Notes on Labor Demand

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One factor competitive benchmark

- The one-factor setup is derived from two:

$$q = F(K, L)$$

- Now, fix one:

$$f(L) \equiv F(\bar{K}, L); f'(L) > 0; f''(L) < 0$$

- Firms are price takers in goods and factor markets:

$$\pi(L) = pf(L) \quad wL$$

  π max f.o.c.'s:

$$\begin{aligned} pf'(L) &= w \\ MR &= MC \end{aligned}$$

- We say: "the demand curve for labor is the VMP_L"

Two factors

- π max becomes:

$$\pi = pF(K, L) \quad wL \quad rK$$

  with f.o.c.s

$$\begin{aligned} pF_L &= w \\ pF_K &= r \end{aligned}$$

- This looks simple, yet it hides something subtle and important: there's action here at both the firm and market level. Firm's face the question of how best to make something, a technological choice. At the market level, there's the question of how much product the industry can expect to sell and at what price. Factor prices factor into this, but the details are tricky. Let's separate and quantify the two pieces of factor demand:

1. We first ask: what's the cheapest way to make q? This generates *conditional factor demands*

$$\{K^c(w,r,q), L^c(w,r,q)\} = \arg\min_{K,L} rK + wL \quad s.t. \quad F(K,L) = q$$

2. Conditional factor demands determine the *cost function* (What piece of consumer theory does this remind you of?):

$$C(w,r,q) \equiv rK^c(w,r,q), +wL(w,r,q)$$

Now choose q to max:

$$\pi(q) = pq \quad C(w,r,q) = pq \quad C^*(q)$$

Output is chosen to solve p=MC, generating the firm's supply curve; price determines *scale*

- CRTS is a commonly-invoked mathematical metaphor for "the long run" because we imagine, given sufficient time, we can clone a production process with all it's inputs (capital, energy, labor, and land). Under CRTS, MC is constant and any one firm's output is indeterminate (the firm's long run supply curve is horizontal). Price is driven to MC by entry, while *industry* scale is determined by market demand at this lowest-possible long-run price.
- Duality in action on the producer side:

1. Differentiate the cost function to generate conditional factor demands (this is Shepherd's lemma for producers)
2. The fact that the cost function is concave in prices establishes that own-price substitution elasticities, reflecting movement along an isoquant, are negative

Technical Substitution Elasticities

- The *elasticity of (technical) substitution* describes movement along an isoquant:

$$= \frac{dln(K^c/L^c)}{dln(w/r)} = \frac{dln(K^c/L^c)}{dln(F_L/F_K)} > 0$$

- Under CRTS, we can show:

$$= \frac{F_L F_K}{qF_{LK}}$$

- Special cases: Cobb-Douglas ($= 1$); Linear ($= \infty$); Leontief ($= 0$); CES ($= \frac{1}{1 \quad \rho}$, where $q = A[\alpha K^\rho + (1 \quad \alpha)L^\rho]^{\frac{1}{\rho}}$)

Factor Demand Elasticities

- *Conditional factor demand elasticities*, also known as substitution elasticities (not the same as , above - watch out!), describe the effects of factor prices on conditional factor demands. Under CRTS, these can be written

$$\eta_{LL} \equiv \frac{\partial L^c}{\partial w}\frac{w}{L} = \quad (1 \quad s_L) \quad < 0$$

$$\eta_{LK} \equiv \frac{\partial L^c}{\partial r}\frac{r}{L} = (1 \quad s_L) \quad > 0$$

  where $s_L = \frac{wL}{pq}$ is the factor share

- *Total factor demand elasticities* include substitution and scale effects. Under CRTS, these can (and will) be shown to be

$$\eta'_{LL} = \quad (1 \quad s_L) \qquad s_L\eta \tag{1}$$

$$\eta'_{LK} = (1 \quad s_L)(\qquad \eta) \tag{2}$$

  where $\eta = \quad \frac{D'(p)p}{q}$ is the absolute value of the product demand elasticity.

- Equations (1) and (2) are Slutsky-like relations for (market-level) demand responses to changing factor prices

- Equation (1) is known to applied micro mavens as the *fundamental law of factor demand*. We'll sketch a derivation, below. But first ...

Hicks-Marshall Laws of Derived Demand

There are four of these; the first two come directly from the fundamental law of factor demand. The third sounds plausible, though few alive today can prove it (the proof is buried in Allen, 1938). The fourth also comes from (1), but it's not really a law, its more like a riddle.

1. The own-price factor demand elasticity increases with ease of substitutability:

$$\uparrow \Rightarrow \ \eta'_{LL} \ \uparrow$$

2. The own-price factor demand elasticity increases as product demand gets more elastic:

$$\eta \uparrow \Rightarrow \ \eta'_{LL} \ \uparrow$$

3. The own-price factor demand elasticity increases as other factors are supplied more elastically (e.g., a highly elastic supply of capital makes it easier [cheaper] to substitute away from labor and towards when wages go up).

4. Increasing the labor share makes labor demand more or less elastic as η is greater than or less than . In other words, because

$$\eta'_{LL} = \quad (1 \quad s_L) \qquad s_L\eta = \qquad s_L(\eta \qquad),$$

η'_{LL} increases with s_L when $\eta >$. It makes sense that a big factor share increases the scale effect, but there's also the substitution term to contend with!

See Hamermesh (1993), Chapter 2, Section III.

Deriving the Fundamental Law of Factor Demand

We start this in class ...

- Under CRTS, the cost function of firm j is

$$C_j(w, r, q) = q_j C(w, r, 1) = q_j c(w)$$

  where we're assuming all firms are identical except possibly for the level of output

- Using Shephard's lemma,

$$L_j^c(w, r, q) = q_j c'(w)$$

- Now sum to get the market demand for labor:

$$L = \sum_j L_j^c(w, r, q) = Qc'(w) = D(p)c'(w)$$

  where $Q = \sum q_j$

- But p=MC, so market demand is

$$D[c(w)]c'(w)$$

- Next, use the chain rule to write

$$\frac{\partial L}{\partial w} = Qc''(w) + D'(p)[c'(w)]^2$$

- With some effort, we get nice elasticity formulas out of this (specifically, equations 1 and 2)
  - To be finished ... in recitation
- Main points:
  - By Shepherd's lemma, the first term is the derivative of (the sum of firms') *conditional* factor demands; that's the substitution effect
  - The second term reflects the elasticity of product demand; that's the scale effect
  - These effects are both negative, yo

Please see Figs. 1-6 and Tables 2 and 4 in:
Angrist, Joshua D. "Short-Run Demand for Palestinian Labor." *Journal of Labor Economics* 14, no. 3 (1996): 425-453.