

## 1.6 Business cycle applications

- Outside finance premium: is it countercyclical?
- Investment volatility: is there amplification/propagation?
- general answer: it depends on the shocks (persistence)
- We focus on amplification

- almost temporary shock  $\rho = .05$

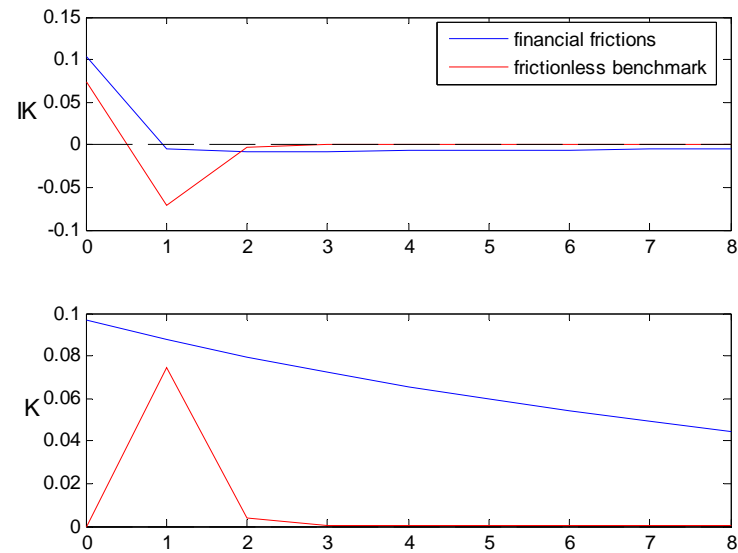


Figure 10:

- more persistent shock  $\rho = .1$

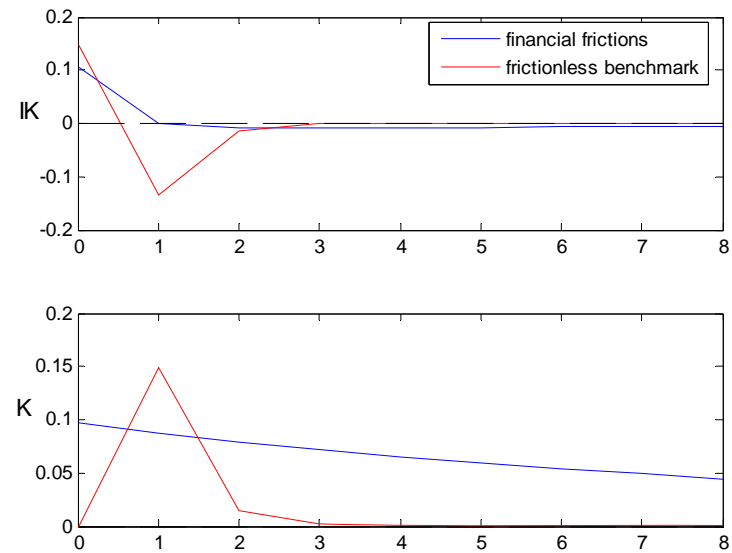


Figure 11:

- persistent shock  $\rho = .5$

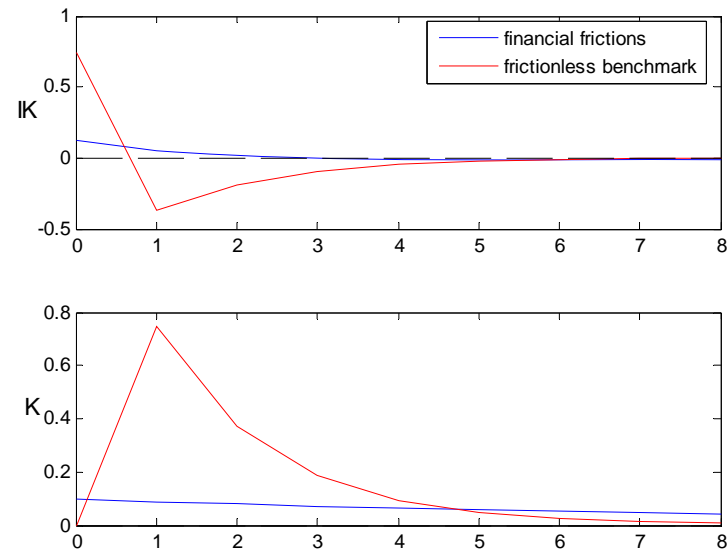


Figure 12:

- Related empirical micro issue: do firms with tighter constraints respond more/less to cash flow shocks?
- Fazzari, Hubbard, Petersen (1998): Yes  
(see table before)
- Kaplan and Zingales:
  - in theory: maybe
  - in empirics: no

- Crucial macro issue: are contracts state-contingent?

- Balance sheet

$$R_t k_t - b_t$$

- Investment

$$k_{t+1} = \frac{R_t k_t - b_t}{q_t^m - \theta \beta_C \mathbb{E} [R_{t+1}]}$$

- question is  $b_t$  sufficiently responsive to shocks?

## 1.6.1 Amplification

- In Kiyotaki-Moore no state contingent contracts
- Feed-back investment-asset prices in KM
- Recall that

$$R_t = A_t F_1(K_t, 1) + q_t^o$$

- Suppose we are in region where  $R_t k_t - b_t$  close to zero/bankruptcy
- with non-state contingent contracts that may happen

- then small positive productivity shock increases  $k_{t+1}$  more than proportionally
- this increases  $q_t^o \rightarrow$  larger increase in  $K_{t+1}$  and so on
- Krishnamurthy (2003): it all depends on ability to condition on aggregate shocks



- Detour on models of Costly State Verification
- Caveat: CSV helps explain why non-state contingent debt is used at the micro level, but it does not really help at the macro level
- In general aggregate shocks seem relatively easy to condition upon: why sometimes balance sheets very exposed?

## 1.6.2 A failure of diversification

- Three period version
- No adjustment costs
- *Risk averse* consumers  $\mathbb{E}[u(c_1 + c_2)]$

- In period 0 no investment, no consumption, only financial contracting ex ante
- Shock in period 1:  $s = H, L$
- In period 1 entrepreneurs have initial endowment  $\omega_s^E \in \{\omega_H, \omega_L\}$
- Consumers have endowment  $\omega_s^C$  in period 1 and work in period 2
- No aggregate shock

$$\omega_s^C + \omega_s^E = 1$$

- Entrepreneurs
- In period 1: Invest  $k_{2,s}$
- In period 2: produce  $F(k_{2,s}, l_{2,s}) - w_s l_{2,s}$
- Balance sheet of the entrepreneur at date 1

$$n_{1,s} = \omega_s^E + z_s^E$$

- state contingent contracts  $z_s$  are available at date 0
- question: will they hedge?

- Consumer problem

$$\begin{aligned} \max \quad & \sum \pi_s u(c_{1,s} + c_{2,s}) \\ \text{s.t.} \quad & \sum q_s z_s^C \leq 0 \\ & c_{1,s} = \omega_s^C + z_s^C \\ & c_{2,s} = w_s \end{aligned}$$

- Entrepreneur problem

$$\begin{aligned}
 \max \quad & \sum \pi_s (c_{1,s}^E + c_{2,s}^E) \\
 \text{s.t.} \quad & \sum q_s z_s^E \leq 0 \\
 & c_{1,s}^E + k_{2,s} = \omega_s^E + z_s^E \\
 & c_{2,s}^E = R_{2,s} k_{2,s}
 \end{aligned}$$

- $\theta = 0$  only internal funds can be used
- Value function of entrepreneur now is simply

$$V(\omega_s^E + z_s^E, s) = R_{2,s} (\omega_s^E + z_s^E)$$

as long as  $R_{2,s} \geq 1$ .

- Reduced form consumer's problem

$$\begin{aligned} \max \quad & \sum \pi_s u \left( \omega_s^C + z_s^C + w_s \right) \\ \text{s.t.} \quad & \sum q_s z_s^C \leq 0 \end{aligned}$$

- Reduced form entrepreneur's problem

$$\begin{aligned} \max \quad & \sum \pi_s R_{2,s} \left( \omega_s^E + z_s^E \right) \\ \text{s.t.} \quad & \sum q_s z_s^E \leq 0 \end{aligned}$$

- market clearing: financial market

$$z_s^C + z_s^E = 0$$

labor market

$$R_{2,s} = F_1(K_s, 1), \quad K_s = \omega_s^E + z_s^E$$

## Equilibrium

$$\frac{\pi_L u'(\omega_L^C + z_L^C + w_L)}{\pi_H u'(\omega_H^C + z_H^C + w_H)} = \frac{q_L}{q_H} = \frac{\pi_L R_{2,L}}{\pi_H R_{2,H}}$$

- multiple equilibria possible
- symmetric equilibrium always exists: full diversification

$$K_H = K_L = \sum \pi_s \omega_s^E$$

- asymmetric equilibrium  $K_H > K_L$  (also the opposite possible!)
- pecuniary externality



## Example

$$\pi_L = \pi_H = 1/2$$

$$u(c) = c^{1-\gamma}$$

$$F(k, 1) = Ak^\alpha$$

$$\omega_L^E = 0, \omega_H^E = 1$$

$$\omega_L^C = 1, \omega_H^C = 0$$

$$z = z_L^E$$

$$q = \frac{q_L}{q_H}$$

Two relations

$$\frac{u'(1 - z + Az^\alpha)}{u'(qz + A(1 - qz)^\alpha)} = q$$
$$\frac{z^{\alpha-1}}{(1 - qz)^{\alpha-1}} = q$$

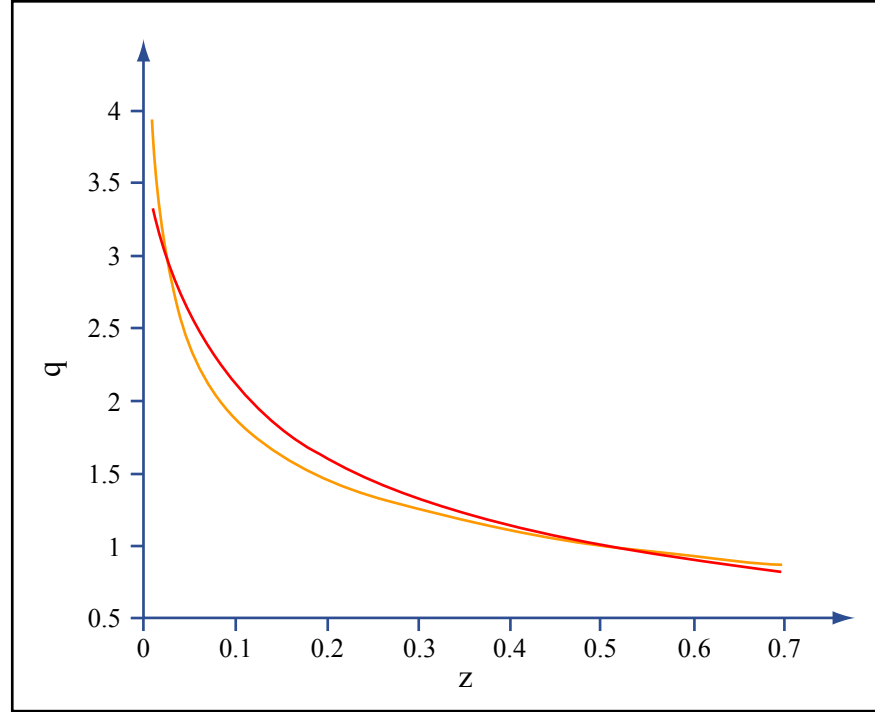


Figure by MIT OCW.

Examples:

- US vs Japan asset price bubble
- real estate concentrated in banks  $\rightarrow$  feed back

- stock market diffused -> no feed back
- Dollarized economies: consumers want deposits in US\$ to be safe, then banks lend in US\$, then companies exposed to XR risk, wages more volatile, consumers want deposits in US\$...
- very different balance sheet effects
- “financial fragility” difficult to assess, credit chains...