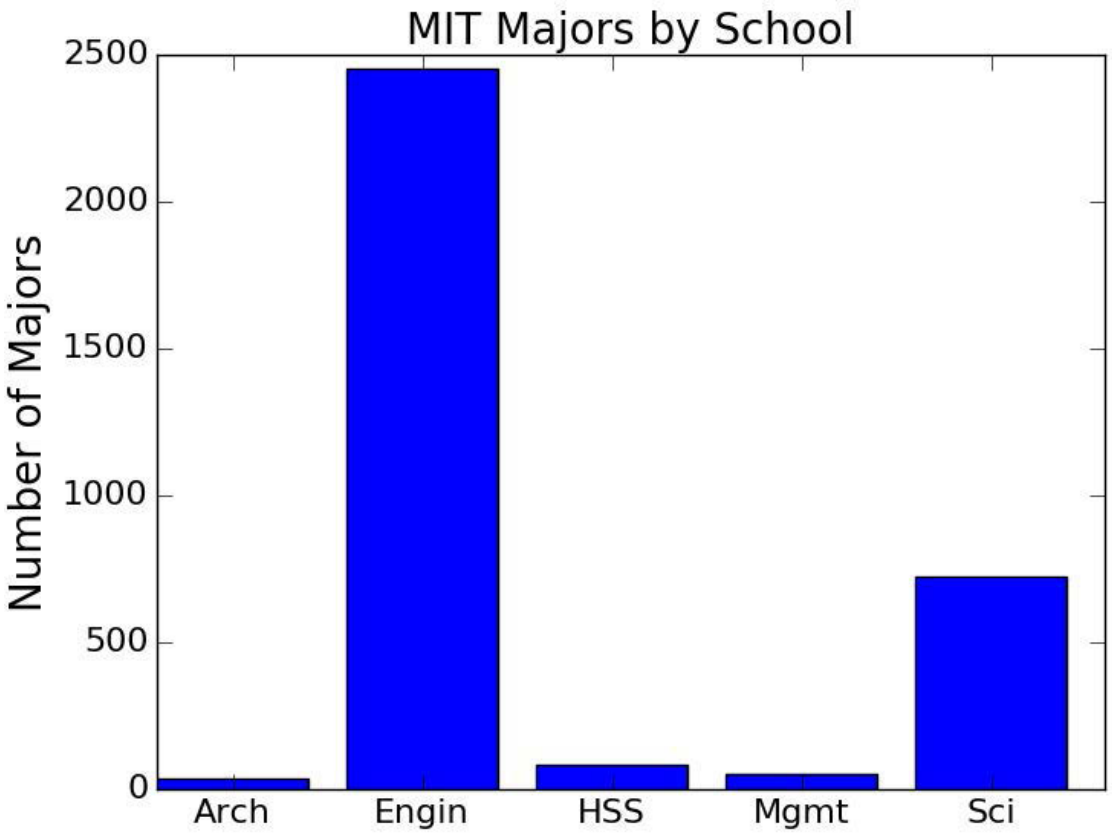
Recall Inferential Statistics

- Inferential statistics: making inferences about a populations by examining one or more random samples drawn from that population
- With Monte Carlo simulation we can generate lots of random samples, and use them to compute confidence intervals
- But suppose we can't create samples by simulation?
  - "According to the most recent poll Clinton leads Trump by 3.2 percentage points in swing states. The registered voter sample is 835 with with a margin of error of plus or minus 4 percentage points." – October 2016

Probability Sampling

- Each member of the population has a nonzero probability of being included in a sample
- Simple random sampling: each member has an equal chance of being chosen
- Not always appropriate
  - Are MIT undergraduates nerds?
  - Consider a random sample of 100 students

Stratified Sampling



- Stratified sampling
  - Partition population into subgroups
  - Take a simple random sample from each subgroup

Stratified Sampling

- When there are small subgroups that should be represented
- When it is important that subgroups be represented proportionally to their size in the population
- Can be used to reduced the needed size of sample
  - Variability of subgroups less than of entire population
- Requires care to do properly
- Well stick to simple random samples

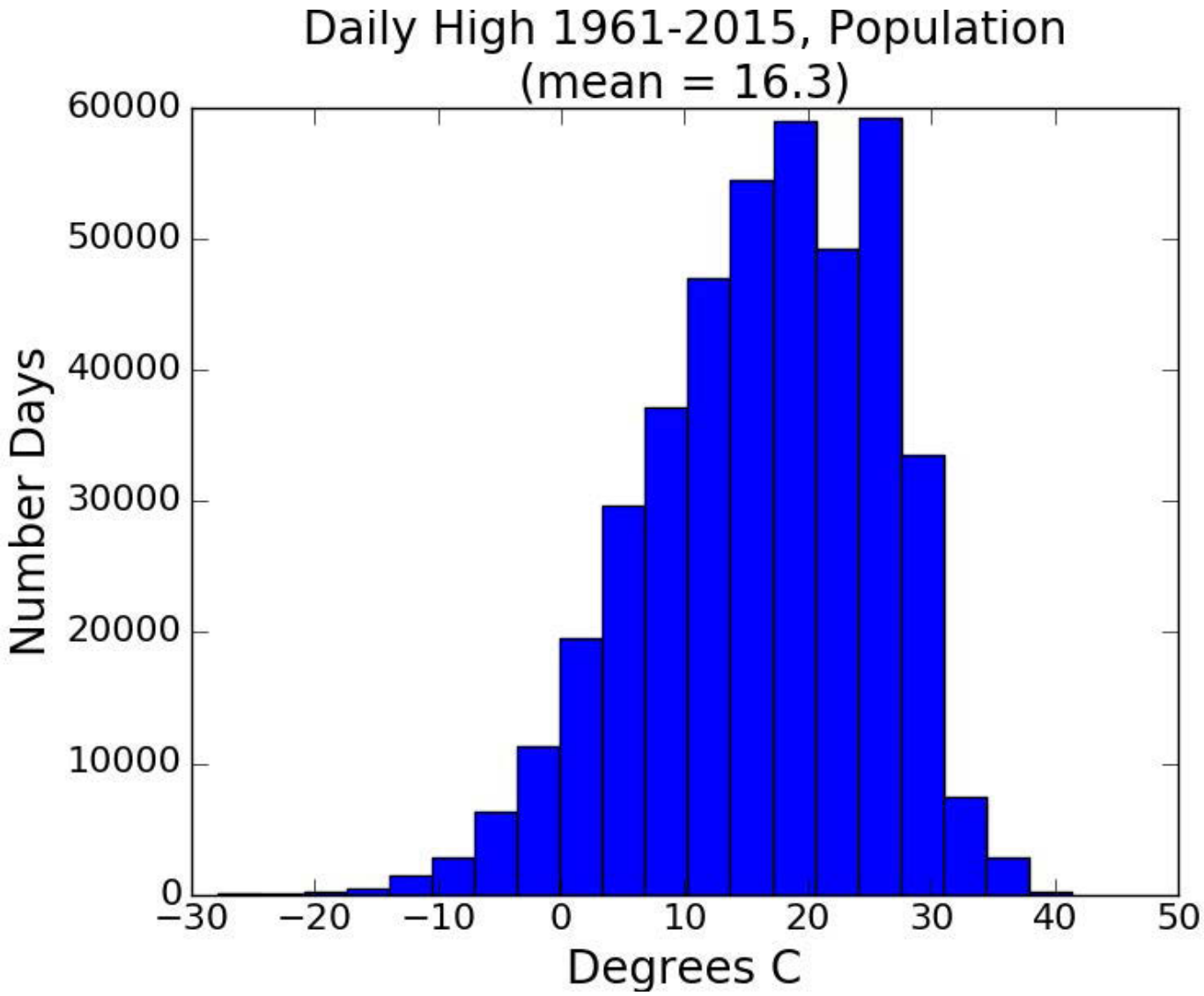
Data

- From U.S. National Centers for Environmental Information (NCEI)
- Daily high and low temperatures for
  - 21 different US cities
    - ALBUQUERQUE, BALTIMORE, BOSTON, CHARLOTTE, CHICAGO, DALLAS, DETROIT, LAS VEGAS, LOS ANGELES, MIAMI, NEW ORLEANS, NEW YORK, PHILADELPHIA, PHOENIX, PORTLAND, SAN DIEGO, SAN FRANCISCO, SAN JUAN, SEATTLE, ST LOUIS, TAMPA
  - 1961 – 2015
  - 421,848 data points (examples)
- Let's use some code to look at the data

New in Code

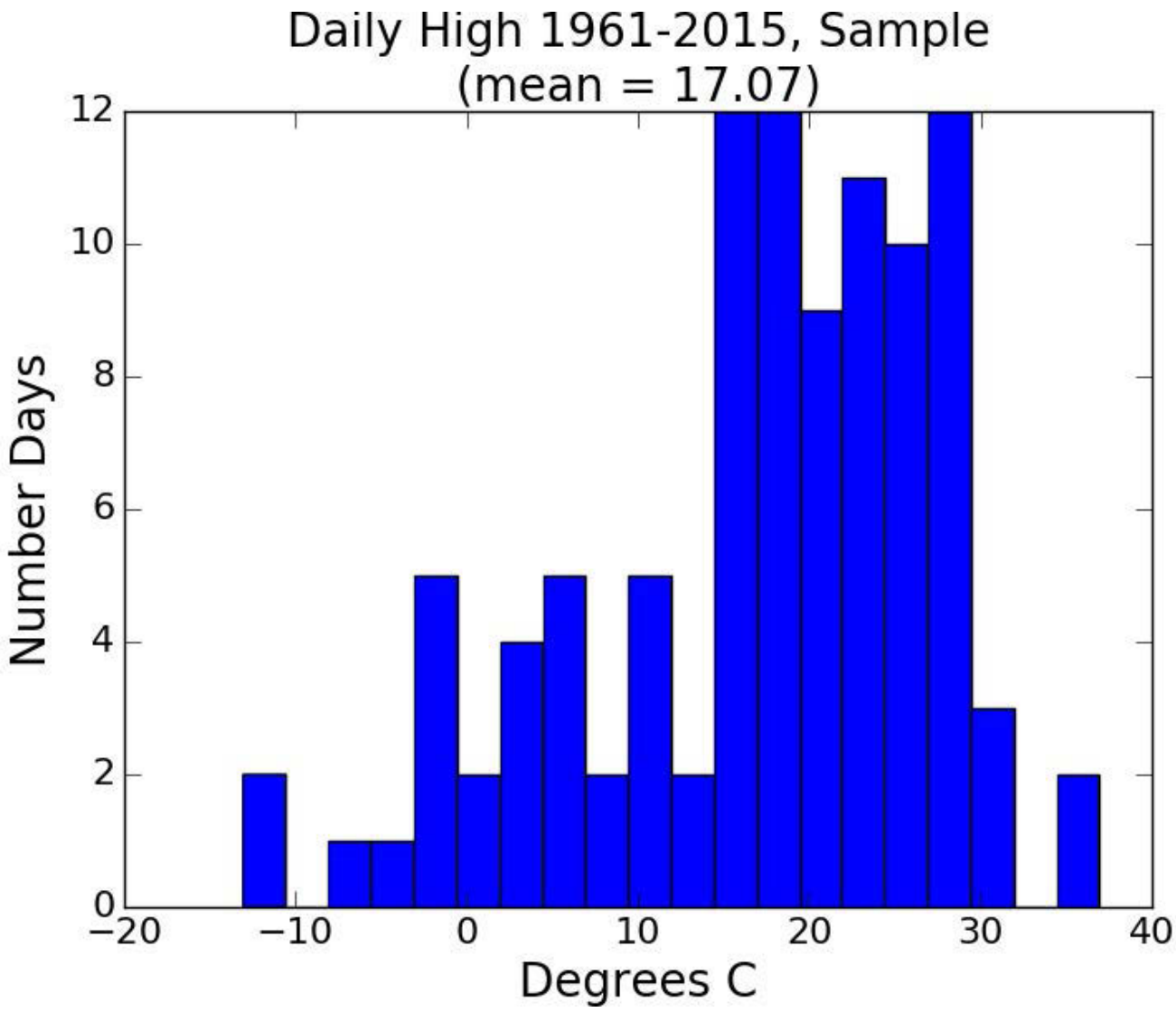
- `numpy.std` is function in the numpy module that returns the standard deviation
- `random.sample(population, sampleSize)` returns a list containing `sampleSize` randomly chosen distinct elements of `population`
  - Sampling without replacement

Histogram of Entire Population



σ = ~9.4

Histogram of Random Sample of Size 100



σ = ~10.4

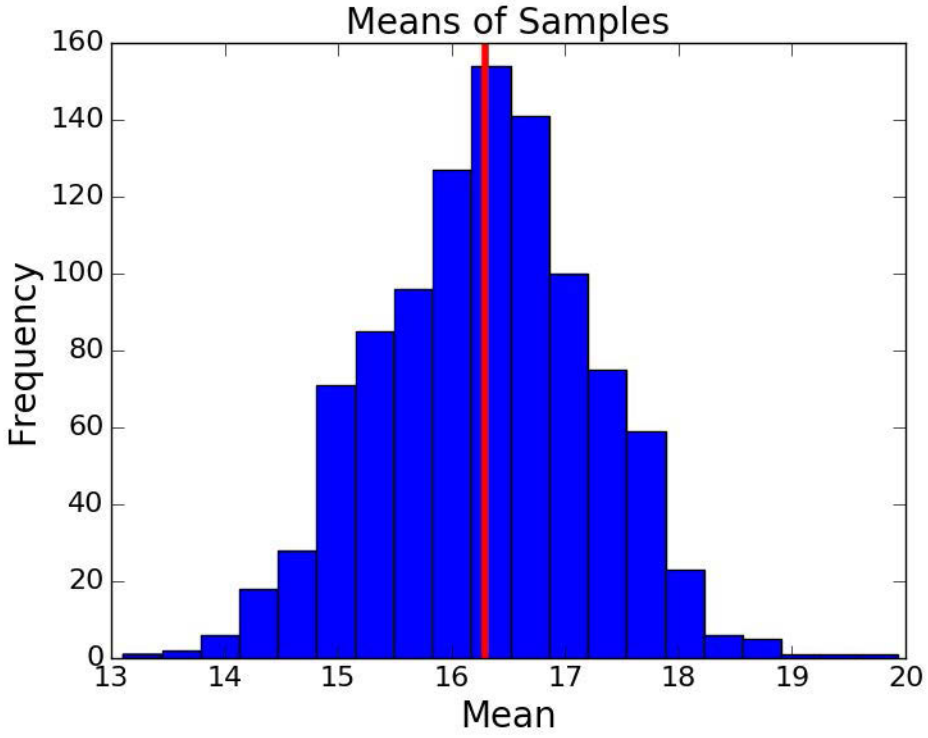
Means and Standard Deviations

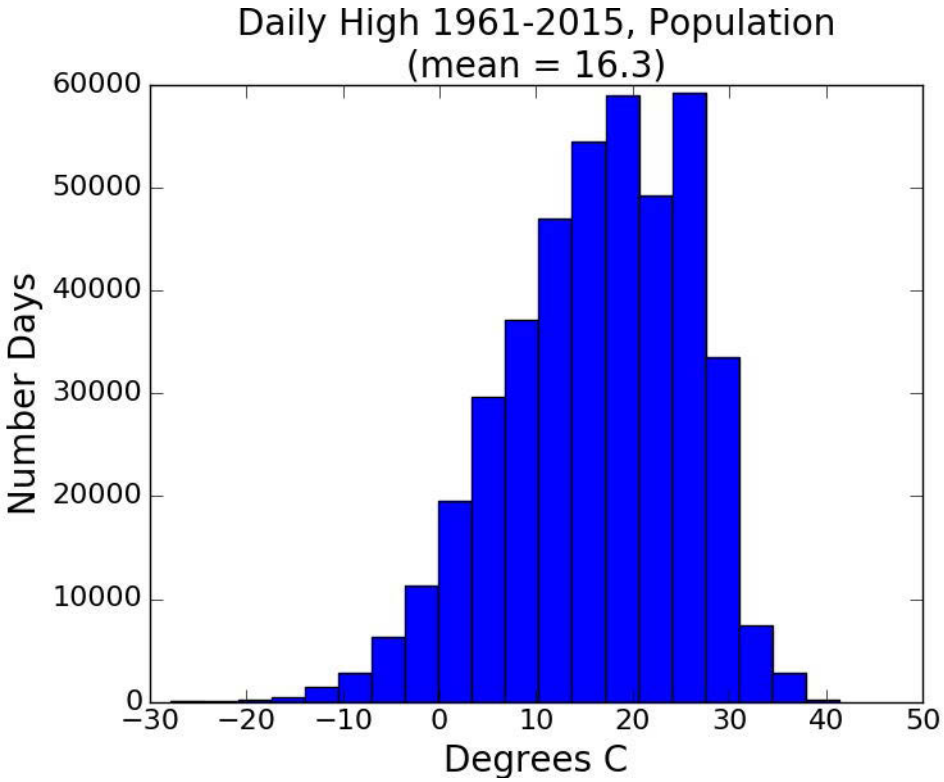
- Population mean = 16.3
- Sample mean = 17.1
- Standard deviation of population = 9.44
- Standard deviation of sample = 10.4
- A happy accident, or something we should expect?
- Let's try it 1000 times and plot the results

New in Code

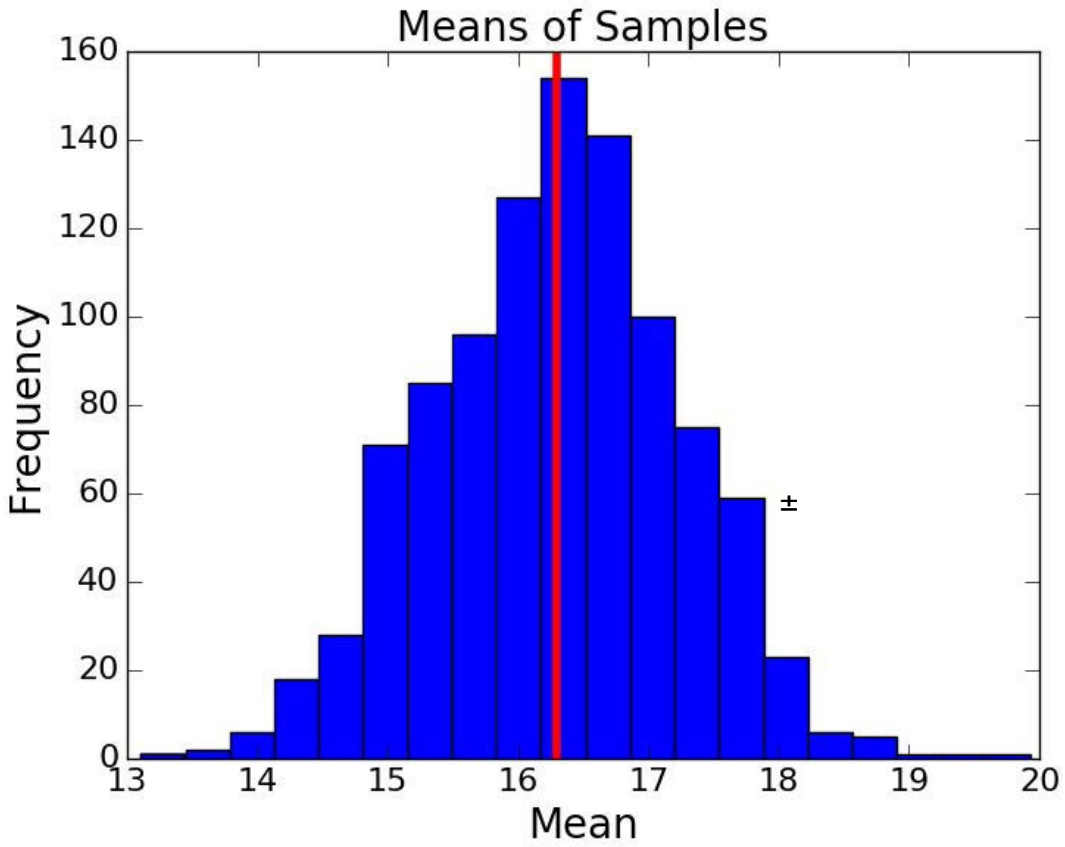
- pylab.axvline(x = popMean, color = 'r') draws a red vertical line at popMean on the x-axis
- There's also a pylab.axhline function

Try It 1000 Times





Try It 1000 Times



Mean of sample Means = 16.3
Standard deviation of sample means = 0.94

What's the 95% confidence interval?

16.28 +- 1.96*0.94
14.5 - 18.1

Includes population mean, but pretty wide

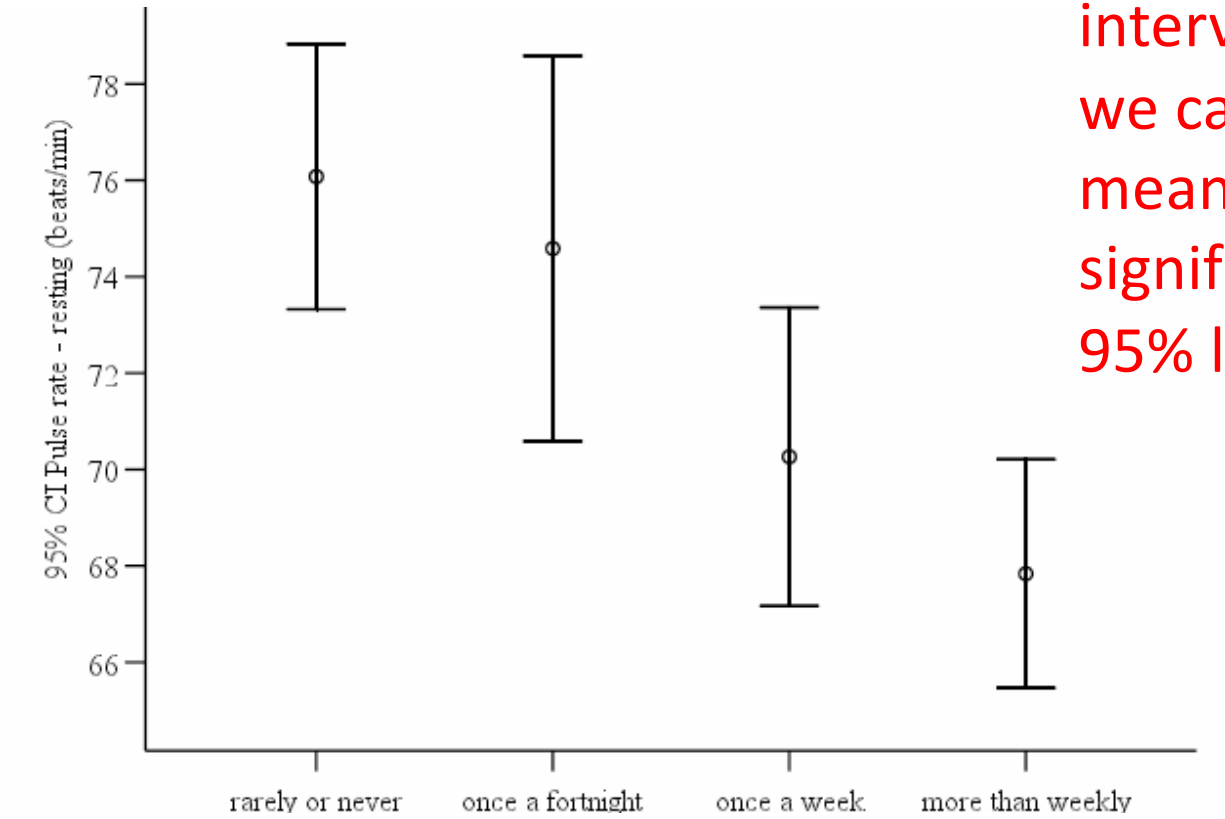
Suppose we want a tighter bound?

Getting a Tighter Bound

- Will drawing more samples help?
  - Let's try increasing from 1000 to 2000
    - Standard deviation goes from 0.943 to 0.946
- How about larger samples?
  - Let's try increasing sample size from 100 to 200
  - Standard deviation goes from 0.943 to 0.662

Error Bars, a Digression

- Graphical representation of the variability of data
- Way to visualize uncertainty



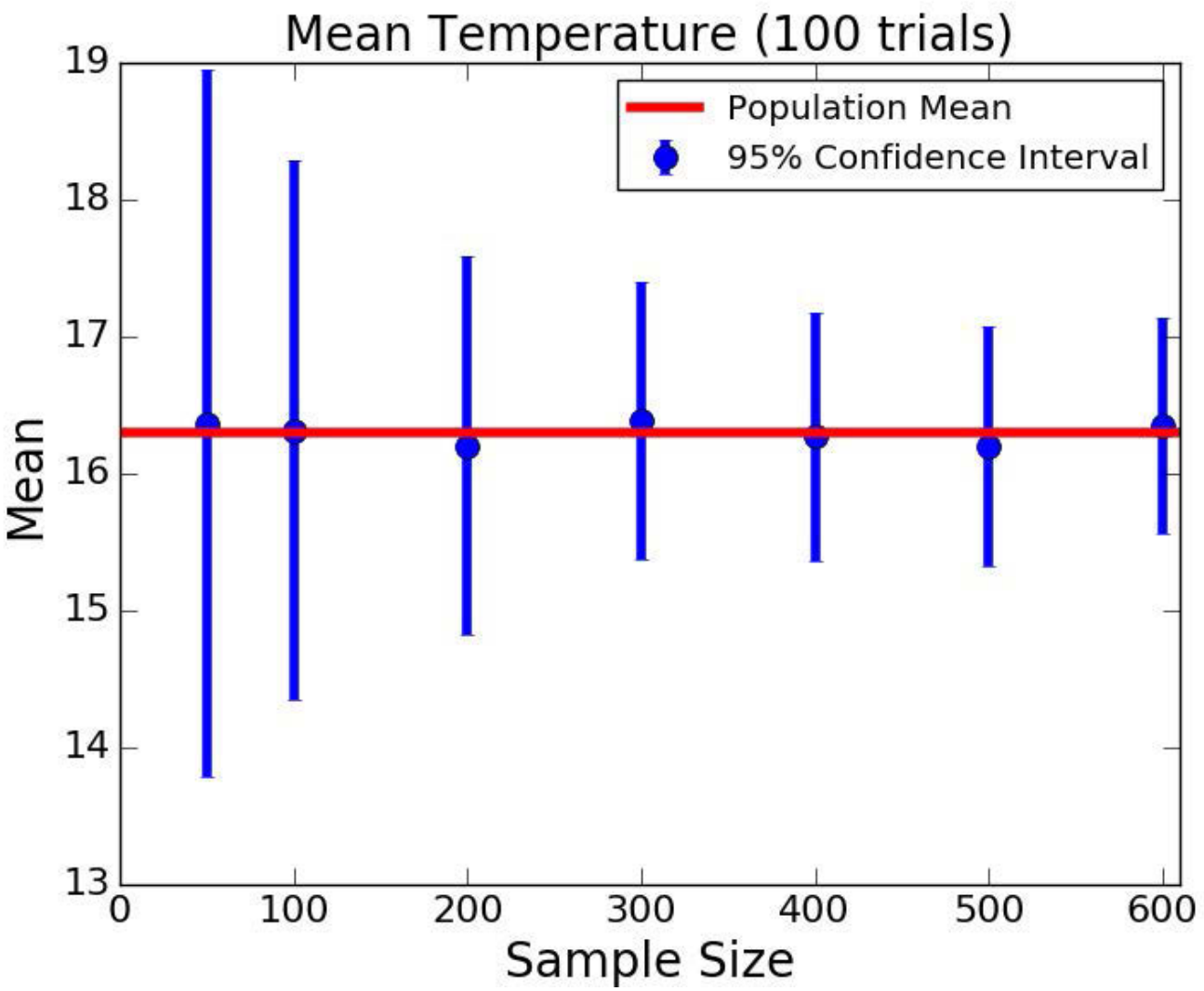
When confidence intervals don't overlap, we can conclude that means are statistically significantly different at 95% level.

https://upload.wikimedia.org/wikipedia/commons/1/1d/Pulse_Rate_Error_Bar_By_Exercise_Level.png

Let's Look at Error Bars for Temperatures

```
pylab.errorbar(xVals, sizeMeans,
               yerr = 1.96*pylab.array(sizeSDs),
               fmt = 'o',
               label = '95% Confidence Interval')
```

Sample Size and Standard Deviation



Larger Samples Seem to Be Better

- Going from a sample size of 50 to 600 reduced the confidence interval from about 1.2C to about 0.34C.
- But we are now looking at 600*100 = 600k examples
  - What has sampling bought us?
  - Absolutely Nothing!
    - Entire population contained ~422k samples

What Can We Conclude from 1 Sample?

- More than you might think
- Thanks to the Central Limit Theorem

Recall Central Limit Theorem

- Given a sufficiently large sample:
  - 1) The means of the samples in a set of samples (the sample means) will be approximately normally distributed,
  - 2) This normal distribution will have a mean close to the mean the population, and
  - 3) The variance of the sample means will be close to the variance of the population divided by the sample size.
- Time to use the 3rd feature
- Compute *standard error of the mean* (SEM or SE)

Standard Error of the Mean

$$SE = \frac{\sigma}{\sqrt{n}}$$

```
def sem(popSD, sampleSize):
      return popSD/sampleSize**0.5
```

- Does it work?

Testing the SEM

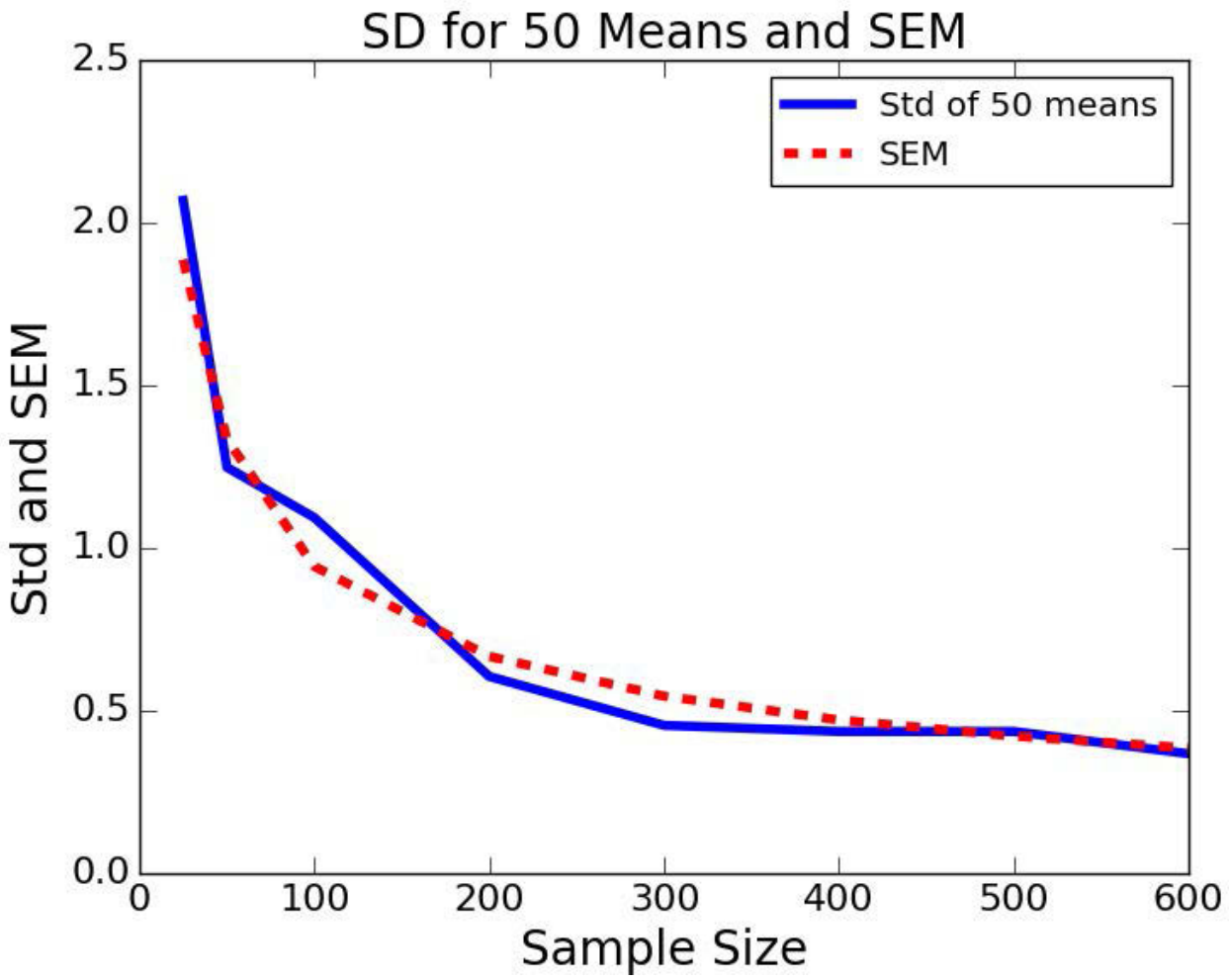
```
sampleSizes = (25, 50, 100, 200, 300, 400, 500, 600)
numTrials = 50
population = getHighs()
popSD = numpy.std(population)
sems = []
sampleSDs = []
for size in sampleSizes:
    sems.append(sem(popSD, size))
    means = []
    for t in range(numTrials):
        sample = random.sample(population, size)
        means.append(sum(sample)/len(sample))
    sampleSDs.append(numpy.std(means))
pylab.plot(sampleSizes, sampleSDs,
           label = 'Std of ' + str(numTrials) + ' means')
pylab.plot(sampleSizes, sems, 'r--', label = 'SEM')
pylab.xlabel('Sample Size')
pylab.ylabel('Std and SEM')
pylab.title('SD for ' + str(numTrials) + ' Means and SEM')
pylab.legend()
```

Standard Error of the Mean

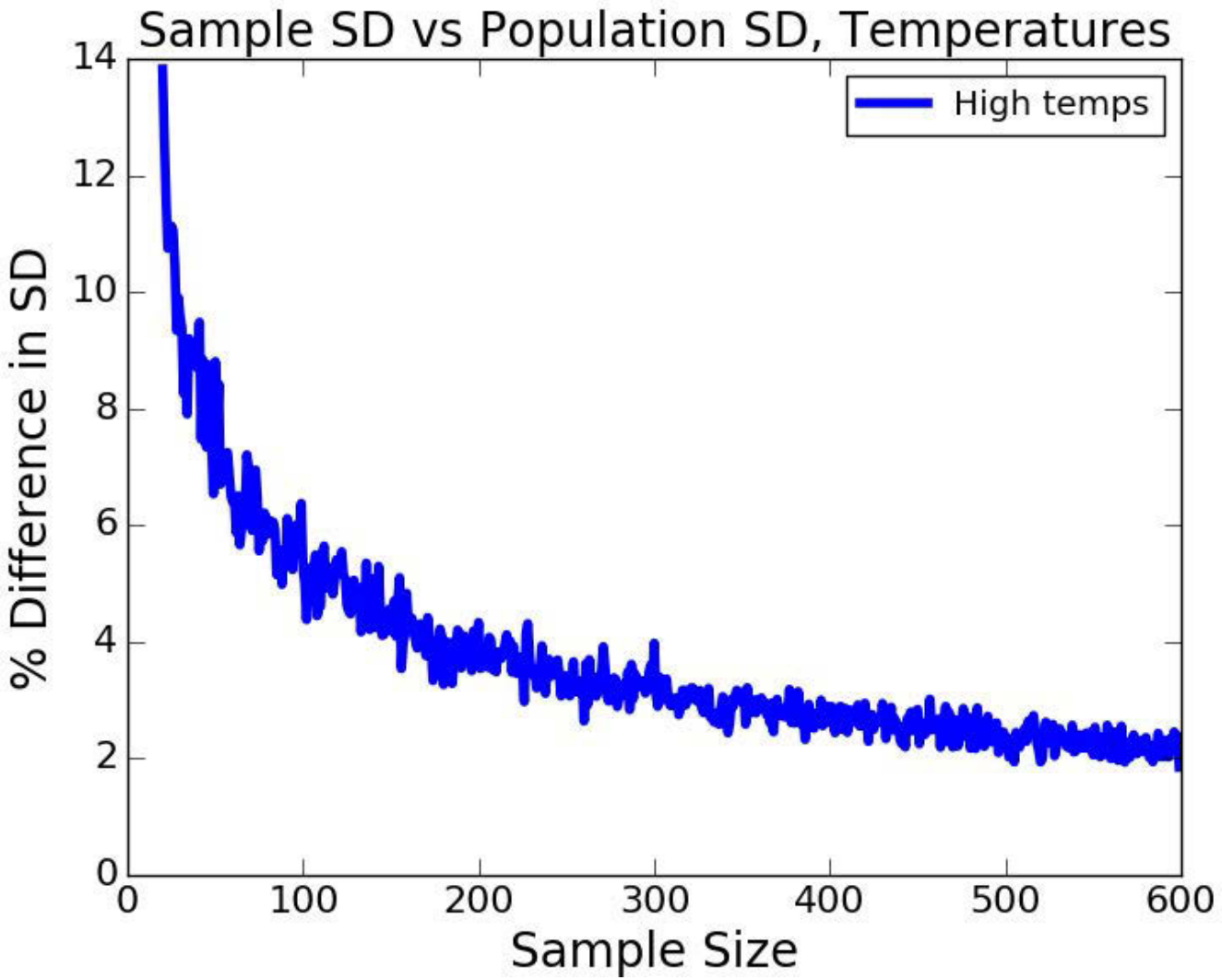
$$SE = \frac{\sigma}{\sqrt{n}}$$

But, we don't know standard deviation of population

How might we approximate it?

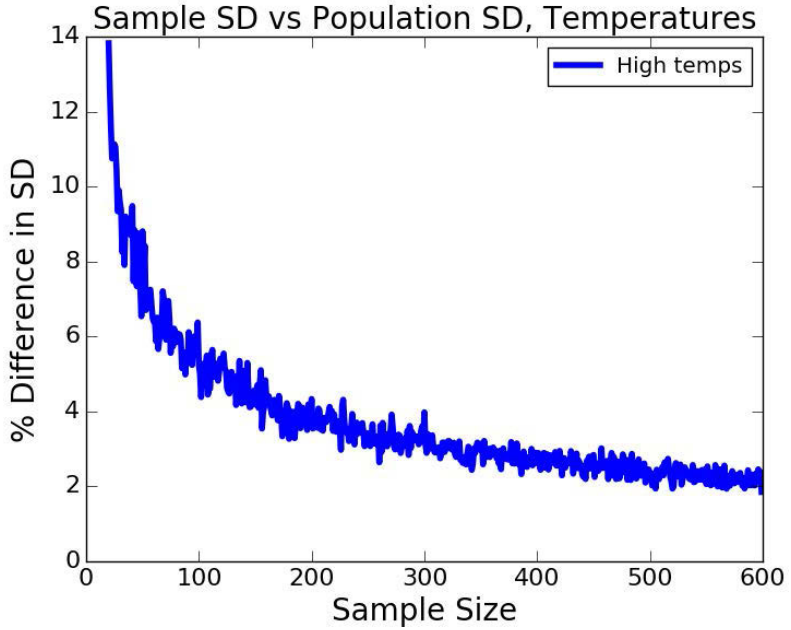


Sample SD vs. Population SD



The Point

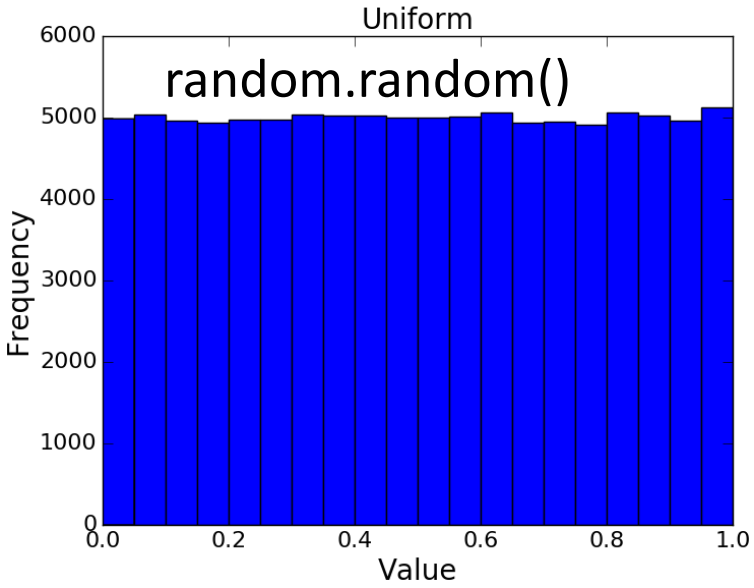
- Once sample reaches a reasonable size, sample standard deviation is a pretty good approximation to population standard deviation
- True only for this example?
  - Distribution of population?
  - Size of population?

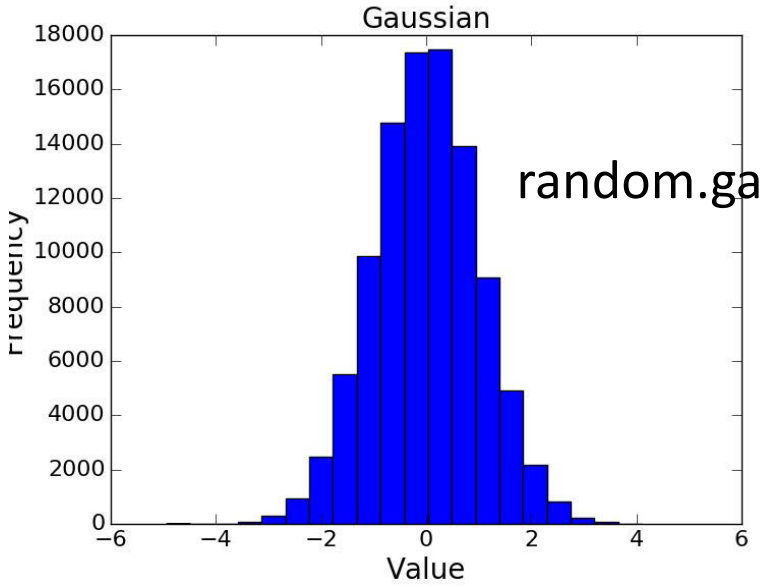


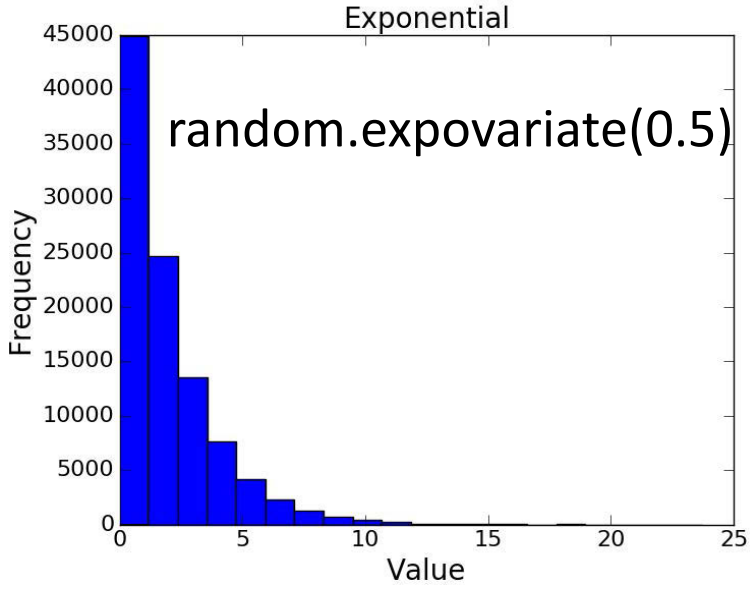
Looking at Distributions

```
def plotDistributions():
    uniform, normal, exp = [], [], []
    for i in range(100000):
        uniform.append(random.random())
        normal.append(random.gauss(0, 1))
        exp.append(random.expovariate(0.5))
    makeHist(uniform, 'Uniform', 'Value', 'Frequency')
    pylab.figure()
    makeHist(normal, 'Gaussian', 'Value', 'Frequency')
    pylab.figure()
    makeHist(exp, 'Exponential', 'Value', 'Frequency')
```

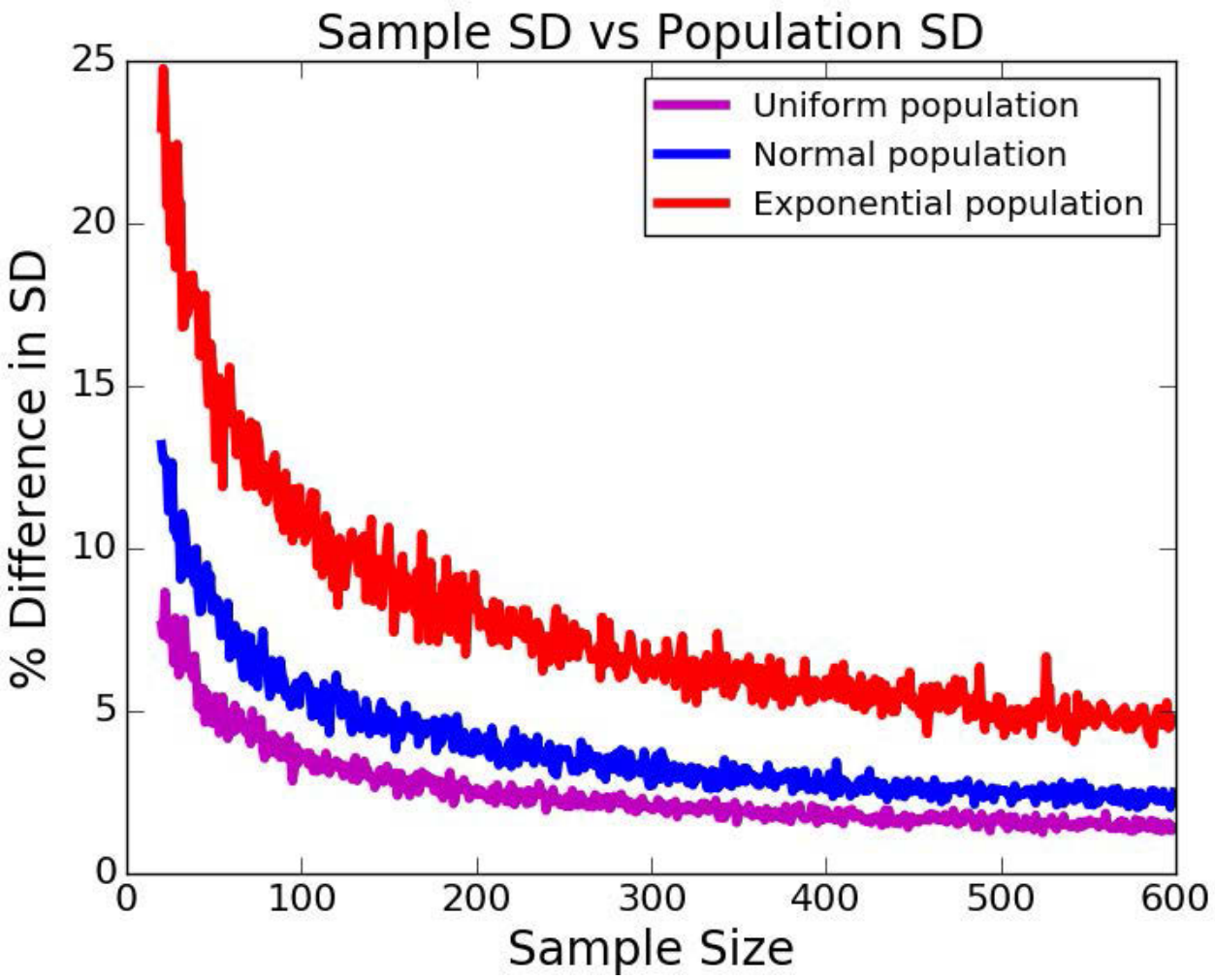
Three Different Distributions





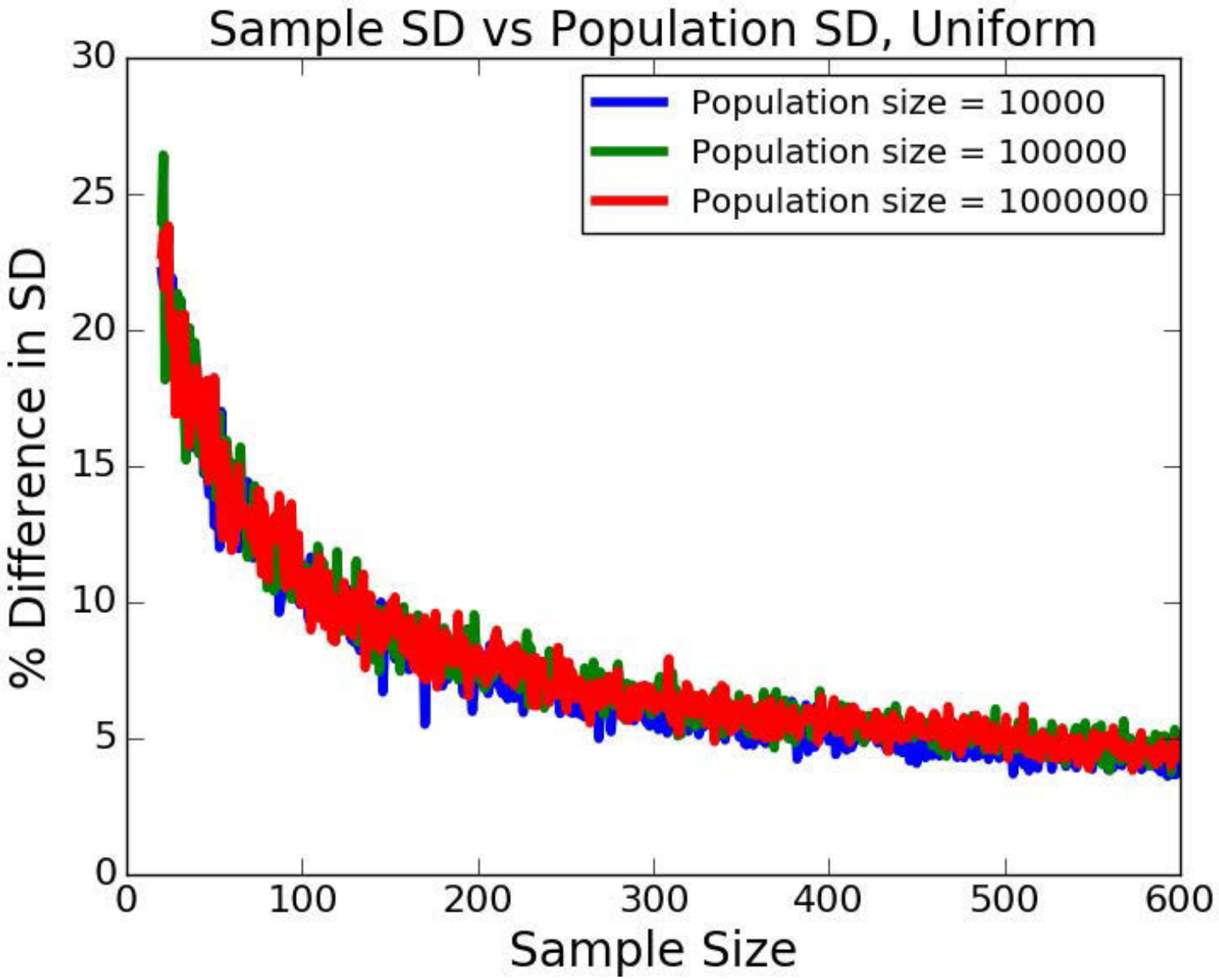


Does Distribution Matter?



Skew, a measure of the asymmetry of a probability distribution, matters

Does Population Size Matter?



To Estimate Mean from a Single Sample

- 1) Choose sample size based on estimate of skew in population
- 2) Chose a random sample from the population
- 3) Compute the mean and standard deviation of that sample
- 4) Use the standard deviation of that sample to estimate the SE
- 5) Use the estimated SE to generate confidence intervals around the sample mean

Works great when we choose independent random samples.

Not always so easy to do, as political pollsters keep learning.

Are 200 Samples Enough?

```
numBad = 0
for t in range(numTrials):
    sample = random.sample(temps, sampleSize)
    sampleMean = sum(sample)/sampleSize
    se = numpy.std(sample)/sampleSize**0.5
    if abs(popMean - sampleMean) > 1.96*se:
        numBad += 1
print('Fraction outside 95% confidence interval =',
      numBad/numTrials)
```

Fraction outside 95% confidence interval = 0.0511

6.0002 Introduction to Computational Thinking and Data Science

Fall 2016