1. a) F - The "Bertrand Paradox" refers to the situation where firms are engaging in perfect competition (price = marginal cost) so could not cover any fixed costs they may have.

   b) T - The monopoly pricing formula is $\frac{P-c'}{P} = \frac{-1}{\epsilon}$

   c) F - Buy-It-Now should be a popular choice for inexpensive items because transaction costs are typically higher for auctions and typically don't scale with the cost of the item, but eBay did not offer that option for their first several years of operation.

   d) F - The Diamond search model produces the equilibrium where all firms charge the monopoly price in the presence of any search costs, so, no, it cannot explain price dispersion.

   e) T (u) - In both the Hotelling & Salop models, we saw firms who could charge above marginal cost because of differentiation. How far above marginal cost is determined in part by whether firms are free to enter or whether there are barriers or delays to entry.

2. a) $\Pi^{0^*} = \Pi^{1^*} = \frac{t}{2}$

   b) Moving closer to the competing firm will gain you market share but will intensify price competition (because you're becoming less differentiated)

   c) 1) If one firm's market length was less than any other firm's half market length, that firm would move, colocate with the second firm, and capture half its market.

      2) If the left-most firm was not collocated with its closest interior firm, it could capture a larger market share by moving closer to its closest interior firm (same with right-most).

      (Note that because prices are fixed in this model, the type of trade off described in part b does not exist.)

   d) Yes, voting is like a linear city model where prices are fixed (a vote is akin to a purchase) and differentiation is not spatial but along a political spectrum.

3. a) $\sqrt[N]{\frac{1}{N+1}} * \frac{N}{N+1}$ (see class notes)

   b) $\frac{N-1}{N+1}$ (see class notes)

   c) Consider three cases: ($x_{(1)}$ is highest valuation, etc.)

if $x_{(1)} < P$	no sale, same as posted price
if $x_{(2)} < P_r < x_{(1)}$	sale at P_r, same as posted price
if $P_r < x_{(2)}$	sale at $x_{(2)}$, greater than posted price

   (This is true for all N except $N = 1$, which is exactly the same as posted price.)

d) Revenue:

$$= \underbrace{\int_0^{P_r} P_r \, \text{Prob}\{x_{(1)} > P_r | x_{(2)} = x\} \, f_{x_{(2)}}(x) \, dx}_{\substack{\text{equal to } \frac{1-P_r}{1-x}, \\ \text{because this is conditional on } x_{(2)} \text{ from} \\ \text{uniform being equal to } x, \, x_{(1)} \text{ has a} \\ \text{has a uniform distribution on } [x, \, 1]}}$$

$$= \frac{1 - P_r}{1 - x} + \int_{P_r}^1 x f_{x_{(2)}}(x) \, dx$$

take the derivative of this expression wrt P_r, set equal to zero, and solve for P_r:

$$\frac{\partial \text{Revenue}}{\partial P_r} = \underbrace{P_r \frac{1 - P_r}{1 - P_r} f_{x_{(2)}}(P_r)}_{\substack{\text{from upper limit} \\ \text{of first term}}} - \underbrace{P_r f_{x_{(2)}}(P_r)}_{\substack{\text{from lower limit} \\ \text{of second term}}} + \int_0^{P_r} \frac{d}{dP_r} P_r \frac{1 - P_r}{1 - x} f_{x_{(2)}}(x) \, dx = 0$$

first two terms cancel and we're left with:

$$\int_0^{P_r} (1 - 2P_r) \frac{f_{x_{(2)}}(x)}{1 - x} \, dx = 0$$

$$\underbrace{(1 - 2P_r)}_{\substack{\text{this must} \\ \text{equal zero}}} \underbrace{\int_0^{P_r} \frac{f_{x_{(2)}}(x)}{1 - x} \, dx}_{\text{strictly positive}} = 0$$

$$\Rightarrow P_r = \frac{1}{2}$$

4. • Groupon has not followed the advice in the article but has rather eliminated the group purchasing features (quietly)

    • I was looking for answers where students pointed out the importance of being able to induce supplier competition and noted specific ideas in the article that would do so.

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