Slutsky for Hours (done in minutes)

Josh Angrist

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A Slutsky derivation

Uncompensated and Compensated Labor Supply

Utility is a function of consumption (x) and leisure (l), where $h = T - l$ is hours worked.

- Uncompensated (Marshallian) demands are a function of wages, prices, and unearned income

$$\begin{aligned} \{x(p,w,y), l(p,w,y)\} &= \arg\max U(x,l) \\ s.t. \quad px &= w(T-l)+y \end{aligned}$$

  This generates uncompensated labor supply:

$$h(p,w,y) = T - l(p,w,y)$$

- Compensated (Hicksian) labor supply is a function of wages, prices and utility

$$\begin{aligned} \{x^c(p,w,\bar{u}), l^c(p,w,\bar{u}),\} &= \arg\min wl + px \\ s.t. \quad \bar{u} &= U(x,l) \end{aligned}$$

  This is the dual problem, generating compensated labor supply:

$$h^c(p,w,\bar{u}) = T - l^c(p,w,\bar{u})$$

  The derivative of the compensated labor supply function is the *substitution effect.*

The *excess* expenditure function

- A consumer *spends* this much to get to $\bar{u}$:

$$\begin{aligned} E[p,w,\bar{u}] &= px^c(p,w,\bar{u}) + wl^c(p,w,\bar{u}) \\ & \quad px^c(p,w,\bar{u}) + w(T - h^c(p,w,\bar{u})) \end{aligned}$$

- I need this much *cash* to get to $\bar{u}$:

$$E^*[p, w, \bar{u}] = E[p, w, \bar{u}] - wT = px^c(p, w, \bar{u}) - wh^c(p, w, \bar{u})$$

  Viewed as a function of prices wages, and my utility target, this is called the *excess expenditure function.*

- The excess expenditure function has the following properties:

  1. Shephard's Lemma

  $$\frac{\partial E^*[p, w, \bar{u}]}{\partial w} = \frac{\partial E[p, w, \bar{u}]}{\partial w} - T = l^c(p, w, \bar{u}) - T = -h^c(p, w, \bar{u})$$

     This is the enevelope theorem in action.

  2. Concavity

  $$\frac{\partial^2 E^*[p, w, \bar{u}]}{\partial w^2} = \frac{\partial^2 E[p, w, \bar{u}]}{\partial w^2} = \frac{\partial l^c(p, w, \bar{u})}{\partial w} < 0$$

     How do we know this? The expenditure function is concave in prices because people reallocate away from the more expensive good when its price increases. In other words, spending (cost) goes up less than linearly in prices. From this, we conclude

  $$\frac{\partial h^c(p, w, \bar{u})}{\partial w} > 0$$

     The substitution effect on hours is positive.

A useful identity

- Compensated and uncompensated labor supply are related as follows

$$h^c(p, w, \bar{u}) = h(p, w, E^*[p, w, \bar{u}]) \tag{1}$$

  In other words, if I adjust your unearned income (compensate you) so as to keep you on $\bar{u}$ while changing your wage, then I learn what happens when your wages change while you're stuck on indifference curve $\bar{u}$.

Slutsky derived

- Differentiate both sides of (1)

$$\begin{aligned}
\frac{\partial h^c(p, w, \bar{u})}{\partial w} &= \frac{\partial h(p, w, E[p, w, \bar{u}] - wT)}{\partial w} + \frac{\partial h(p, w, E[p, w, \bar{u}] - wT)}{\partial y} \left[\frac{\partial E[p, w, \bar{u}]}{\partial w} - T \right. \\
\frac{\partial h^c}{\partial w} &= \frac{\partial h}{\partial w} + \frac{\partial h}{\partial y}[-h^c(p, w, \bar{u})]
\end{aligned}$$

- Re-arrange to get the Slutsky equation for hours:

$$\frac{\partial h}{\partial w} = \frac{\partial h^c}{\partial w} + \frac{\partial h}{\partial y} h$$

  Now, use it!

B Ashenfelter (1978) puts Slutsky to work

Parameterizing policy: the Negative Income Tax

- A stylized negative income tax (NIT) or similar welfare program provides a subsidy of G, reduced by amount t, for every dollar of earnings. Many such program emerged in the 1970s as an alternative to traditional welfare (we'll come back to this)
  - Assuming (as is often the case) that only earnings are taxed, the *program subsidy* is
    $$S = G - twh$$
    when
    $$wh < \frac{G}{t} = B$$
    and zero otherwise
  - B is the *program breakeven*
- Ashenfelter (1978) asks: what does basic theory say about program effects on labor supply? The paper then uses data from the Rural NIT to estimate key labor supply paremeters

Theoretical NIT effects

- Define
  $$\begin{aligned} h(p,w,y) &= \textit{uncompensated .l.s} \\ h^c(p,w,\bar{u}) &= \textit{compensated .l.s} \end{aligned}$$
  - Uncompensated differential
    $$dh = \frac{\partial h}{\partial w}dw + \frac{\partial h}{\partial y}dy$$
    Substituting with Slutsky
    $$dh = \left[\frac{\partial h^c}{\partial w} + \frac{\partial h}{\partial y}h \;\; dw + \frac{\partial h}{\partial y}dy \qquad (2)$$
- Rearranging and inserting program parameters
  $$\begin{aligned} dh &= \frac{\partial h^c}{\partial w}dw + \frac{\partial h}{\partial y}[hdw + dy] \\ &= \frac{\partial h^c}{\partial w}(-tw) + \frac{\partial h}{\partial y}[-twh + G] \\ &= \frac{\partial h^c}{\partial w}(-tw) + \frac{\partial h}{\partial y}S \end{aligned}$$

so

$$dlnh = \eta_c(-t) + \eta_y \frac{S}{y} \tag{3}$$

This is Ashenfelter (1978) "parameter scheme 2," where S is replaced by S_0, an ex ante subsidy value based on pre-program earnings. *Note that labor supply must fall.*

- "Parameter scheme 1" generates an estimation scheme for uncompensated elasticities by working directly with 2 to produce

$$dlnh = \eta_u(-t) + \eta_y \frac{G}{y}$$

Estimates

Rural NIT: 5 experimental treatments (G,T) plus controls in each of IA and NC. About 800 families exposed for 3 years; Average G=3,200 (50-100% pov level) and t=1/2 (.3-.7). Ashenfelter pools data for the three years and regresses change in log earnings on year effects, tax rates, and guarantee or subsidies as proportion of unearned income.

- See **Tables 3-6**.
- Beware sinister misreporting and attrition! (Greenberg and Halsey, 1983; Ashenfelter and Plant, 1990)