# 14.454 - Macroeconomics IV
# Problem Set 3

05/04/2011

# 1 Question 1 - Jacklin's Critique to Diamond-Dygvig

Take the Diamond-Dygvig model in the recitation notes, and consider Jacklin's implementation of the social optimum via a firm that pays dividends and whose shares can be sold in a spot market at $t = 1$. Suppose now that consumers can also directly invest in the long technology, without having to invest in the firm (that is, firms do not have the exclusivity of access to the projects). Show then that in this setting, the social optimum $(c_1^*, c_2^*)$ is not implantable. In particular, show that if the firm offers a contract with dividend $D = \pi c_1^*$, a single agent may deviate by investing all his resources in the long technology at $t = 0$ and obtain higher utility (if the rest of the agents are actually investing all their income in shares from this firm).

# 2 Question 2 - Public Debt and Bursting Bubbles

There's an OLG economy of agents that live only two periods. Generation born at $t$ has preferences $U\left(c_t^y, c_{t+1}^o\right) = \alpha \ln\left(c_t^y\right) + (1 - \alpha) \ln\left(c_{t+1}^o\right)$ where superscript $y$ stands for "young" and $o$ for "old". Each agent is endowed with one unit of consumption good at birth, and has no endowment when old. Let $N_t$ be the number of agents born at time $t$: we assume that

$$N_t = (1 + g) N_{t-1}$$

where $g > 0$ is the growth rate of the economy (and in particular, of the endowment). Also suppose that agents can only transfer resources from $t$ to

$t + 1$ in a storage technology that pays 1 unit of consumption at $t + 1$ by unit invested of consumption at $t$. Borrowing and lending among consumers does not exist, since old people cannot repay young people when these are old.

(a) Characterize the equilibrium allocation of the economy. Show that the allocation is not Pareto Optimal, by finding a Pareto Optimal scheme . In particular, find the "pay as you go" social insurance scheme in this economy, and show that it Pareto Dominates the equilibrium allocation.

(b) Imagine now that there exist an infinitely lived government that can issue debt contracts with consumers. In particular, the government offers the following contract

- At $t = 0$, sell $D_0 > 0$ bonds at a price of 1 (this is a normalization).

- At $t = 1$, pay $R_0 D_0$ to bond holders (with $R_0 > 1$). To do this, the government issues $D_1$ bonds to pay interest, and the scheme keeps on going.

In general, the budget constraint of the government at time $t$ is then

$$D_t - D_{t-1} R_{t-1} \geq 0$$

The government in this case is creating a new asset, that was not in the economy before (also note that this is exactly a Ponzi scheme, but run by the government!). Show that by picking a constant debt per capita $d \equiv \frac{D_t}{N_t}$ and a constant gross interest rate $R > 1$ the government can implement the same as in the Social Security Scheme

(c) Imagine now that there is no public debt issued by the government, but rather that there exist a "bubble" asset $B$ that pays no dividend. Let $\{p_t\}_{t=0}^{\infty}$ be the equilibrium prices of the bubble. Imagine too that the bubble may burst: in every history in which the bubble did not burst, the bubble burst in the next period with probability $(1 - \lambda)$ where $\lambda \in (0, 1)$. If the bubble bursts, then the bubble has no value from tomorrow on (i.e. $p_T = 0$ for all $T \geq t$)

Consider the consumption problem of a young agent born at time $t$, and that the bubble has not bursted until now. Let $S_t$ be the savings of generation $t$ in the storage technology, and $B_t$ be the demand for the bubble asset. Find the optimal asset demands

$$
\begin{aligned}
S_t &= S\left(p_t, p_{t+1}^B\right) \\
B_t &= B\left(p_t, p_{t+1}^B\right)
\end{aligned}
$$

where $p_{t+1}^B$ is the price of the bubble at $t+1$ if the bubble does not burst (if it does, then the price is 0). Do this under the assumption that $p_{t+1}^B > p_t$

**(d)** Find a stationary equilibrium with bubbles in this economy, in which

$$\frac{p_{t+1}^B}{p_t} = 1 + \gamma > 1 \text{ for all } t : \text{ bubble did not burst at } t$$

And show that in any such equilibrium, $\gamma = g$. Does this equilibrium implement the "pay as you go" social insurance allocation? If it does not: which one is better?

**(e)** Suppose now that on top of the bubble, the government issues the debt contract specified in part (b). Will there be bubbles in equilibrium? Think about policy implications of your findings

# 3    Question 3 - Bubbles and Investment

Consider an economy with $2-$period lived overlapping generations of agents. Population is constant. When young, agents have a unit endowment of labor, which they supply inelastically on the labor market at the wage $w_t$. They consume $c_{t,t}$ and save $w_t - c_{t,t}$. For the moment, assume all their savings go into physical capital $k_{t+1}$. When old, they rent capital at the rate $r_t$ and consume $c_{t,t+1} = r_t k_t$. Their preferences are

$$\ln\left(c_{t,t}\right) + \beta \ln\left(c_{t,t+1}\right)$$

The production function is Cobb-Douglas:

$$y_t = k_t^\alpha l_t^{1-\alpha}$$

**(a)** Solve the optimal savings problem of the consumer born at time $t$, taking as given the prices $w_t$ and $r_t$

**(b)** Solve the problem of the representative firm and use market clearing in the labor market to derive expressions for $w_t$ and $r_t$ as functions of $k_t$

**(c)** Substitute the result in (b) in the optimal savings rule derived in (a) and obtain a law of motion for $k_t$

**(d)** Find a steady state with constant capital stock $k_t = k_{SS}$. Show that if

$$\frac{\alpha}{1-\alpha}\frac{1+\beta}{\beta} < 1 \tag{1}$$

then $\alpha k_{SS}^{\alpha-1} < 1$

**(e)** Suppose the economy begins at $t = 0$ at the steady state capital stock. Write down the resource constraint of the economy in steady state and argue that if $\alpha k_{SS}^{\alpha-1} < 1$ it is possible to make all agents better off by reducing the capital stock in all periods.

**(f)** Suppose now agents are allowed to trade a useless, non-reproducible asset, in fixed unit supply, which trades at the price $p_t$, the "bubble" asset. Argue that if $p_t > 0$ and $k_t > 0$, the agent must be indifferent between holding capital and the bubble asset, and derive the associated arbitrage condition.

**(g)** Show that if (1) holds, then there exists a steady state equilibrium with $p_t = p_{SS} > 0$ and $\alpha k_{SS}^{\alpha-1} = 1$

# 4    Question 4: Allen & Gale (2000) - Fundamental Values

Take the "Bubbles and Crisis" model seen in class (Lecture notes 5). We want to get the fundamental price of investing in the risky asset without risky asset.

**(a)** Consider the "complete contracts" setting (i.e. with no bankruptcy or default), in which a single risk neutral agent endowed with wealth $B > 0$ has to decide how much to invest in the safe asset $(X_S)$, and how much to invest in the risky asset $(X_R)$ to maximize expected profits (minus non-pecuniary costs), subject to the constraint that $X_S + PX_R \leq B$. Set up the problem and set the first order condition for $X_R$

**(b)** Setting $X_R = 1$ and using the FOC found in (a), find $P^f$ as the price at which an agent who invests his own money would be willing to hold one unit of the risky asset (that is, the marginal utility of having an extra unit of risky asset in the optimum plan).

**(c)** Under what conditions will we have that $P^f > P$, where $P$ is the equilibrium price?

# 5    Question 5: Caballero & Krishnamurthy (2006) - Welfare and Bubbles

Consider the model in Caballero and Krischnamurthy (JME, 2006) that we studied in class. Compute equilibrium welfare in the case where there is no

bubble and in the case where there is a bubble. Compare and determine whether:
**(a)** it is always better to have a bubble; **(b)** it is always better to have no bubble;
or **(c)** it depends on parameters. Throughout, you must assume the parameter
restrictions imposed in the paper are satisfied.

# 6 Question 6: Stock Prices, Dividends and Bubbles (Exam question, 2004)

Assume you are in an economy where the stock price $p_t$ is given by the standard
arbitrage equation

$$p_t = \frac{1}{1+r}\mathbb{E}_t\left(p_{t+1} + d_{t+1}\right) \tag{2}$$

where

$$d_t - \bar{d} = \rho\left(d_{t-1} - \bar{d}\right) + \varepsilon_t \text{ where } \varepsilon_t \sim_{i.i.d} f\left(\varepsilon\right) \text{ where } \mathbb{E}_{t-1}\left(\varepsilon_t\right) = 0$$

**(a)** Use iterated expectations to solve for the price $p_t$ as a function of ONLY
future expected dividends. What assumption do you implicitly need to do this?

**(b)** Assume that $\rho < \frac{1}{1+r}$. Use iterated expectations to find an expression
for the expectation (as of time $t$) for dividends at time $t + i = \mathbb{E}_t\left(d_{t+i}\right)$ that is
a function of only $\bar{d}, \rho$ and $d_t$

**(c)** Use your answers from (a) and (b) to find an expression for $p_t$ as a func-
tion of $\bar{d}, \rho$ and $d_t$. Call this solution the arbitrage equation (2) the "fundamental
price" $p_t$

**(d)** Now assume that the price of the stock has a bubble component $b_t = \left(1+r\right)^t b_0$ with $b_0 > 0$. Prove that the price $p_t = p_t^* + b_t$ is also a solution to
the arbitrage condition (2) and that our assumption from part (a) is no longer
necessary

**(e)** Why are individiduals willing to pay a higher price $p_t$ for the stock than
the fundamental price corresponding to the present value of the dividends, $p_t^*$?

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