

Due Date: Weds, March 10, 2004

We will consider an economy with two classes of workers: $i = H, L$. Assume that workers work until some retirement age and do no work thereafter (that is, we ignore indifference to the timing of work). Assume that there are equal numbers of the two types. Assume that labor supply is a zero-one choice. Assume that there are no liquidity constraints on consistent lifetime savers. Assume that the interest rate and the utility discount rates are zero.

Notation

V^i	Lifetime utility, $i = H, L$, equal to the undiscounted integral of flow utility
$u^i [x_z^i] - a_z^i$	flow utility at age z if working
x_z^i	consumption when working
$v^i [c_z^i]$	flow utility at age z if not working
c_z^i	consumption when not working
w_z^i	wage at age z
R^i	retirement age
T^i	age at death
I^i	lump-sum income
t, b	social security taxes and benefits
τ, β	flow income tax rate and benefit

Assumptions

$$u^i [x_z^i] = \log [x_z^i]$$
$$a_z^i = a^i$$
$$v^i [c_z^i] = \log [c_z^i]$$
$$w_z^i = w^i$$
$$a^L \geq a^H, T^L \leq T^H, w^L \leq w^H, I^L \leq I^H$$

Part A: consistent lifetime utility maximization

Assume both types of workers do consistent lifetime utility maximization.

Question 1.

Assuming no social security and no income taxes, solve for the equations for individually optimal retirement age and consumption at each age in terms of the parameters assuming time-consistent optimal savings and retirement decisions. Solve for lifetime utility. Show the comparison of marginal utilities of consumption of the two types.

Question 2.

Assume that there is a linear tax on earnings at rate τ . Assume that each person receives a flow benefit from the income tax authority of size β whether working or not. Assume that the income tax authority has a PDV break-even budget constraint.

Solve for the equations for individually optimal retirement age and consumption at each age in terms of the parameters assuming time-consistent optimal savings and retirement. Solve for lifetime utility. Show the comparison of marginal utilities of consumption of the two types.

State the government budget constraint that relates the benefit to the tax rate.

Question 3.

Now assume that there is no income tax, but there is a social security system. There is a payroll tax at rate t . Assume that workers have individual accounts. Assume that the fraction α of an individual's taxes goes into that individual's account. The rest of payroll tax revenue will be used to finance a flow contribution of b into the account of any worker who has not yet retired. Since there are no liquidity constraints and workers do time-consistent lifetime saving, it does not matter when or how benefits are paid.

Solve for the equations for individually optimal retirement age and consumption at each age in terms of the parameters assuming time-consistent optimal savings and retirement. Solve for lifetime utility. Show the comparison of marginal utilities of consumption of the two types.

In terms of the two retirement ages, write the social security budget constraint, assuming that social security must break even for the cohort. You do not need to incorporate the endogeneity of the retirement ages.

Question 4.

Discuss the differences in incentives between the economies in questions 2 and 3.

Discuss how the analysis of incentives would be different if accounts had to be annuitized and annuitization were individually fair. (The lack of uncertainty about length of life does not prevent annuitization.)

Discuss the differences between incentives if instead of benefits being based on fair annuities separately for each type, they are fair for the cohort as a whole. Note the ambiguity in the question based on possible differences in retirement ages.

Part B: no savings before retirement, rational retirement decision and rational savings after retirement.

Assume $I^L = I^H = 0$

Question 5.

Assuming no social security and no income taxes, solve for the equations for individually optimal retirement age and consumption at each age in terms of the parameters assuming no savings. Solve for lifetime utility.

Question 6.

Assume that there is a linear tax on earnings at rate τ . Assume that each person receives a flow benefit from the income tax authority of size β whether working or not. Assume that the income tax authority has a PDV break-even budget constraint.

Solve for the equations for individually optimal retirement age and consumption at each age in terms of the parameters assuming no savings. Solve for lifetime utility. Show the comparison of marginal utilities of consumption of the two types. Note that you need to consider different regimes of government parameters.

Solve for the government budget constraint that relates the benefit to the tax rate. Be careful – there may be different equations in different regimes.

Question 7.

Now assume that there is no income tax, but there is a social security system. There is a payroll tax at rate t . Assume that workers have individual accounts which they can not access until retirement. Assume that the fraction α of an individual's taxes goes into that individual's account. The rest of payroll tax revenue will be used to finance a flow contribution of b into the account of any worker who has not yet retired. Assume that accounts have to be annuitized and annuitization is individually fair.

Derive the first order conditions for worker consumption and retirement.

In terms of the two retirement ages, write the social security budget constraint, assuming that social security must break even for the cohort.

Question 8.

Discuss the differences in incentives between the economies in questions 6 and 7.

Part C: no savings before retirement, rational savings after retirement, rational retirement decision for type one, retirement at the early entitlement age for type 2. Assume that the two types differ ONLY in the disutility of labor. Consider separately the answers depending on whether the rational retirement type has higher or lower disutility of labor.

Assume $I^L = I^H = 0$

Question 9

Assume that there is no income tax, but there is a social security system. There is a payroll tax at rate t . Assume that workers have individual accounts which they can not access until retirement. Assume that the fraction α of an individual's taxes goes into that individual's account. The rest of payroll tax revenue will be used to finance a flow contribution of b into the account of any worker who has not yet retired. Access to the accounts is restricted to those who have stopped working and are at least E years of age.

Derive the first order conditions for worker consumption and retirement for different values of E .

Discuss how the social security budget constraint varies with E , assuming that social security must break even for the cohort. How would this be different if the two types differed in wage levels?

Discuss the determination of the optimal E .