14.381: Statistics

Problem Set 6

No due date

This problem set is intended for your own practice.

1. Let $X_1, \ldots, X_n$ be iid Poisson (λ) and let λ have a Gamma (α, β) distribution (the conjugate family for Poisson)

$$\pi(\lambda) = \lambda^{\alpha-1} \frac{\exp\{-\lambda/\beta\}}{\Gamma(\alpha)\beta^{\alpha}}$$

   (a) Find the posterior distribution for λ.

   (b) Calculate posterior mean and variance. *Hint:* mean of Gamma (α, β) is $\alpha\beta$; the variance is $\alpha\beta^2$.

   (c) Discuss whether the prior vanishes asymptotically.

   (d) Assume that α is an integer. Show that the posterior for $\frac{2(n\beta+1)}{\beta}\lambda$ given X is $\chi^2(2(\alpha + \Sigma X_i))$.

   (e) Using result of (d), suggest a 95%-credible interval for λ.

2. Suppose that the random variables $Y_1, ..., Y_n$ satisfy

$$Y_i = \beta x_i + e_i, \quad i = 1, ..., n,$$

   where $x_1, ..., x_n$ are fixed constants and $e_1, ..., e_n$ are i.i.d. normals with mean 0 and known variance σ^2. The prior for β is normal $N(\beta_0, \tau^2)$.

   (a) Find the posterior for β.

   (b) The maximum likelihood estimator is the OLS estimator, $\hat{\beta}_{OLS} = \frac{\sum_{i=1}^n Y_i x_i}{\sum_{i=1}^n x_i^2}$. What is the variance of the OLS estimator? How is $\hat{\beta}_{OLS}$ distributed?

   (c) What is the posterior mean of β? How is it related to $\hat{\beta}_{OLS}$?

   (d) Construct posterior credible set for β.

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