Exam Practice Questions

Problem 1: Consider the game below.

	A	B
A	$(2,1)$	$(0,0)$
B	$(0,0)$	$(1,2)$

(a) What is player 1's minimax payoff?

(b) Describe the set of all Nash equilibria (pure and mixed).

Suppose player 1 (the row player) is uncertain about player 2's preferences. The actual game is one of the following two games:

	A	B
A	$(2,1)$	$(0,0)$
B	$(0,0)$	$(1,2)$

	A	B
A	$(2,-1)$	$(0,0)$
B	$(0,0)$	$(1,4)$

Player 2 learns her preferences before making a choice, while player 1 must make a choice without further information. The common prior is that the first game is played with probability p, and the second with probability $1-p$.

(c) As a function of p, describe the set of all Bayes-Nash equilibria.

Problem 2: Each player i in a population of size N makes a non-negative contribution $x_i \in \mathbb{R}$ to a public good. If the vector of investments is $(x_1, x_2, ..., x_N)$, player i's payoff is

$$u_i(\mathbf{x}) = 2\sqrt{\sum_{j=1}^{N} x_j} - x_i.$$

(a) Write down player i's best response map. What is the total level of investment $\sum_{j=1}^{N} x_j$ in any Nash equilibrium? What is the efficient level of investment?

Suppose instead that spillovers from other players' investments are imperfect, and player i's payoff is

$$u_i(\mathbf{x}) = 2\sqrt{x_i + \delta \sum_{j \neq i} x_j} - x_i$$

for some $\delta \in (0, 1)$.

(b) Write down player i's best response map. What is the total level of investment $\sum_{j=1}^{N} x_j$ in any Nash equilibrium? What is the efficient level of investment?

Suppose $N = 2$ and now the game takes place in two stages. In the first stage, players 1 and 2 invest efforts $s_1, s_2 \geq 0$ at constant marginal cost c to establish a relationship. These investments result in the tie strength $\delta(s_1, s_2) = \min\{s_1 + s_2, 1\}$. Once these investments are made, the resulting tie strength is observed, and we move to the second stage. In the second stage, the players invest as above. The payoff to player i is

$$u_i(\mathbf{x}, \mathbf{s}) = 2\sqrt{x_i + \delta(s_i, s_{-i})x_{-i}} - x_i - cs_i.$$

(c) As a function of c, what tie strength forms in equilibrium? What is the efficient outcome?

Problem 3: Consider a variant of the mean-field diffusion model from the first lecture on diffusion. Each agent in a large population chooses between two actions 0 and 1. Agents have degrees drawn independently from the distribution D and private values drawn independently from a uniform distribution on $[0, 1]$. If an agent has degree d and value v, and a neighbors adopt, the payoff to adoption is

$$u(d, v, a) = av - c.$$

That is, the payoff to adoption increases linearly in the number of neighbors who end up adopting. Recall the neighbor degree distribution $\tilde{D}$ that corrects for the friendship paradox:

$$\mathbb{P}(\tilde{D} = d) = \frac{\mathbb{P}(D = d) \cdot d}{\sum_{k \in \mathbb{N}} \mathbb{P}(D = k) \cdot k}.$$

Time is discrete. Let $\sigma_{t,d}$ denote the fraction of degree d agents adopting at time t. At time $t + 1$, each agent chooses an action to maximize expected utility, assuming that neighbors will each adopt with independent probability

$$\sigma_t = \sum_{d \in \mathbb{N}} \mathbb{P}(\tilde{D} = d)\sigma_{t,d}.$$

(a) Suppose the degree distribution D takes the value 3 with probability one. Compute $\sigma_{t+1,3}$ as a function of $\sigma_{t,3}$. Find the steady state adoption levels. Which are stable?

(b) Suppose now that D takes the values 2 and 4 with equal probability. Write down the neighbor degree distribution $\tilde{D}$. Compute the best response maps $\sigma_{t+1,2}$ and $\sigma_{t+1,4}$ as a function of $\sigma_{t,2}$ and $\sigma_{t,4}$. Find the steady state adoption levels.