

14.451 Midterm

Spring 2005

You have **1.5 hours**. You must answer **all** questions. No books or notes are allowed. **Good luck!**

Question 1 [60 points/100]

Consider the social planner's problem in the standard neoclassical growth model. Preferences are given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \quad \text{with} \quad u(c) = \frac{c^{1-1/\theta}}{1-1/\theta},$$

where $\beta \in (0, 1)$ and $\theta > 0$. The resource constraint is

$$c_t + k_{t+1} = Af(k_t)$$

where $f(k) = k^\alpha$, $\alpha \in (0, 1)$, and $A > 0$. (For simplicity, I have assumed full depreciation, $\delta = 1$.)

Let k^* the solution to $Af'(k^*) = \beta^{-1}$ (that is, the steady-state level of the capital stock) and suppose that the initial capital stock is zero ($k_0 = 0$).

Consider now the following variation of the model. In period 0, and only in that period, the economy (the planner) can borrow some resources from an international lender. If the economy borrows an amount $D \geq 0$, it has to pay back to the international lender an amount RD in period 1, where R denotes one plus the interest rate at which international borrowing is available. Borrowing from abroad in any other period, or lending abroad in any period, is not possible; and R satisfies $R > \beta^{-1}$, or equivalently $R > Af'(k^*)$.

(a) Reconsider the problem of the social planner given the availability of international borrowing. Write out the resource constraint of the economy in periods 0, 1, and $t \geq 2$. Write out the first-order conditions that characterize the optimal allocation, including the optimal choice of D .

(b) Compute in closed form the optimal k_1 as a function of R, A, α . Does k_1 increase or decrease with R and/or A ? Explain.

(c) Is D higher than k_1 , equal, or lower? Explain.

(d) Draw the paths of optimal c_t and k_t against t for two different international borrowing rates, $R_H > R_L (> \beta^{-1})$. Describe how the increase in R affects consumption and capital in all dates; explain why.

(e) Finally, consider the following business-cycle exercise. The economy starts in period 0 with $k_0 = k^*$ (rather than $k_0 = 0$) and is hit by a temporary recession: in periods 0 and 1 productivity falls from A to \tilde{A} , where $\tilde{A} < A$; in periods $t \geq 2$, productivity is back to its initial level A .

Consider then the response of the economy under two alternative scenarios. In scenario "Closed", the economy can not borrow abroad; in scenario "Open", the economy can borrow during periods 0 and 1 at a rate R such that $\tilde{A}f'(k^*) < R < Af'(k^*)$.

Under both scenarios consumption c_0 and investment k_1 fall in response to the recession, while domestic interest rates increase; but, under which scenario is the reaction of consumption, investment and interest rates higher?

(You do not need to provide a formal result; just a concise explanation.)

Question 2 [40 points/100]

(a) Consider a simple endogenous growth model in which utility is logarithmic, output is linearly homogenous in capital and government services, and government services are financed with a distortionary tax on capital holdings:

$$\begin{aligned} U_0 &= \sum \beta^t u(c_t) \\ u(c_t) &= \log c_t \\ y_t &= k_t^\alpha g_t^{1-\alpha} \\ g_t &= \tau k_t. \end{aligned}$$

Consider the competitive equilibrium of this economy (*not* the planner's solution). How does the growth rate depend on τ ? Explain.

(b) A tax on consumption increases the incentive to save and therefore boosts growth. True/False?

(c) If agents are risk neutral, incomplete financial markets have no effect on savings and growth. True/False?

(d) Consider an economy in which firms engage in R&D to produce new products. If the private return to innovation is lower than the social return, a subsidy to R&D activity is both necessary and sufficient for implementing the efficient growth rate. Moreover, if the efficient growth rate is obtained with such a subsidy, the entire allocation (i.e., the level of output, consumption, etc.) is efficient. True/False? Explain.

Good Luck!