14.124
Holmstrom
Spring 2017

HOMEWORK 2

*QUESTION 1

Let x be a random variable with n discrete outcomes $x_1,...,x_n$. The probability of each outcome depends on a privately chosen action by an agent. The agent can either choose L or H. If the agent chooses L, the probability vector for x is $p_L = (p_L(1),..,p_L(n))$. If the agent chooses H, the probability vector for x is $p_H = (p_H(1),..,p_H(n))$. Assume that the Monotone Likelihood Ratio Property (MLRP) holds strictly, that is, $p_H(i)/p_L(i)$ is strictly increasing in i.

The agent's preferences can be represented by the separable utility function u(s,e) = s - c(e), where s is the payment by the principal to the agent and e is the agent's unobserved action (e = L or H). Assume c(H) > c(L) = 0. The principal's utility is v(x-s) = x - s. Thus, both the agent and the principal are risk neutral. However, the agent has no wealth, which constrains the principal's payment to be non-negative: $s_i \geq 0$ for every outcome x_i.

a. Set up the program that implements H at the minium expected cost to the principal (accounting for the agent's limited liability constraint).

b. Show that in the minimum cost contract, $s_i = 0$ for i = 1,..,n-1 and $s_n > 0$.

QUESTION 2.

Adam is thinking about buying fire insurance for his summer house. The probability that the house is going to catch fire is assessed to be one in ten (.10). This probability is exogenous and not affected by anything Adam does. If the house catches fire, the damage is going to be either 1000 or 2000 with probabilities depending on whether Adam has prepared himself for a fire or not (bought fire extinguisher, etc). Adam's opportunity cost of preparing himself for a fire is c > 0; assume Adam's level of preparation cannot be observed by the insurance company. If Adam is prepared, the probability that the damage from the fire is 2000 equals (.5). If Adam isn't prepared, that same probability is (.7).

Adam's utility function takes the form: u(m) – {cost of preparing for fire}. Here u is a strictly concave utility function over money m.

a. Set up a program that identifies the second-best insurance plan in which Adam prepares himself for a fire and **the insurance company makes zero profits**.

b. Solve for the second-best solution in part (a) in terms of payoffs to Adam in the three cases: no fire, fire and damage is 1000, fire and damage is 2000 (use notation S_1, S_2, S_3 for Adam's payoffs in these three cases). You will not be able to solve payoffs in closed form, but you should be able to provide a characterization and interpretation of the second-best contract.

Suppose now that Adam has a third option: he can prepare for a fire in a manner that would entirely eliminate the chance of incurring the 2000 damage (at some higher cost C > c). Describe qualitatively the optimal insurance contract that implements this new level of preparedness in the most efficient manner?

*QUESTION 3.

In the linear incentive model, assume the cost function of the agent is $c(e, s) = ½(e + s)^2 – s$, where e is time put into working for the principal and s is time spent with family and friends. The negative sign in front of s comes from the fact that the agent values time spent with family and friends. The agent's utility function is exponential with risk aversion coefficient r = 2. Output from work is $x = e + \theta$, where θ is Normally distributed with mean zero and variance 4.

a. Suppose the incentive coefficient $\alpha = .3$. How much time will the agent spend on family and friends?

b. Suppose the principal can set s = 0 by asking the agent to come to the office each day. What is the optimal choice of α in that case?

c. Suppose the agent's best alternative to working for the principal is spending time with family and friends. What will the principal have to pay the agent as a fixed salary (β) in part b?

QUESTION 4.

This question is about optimal contracting in a principal-agent model with uniform distribution function. Let the production technology be described by $x = e + \theta$. The agent chooses "effort" e privately (the principal cannot observe e). The principal observes x, which therefore can be used as a basis for contracting. Assume θ is distributed uniformly on [0,1]. The agent's utility function takes the form U(m,e) = u(m) - c(e), over money m and effort e. Here u is concave, c is convex, and both are increasing functions. The principal is risk neutral.

a. What is the first-best contract (ie the contract when e can be observed and contracted on)?

b. Show that the first-best outcome can be achieved even when e is not observed by the principal.

That is, show that in the special case of a uniform distribution the principal-agent problem does not entail any welfare loss relative to the first-best.

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