14.471: Fall 2012: Recitation 2: Impact of factor tax in 1 sector/2 factors GE model

Daan Struyven

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Question: Who bears the tax on a factor in an economy with 2 factors and 1 sector in general equilibrium? How do factor prices respond to taxes?

1 Taxes on factors

- In the US most revenue comes from production factors.
- Sectors can be industries (e.g. manufacturing/non-manufacturing), states, ...
- Begin with 1 sector model, then develop 2.
- Labor supply is given by $L = L(w)$ and capital is in fixed supply $K = \bar{K}$:

- Denote the tax rates on labor and capital respectively τ and θ. In perfect competition factors are rewarded their marginal product:

$$w = (1 - \tau)F_L(L, \bar{K})$$

$$r = (1 - \theta)F_K(L, \bar{K})$$

- Total differentiation of labor, the wage and the interest rate gives:

$$dL = L'dw$$

$$dw = -F_L d\tau + (1 - \tau)F_{LL}dL = -F_L d\tau + (1 - \tau)L'dw \tag{1}$$

$$dr = -F_K d\theta + (1 - \theta)F_{KL}dL = -F_K d\theta + (1 - \theta)F_{KL}L'dw$$

- Isolating dw in (1) allows to compute how the wage rate reacts to the tax on labor:

$$\frac{dw}{d\tau} = \frac{-F_L}{1 - (1 - \tau)F_{LL}L'} \tag{2}$$

- To simplify, let us assume that the inital taxes are zero and get the elasticity of labor supply $\eta_L^S = \frac{dL}{dw}\frac{w}{L}$. Under $\tau = 0$, we have $w = F_L$ and (2) becomes:

$$\frac{dw}{d\tau} = \frac{-w}{1 - F_{LL}L'} = \frac{-w}{1 - \frac{F_{LL}}{F_L}L'w\frac{L}{L}} = \frac{-w}{1 - \frac{LF_{LL}}{F_L}\eta_L^S} \tag{3}$$

- Assume homogeneity of degree 1 of the production function (Definition $F(\lambda L, \lambda K) = \lambda^1 F(L, K)$). If $F(L, K)$ is homogeneous of degree 1, then by Euler's theorem $1 * y = F_K K + F_L L$. Since $F(K, L)$ is HDG1, $F_L(K, L)$ is HDG0 (i.e. doubling inputs has no effect on F_L) and thus by Euler:

$$F_{LK}K + F_{LL}L = 0 * F_L = 0$$

Hence,

$$F_{LL}L = -F_{LK}K \tag{4}$$

- Why is this helpful? The cross-derivative is used in defining the elasticity of substitution (measuring how easily factors can be substituted for one another, i.e. the percentage change in the capital intensity as reaction to a change in the marginal rate of substitution between capital and labor). Since output is $F(L, K)$, we have:

$$\sigma = \frac{d\ln(\frac{K}{L})}{d\ln MRTS_{LK}} = \frac{d\ln(\frac{K}{L})}{d\ln(\frac{p_L}{p_K})}$$

- Since the production function is linear and homogeneous, we have:

$$\sigma = \frac{F_L F_K}{F_{LK}F} \tag{5}$$

- Now, we can combine (5) , (4) and (3), to get:

$$\boxed{\frac{dw}{d\tau} = \frac{-w}{1 - \frac{LF_{LL}}{F_L}\eta_L^S} = \frac{-w}{1 + \frac{F_{LK}K}{F_L}\eta_L^S} = \frac{-w}{1 + \frac{KF_K}{F}\frac{F_{LK}F}{F_LF_K}\eta_L^S} = \frac{-w}{1 + \frac{KF_K}{F}\frac{1}{\sigma}\eta_L^S} = \frac{-w}{1 + \frac{\alpha}{\sigma}\eta_L^S}} \tag{6}$$

- Interpretation:

1. Elasticity of labor supply η_L^S reduces the drop in the wage dw
2. A high elasticity of substitution increases the drop in the wage
3. A high labor share increases the drop in the wage.

2 Afternotes on 1 sector GE model

2.1 Marshall's rules:

The own-wage elasticity of demand for a category of labor is high if:

1. The elasticity of demand for the final product is high: An increase in the wage/cost of the product strongly reduces final product demand. Hence labor used in equilibrium for that product strongly drops. ("Scale effect")
2. The elasticity of supply of other factors is high: If the wage of the studied factor inceases, then other factors can be expanded without increasing their prices too much and hence the studied factor can be used much less. ("Substitution Effect")
3. The elasticity of substitution is high: The factor for which the price goes up can be easily substituted. ("Substitution Effect")
4. The share of the factor in the cost of production is high ("Scale effect")

The 4 respective rules are related to (6) in the following ways:

1. Only 1 product/sector, thus no effect in (6)

2. Since Capital is in fixed supply $K = \bar{K}$, this effect does not show up in (6)
3. $\mid \frac{dw}{d\tau} \mid$ (6) clearly increases in σ
4. If the share of capital α in (6) is low, then the share of labor $1 - \alpha$ is high and then $\mid \frac{dw}{d\tau} \mid$is large

2.2 What does it mean to "bear the whole tax"?

- Bearing the whole tax means that $d(income_{factor}) = -dRevenue = -dR$
- Since we perform an experiment where we introduce an infinitesimal tax rate , we have $dR = d(wL\tau) = wLd\tau$
- Exemple for labor: Rewrite the percentage change in the wage on the basis of (6)) by multiplying by σ as

$$\frac{dw}{w} = d\tau \left\{ \frac{-\sigma}{\sigma + \alpha\eta_L^S} \right\} \tag{7}$$

- Impact on labor income with (7):

$$d(wL) = wL'dw + Ldw = (1 + \eta_L^S)Ldw = Lwd\tau \left\{ \frac{-\sigma}{\sigma + \alpha\eta_L^S} \right\} = wLd\tau \left\{ \frac{-\sigma(1 + \eta_L^S)}{\sigma + \alpha\eta_L^S} \right\} = dR \left\{ \frac{-\sigma(1 + \eta_L^S)}{\sigma + \alpha\eta_L^S} \right\}$$

- Extreme cases:

1. If $\eta_L^S = 0$, then $\frac{d(wL)}{dR} = -1$: Labor bears 100% of the tax.
2. If $\sigma = 0$, then $d(wL) = 0$: Capital bears 100% of the tax.
   (a) labor and capital approach perfect complementarity.
   (b) Their input is a fixed combination of capital and labor. A tax on labor is identical to firms to a tax on capital as long as it raises the cost of the fixed capital and labor input bundle by the same percentage. Since any reduction in the wage, received by labor, will reduce labor supply, and make part of the capital stock redundant, the equilibrium involves no change in the wage but a fall in the interest rate s.t. the loss to capital owners equals the tax revenue.
3. If $\sigma = \infty$, then $d(wL) = -(1 + \eta_L^S)dR$: Labor bears more than 100% of the tax burden.
   (a) $\sigma = \infty$ means that capital and labor are perfect substitutes. Hence, their after-tax user costs to competitive firms must be identical.
   (b) Assuming CRS $F = \alpha(K + L)$ where α equals F_K and F_L. Since $F_K = F_L = \alpha = w/(1 - \tau)$, the pre-tax wage$w$ falls by the full amount of the tax. Now, a lower after-tax wage reduces labor supply.
   (c) Hence, labor bears more than 100% of the burden.

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