14.122 Problem Set #1

1. Find a game which is not solvable by pure strategy iterated strict dominance but which does have an unique pure strategy Nash equilibrium.

2. Consider the model of Cournot competition discussed in class where the inverse demand function is $P(q) = 1 - q$ and the firms have zero marginal costs. Show that it is strictly dominated for the firms to produce any quantity greater than $\frac{1}{2}$ Write the set of strategies which are not strictly dominated for the firms as an interval $[\underline{S}^1, \overline{S}^1]$. Find the interval of strategies $[\underline{S}^2, \overline{S}^2]$ which are not strictly dominated when a firm's opponent chooses a quantity in $[\underline{S}^1, \overline{S}^1]$. Prove by induction that when the iterated strict dominance process is continued the set of strategies remaining at stage $2k$ is the interval $[\underline{S}^{2k}, \overline{S}^{2k}]$ where

$$\underline{S}^{2k} = \sum_{j=1}^{k} \frac{1}{4^j}.$$

$$\overline{S}^{2k} = \frac{1}{2} - \frac{1}{2} \sum_{j=1}^{k-1} \frac{1}{4^j}.$$

Conclude that the game is solvable by iterated strict dominance.

3. Consider the following game-theoretic model of the equilibrium determination of the cleanliness (and effort distribution) of an apartment shared by two roommates. In the game, the two roommates simultaneously choose the effort, e_1 e_2, to spend on apartment cleaning. They each get utility from the cleanliness of the apartment (which is a function of the sum of the efforts) and disutility from the effort they personally expend. Player 1 places a higher valuation on cleanliness. Specifically, assume that e_1 and e_2 are chosen from the set of nonnegative real numbers and that

$$u_1(e_1, e_2) = k \log(e_1 + e_2) - e_1$$
$$u_2(e_1, e_2) = \log(e_1 + e_2) - e_2,$$

where $k > 1$

(a) Find the two players best response functions.

(b) Find the pure strategy Nash equilibria of the game. How does the equilibrium distribution of effort reflect the differences in the players' tastes.

(c) Try to write down a modification of the model above in which the outcome seems more fair.

4. Write out a formal specification (strategy sets, payoff functions, etc.) of the two player version of the game described in problem 1.8 of Gibbons. What is the pure strategy Nash equilibrium of the two player game?

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5. Find all of the Nash equilibria of the following games.

	L	M	R
A	2,0	1,1	4,2
B	3,4	1,2	2,3
C	1,3	0,2	3,0

	W	X	Y	Z
A	6,1	5,3	4,0	3,2
B	4,0	4,2	4,0	4,2
C	1,6	4,5	2,8	0,2
D	3,9	3,0	5,0	3,2

1.8. Consider a population of voters uniformly distributed along the ideological spectrum from left $(x = 0)$ to right $(x = 1)$. Each of the candidates for a single office simultaneously chooses a campaign platform (i.e., a point on the line between $x = 0$ and $x = 1$). The voters observe the candidates' choices, and then each voter votes for the candidate whose platform is closest to the voter's position on the spectrum. If there are two candidates and they choose platforms $x_1 = .3$ and $x_2 = .6$, for example, then all voters to the left of $x = .45$ vote for candidate 1, all those to the right vote for candidate 2, and candidate 2 wins the election with 55 percent of the vote. Suppose that the candidates **care only about being elected-they do not really care about their** platforms at all! **If there are two candidates, what is the pure-strategy Nash equilibrium?** If there are three candidates, exhibit a pure-strategy Nash equilibrium. (Assume that any candidates who choose the same platform equally split the votes cast for that platform, and that ties among the leading vote-getters are resolved by coin flips.) See Hotelling (1929) for an early model along these lines.