Introduction to Political Economy 14.770
Problem Set 3

Due date: October 27, 2017.

Question 1:

Consider an alternative model of *lobbying* (compared to the Grossman and Helpman model with enforceable contracts), where lobbies have to make up-front contributions to the politician and the politician chooses the favorite policy of the lobby which made the highest contribution. One way to formalize this is to model it as an *all-pay auction*. Formally, an all-pay auction is "an auction in which every bidder must pay regardless of whether they win the prize, which is awarded to the highest bidder as in a conventional auction."

Suppose there are N lobbies competing to get the politician's support to have the legislation in their favor. Assume that the value of having legislation in one's favor is worth $\bar{x}$ for each lobby. Each lobby makes a contribution the politician before the legislation is decided, and the contribution is nonrefundable. The lobbies don't observe other lobbies' contributions before the legislation passes. The politician passes the legislation in favor of the lobby which pays the highest contribution. (If there are multiple lobbies which pay the highest contribution, the politician decides randomly.) If a lobby pays x and gets the legislation in its favor, then its payoff is $\bar{x} - x$. If the legislation is not in one's favor, then the payoff is $-x$. For simplicity, normalize $\bar{x} = 1$.

1. Assume $N = 2$. Does this game have any pure strategy Nash Equilibrium? Explain.

2. Assume $N = 2$. Find a symmetric mixed strategy equilibrium where both lobbies randomize over possible contributions according to a c.d.f. $F(x)$.

3. Now, consider a general case where N can be any integer. Find a symmetric mixed strategy equilibrium where each lobby (independently) randomizes over possible contributions according to a c.d.f. $F(x)$.

4. How do the equilibrium distributions change with N? Can you suggest an economic intuition on why the equilibrium changes in this way? Calculate the expected total contribution that politician receives. How does it change with N? Are more lobbies better or worse for the politician? What if the politician is risk averse/risk loving?

Question 2:

This question will walk you through a version of the Lizzeri and Persico (2005) model of *vote buying* – a model we partly covered in Lectures 6 and 7.

Assume that there is a population of voters whose measure is normalized to 1 (indexed by $v \in [0, 1]$). Everyone has 1 unit of resources and have linear utility over goods.

There are 2 parties, and they make binding promises to voters concerning their policy conditional on winning the election. A party can:

- Offer different taxes and transfers to different voters (it is possible to target resources to individuals), or,

- Offer to provide a public good (to all voters). The public good costs 1 unit of resources per head (i.e. requires taxing everyone fully),[1] and generates a utility G for each voter.

Each voter votes for the party who promises her the greatest utility. Parties maximize their expected vote share.

Before you begin the analysis, note that when $G > 1$, the public goods are efficient (in the sense of utilitarian welfare maximization). We will observe that this is not sufficient to ensure that they are always offered in equilibrium.

1. Suppose $G > 2$. Show that the only equilibrium is one with both parties offering public goods.

---

[1]Note that, due to this assumption, a party cannot offer both the public good and transfers at the same time.

2. Now suppose $G < 2$. Show that there is not an equilibrium in which a party offers the public good with probability one.

3. Suppose $G < 2$. Show that there is not an equilibrium in which a party offers a transfer scheme in pure strategies, either. Conclude that there is no pure strategy equilibrium.

4. Now, consider the case $G < 1$. Show that none of the parties offer public good in equilibrium. Find a symmetric mixed strategy equilibrium where each party offers each voter v a transfer drawn from a distribution with c.d.f. $F(.)$. [Hint: Going over Question 1 first would make this part easier.]

5. Now, consider the case $1 < G < 2$. Show that the public good must be provided with positive probability in equilibrium. Find a symmetric mixed strategy equilibrium where each party offers the public good with probability β, offers transfers with probability $1-\beta$, and if it offers transfers, each voter v is offered a transfer drawn from a distribution with c.d.f. $F(.)$.

6. For the case $1 < G < 2$, what is the probability that the public good is offered in equilibrium? Comment on what features of this model lead to the inefficiency result.

Question 3:

This question will walk you through a political agency model with an interesting implication: with sufficiently strong re-election incentives, even "honest" politicians may choose *pander* to the voters by taking an action which may not be in the electorate's best interest.

In order to motivate this model, here is an excerpt from Besley's Principled Agents (2006), Section 3.4.3:

> ...A small emerging literature, however, is concerned with the possibility that agency can lead to poorer quality social decisions because politicians tend to choose outcomes that are too close to what voters want. This is most relevant when politicians have better information than voters. A conflict arises when this information goes against what voters would most likely think to be optimal. Re-election incentives may then lead to politicians to choose excessively popular politics.

Consider the following model: There are two periods $t \in \{1, 2\}$. The discount factor is $\delta \in (0, 1]$.

A politician has a (persistent) type $i \in \{c, nc\}$, where c is corrupt and nc is noncorrupt. Each politician's type is drawn independently from an distribution with $Pr\{i = nc\} = \pi \in (0, 1)$.

In each period $t \in \{1, 2\}$, there is a state of the world $s_t \in \{0, 1\}$, privately observed by the politician. Each period, the state of the world is drawn independently from a distribution with $Pr\{s_t = 1\} = \frac{1}{2}$.

In each period $t \in \{1, 2\}$, the elected politician of type i observes the state s_t and picks a policy $e_t(s_t, i) \in \{0, 1\}$. The citizens have a payoff of

$$u_t(s_t, e_t) = \begin{cases} V, & \text{if } e_t = s_t \\ 0, & \text{if } e_t \neq s_t \end{cases}$$

Each period, a non-corrupt politician receives a payoff of

$$u_t^{nc}(s_t, e_t) = u_t(s_t, e_t) + \mathbf{1}_{\{inofficeatperiodt\}} W$$

Where $W > 0$ is the "ego rents" from being in the office in period t. (Note that the non-corrupt politician cares about the voter welfare, even when she is not in the office. She is truly a considerate politician!)

A corrupt politician's per period payoff is:

$$u_t^c(s_t, e_t) = \begin{cases} \mathbf{1}_{\{inofficeatperiodt\}} W, & \text{if } e_t = 0 \\ \mathbf{1}_{\{inofficeatperiodt\}} (r_t + W) & \text{if } e_t = 1 \end{cases}$$

where r_t is the "private benefit" from setting $e = 1$. Each period, r_t is drawn independently from a distribution $G(r)$ with mean μ and support $[0, R]$. The timing of the game is as follows:

i. An incumbent politician is in the office. The incumbent's type is drawn, and she privately observes her type.

ii. s_1 is drawn and observed by the politician.

iii. If the incumbent is corrupt, r_1 is drawn and observed by the politician.

iv. The incumbent chooses e_1, and it is observed by the citizens.

v. Citizens decide whether to keep the incumbent or elect a new politician. If they elect a new politician, her type is drawn randomly from the same distribution.

vi. Citizens observe their payoffs from period 1.

vii. In the second period, s_2 is drawn and observed by the elected politician, (if she is corrupt) r_2 is drawn and observed by the politician, and the elected politician chooses e_2. Payoffs are realized.

Note, in particular, that the citizens observe the first period payoffs only *after* the election.

1. What does this timing imply for the role of *retrospective voting* in this model? Is this timing a realistic assumption?

2. Find a Perfect Bayesian Nash Equilibrium of the game where

   - A non-corrupt incumbent picks $e_1 = 0$ regardless of s_1,
   - A corrupt incumbent picks $e_1 = 1$ only if r_1 is sufficiently high, and,
   - An incumbent is re-elected only if $e_1 = 0$.

   Note: you must verify that each politician and the voters are optimizing, and Bayes' rule is used whenever possible.

3. When does a corrupt incumbent choose $e_1 = 0$? What is the ex ante probability of this event? How does it depend on W, μ and δ?

4. What is the condition on non-corrupt incumbent's period one incentives to sustain such an equilibrium? How does it depend on V, W, δ and π? Discuss.

Further reading: if you're interested in the general idea of *pandering*, Morris' Political Correctness (2001, JPE) is a good resource to look at, even though it's framed as a different model.

Question 4:
This question is designed to give you an opportunity to work with different models of bargaining. Consider the the alternating-offers bargaining model of by Rubinstein (1982), which we covered in Lecture 11. We'll denote Player 1's share as $x_1 \in [0, 1]$ and Player 2's share as $x_2 \in [0, 1]$, so that $x_1 + x_2 = 1$.

(Warm-Up). First, consider the *ultimatum bargaining* game. Player 1 moves first and offers $x_1 \in [0,1]$. After observing the offer, Player 2 either accepts (Y) or rejects (N). If Player 2 accepts, the payoffs are $(x_1, 1-x_1)$. If she rejects, the game ends with payoffs $(0,0)$. Find the backward induction equilibria of this game. (For simplicity, you can assume that a player accepts an offer when she is indifferent between accepting and rejecting.)

1. Now, take it one step further and assume there are two periods in which players can make offers. Once again, Player 1 begins by offering $x_1 \in [0,1]$ and Player 2 either accepts (Y) of rejects (N). If Player 2 accepts, the payoffs are $(x_1, 1 - x_1)$. If Player 2 rejects, then Player 2 moves to offer $x_2 \in [0,1]$. In this case, Player 1 responds by either accepting (Y) or rejecting (N). If Player 1 accepts, the payoffs are $(\delta(1 - x_2), \delta x_2)$, where $\delta \in (0,1)$. If Player 1 rejects, then the game ends with payoffs $(0,0)$. Find the backward induction equilibria of this game.

2. Now, generalize the result to $T \geq 2$ periods. Player 1 makes offers in odd periods and Player 2 makes offers in even periods. Receiving a share of x_i in period t gives a payoff of $\delta^{t-1}x_i$ for player $i \in \{1,2\}$. Assuming T is even, find the payoff vectors in subgame perfect equilibrium.

3. What is the payoff vector if T is odd?

4. Comparing the results in parts 3 and 4, you should be able to observe the phenomena called *last-mover advantage* and *first-mover advantage*. Can you observe how they are reinforced/weakened as $T \rightarrow \infty$ and $\delta \rightarrow 1$? Can you offer an economic intuition on why the changes occur that way?

14.770 Introduction to Political Economy
Fall 2017