

Lecture 3

Representation of Games

14.12 Game Theory

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Game: Ingredients

- Who are the players (decision makers)?
- What moves are available to each player and when?
- What does each player know at the time of each of his decisions?
- What are the outcomes and payoffs at the end?

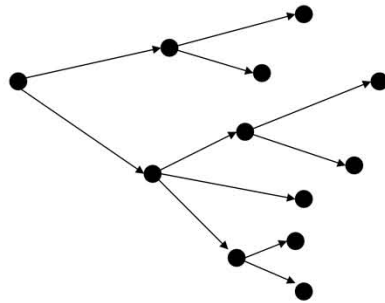
Road Map

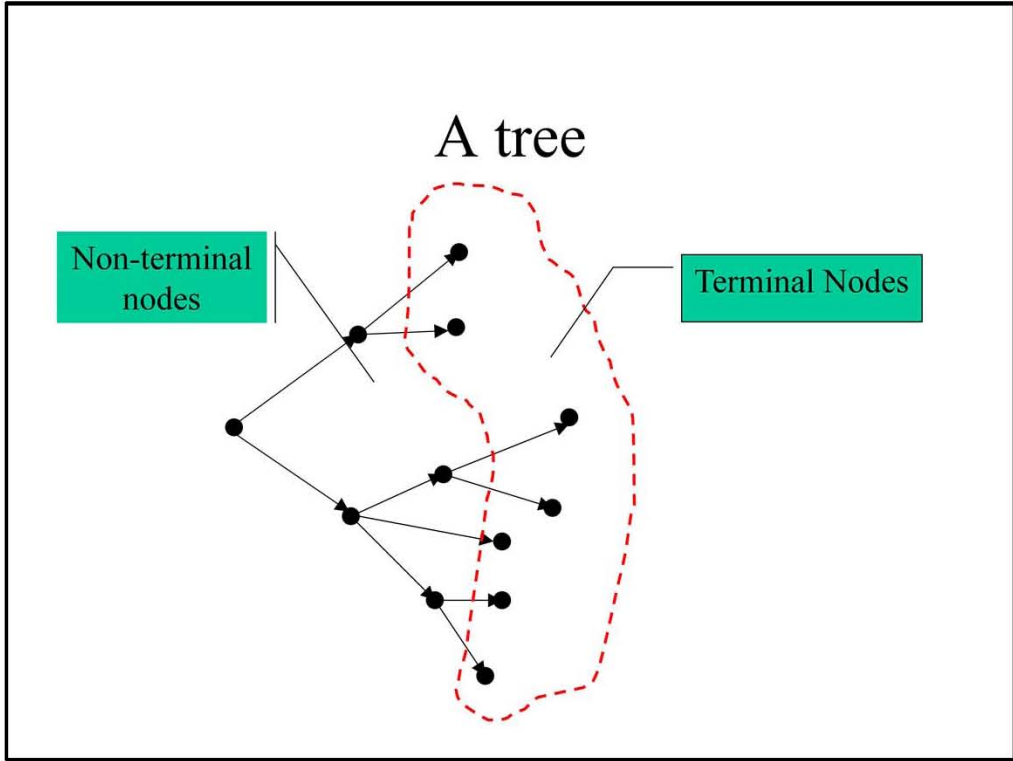
1. Extensive form representation
2. Strategy
3. Normal form representation
4. Mixed strategy

Extensive-form representation

Definition: A **tree** is a set of nodes connected with directed arcs such that

1. There is an initial node;
2. For each other node, there is one incoming arc;
3. each node can be reached through a unique path.





Extensive form – definition

Definition: A game consists of

- a set of players
- a tree
- an allocation of each non-terminal node to a player
- an informational partition (to be made precise)
- a payoff for each player at each terminal node.

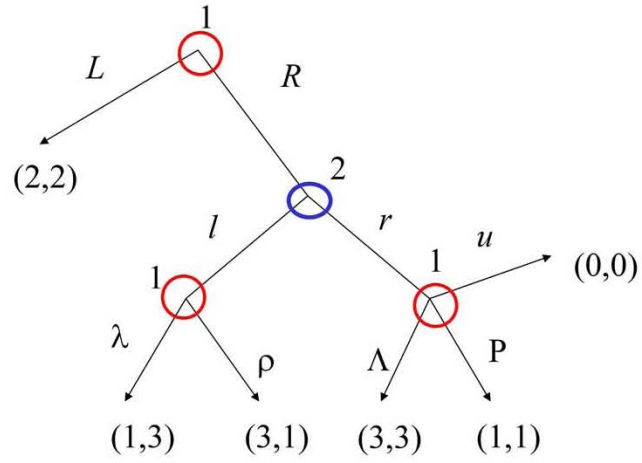
Information set

An **information set** is a collection of nodes such that

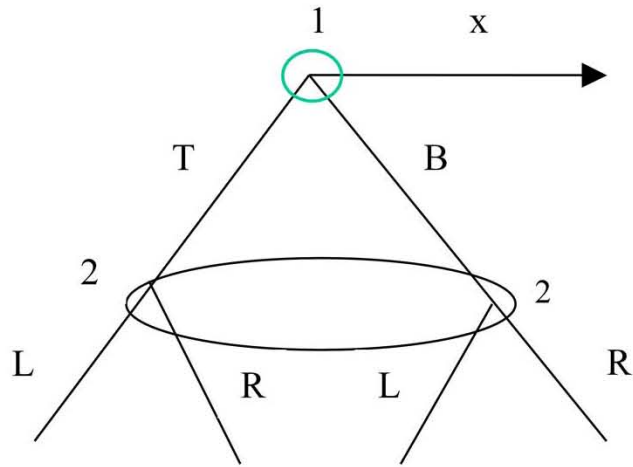
1. The same player is to move at each of these nodes;
2. The same moves are available at each of these nodes.

An **informational partition** is an allocation of each non-terminal node of the tree to an information set.

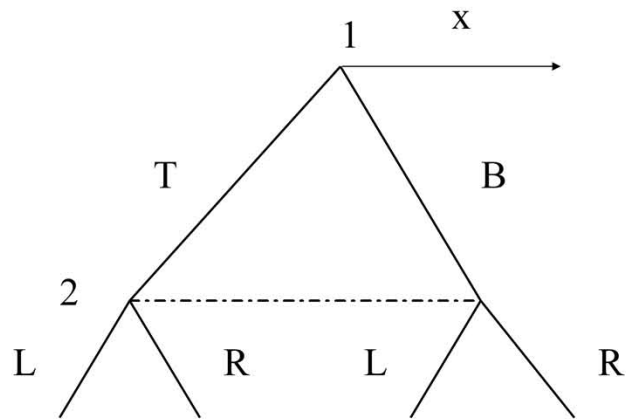
A game



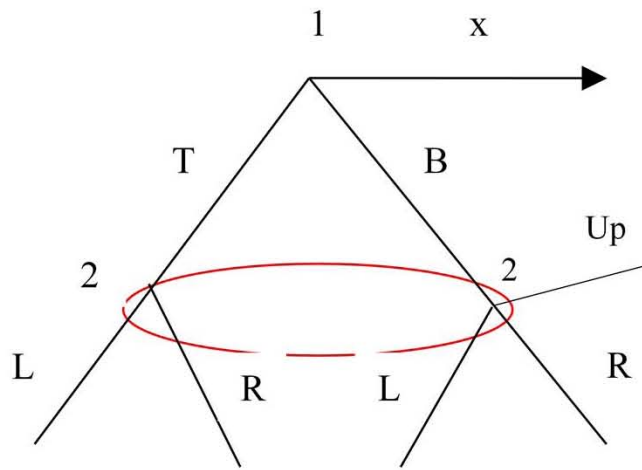
Another game



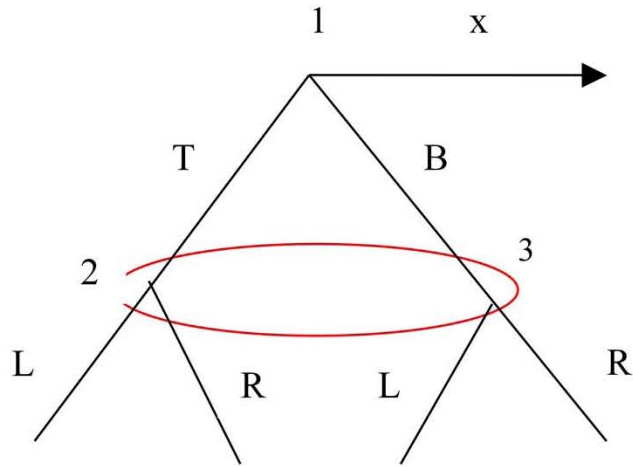
The Same Game



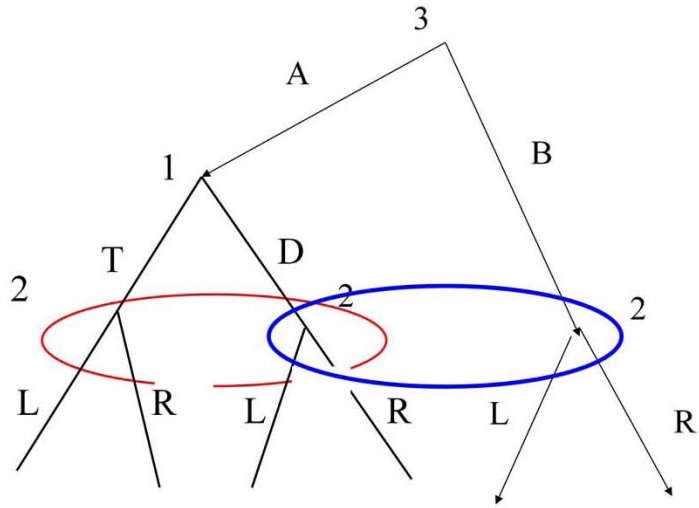
What is wrong?



What is wrong?



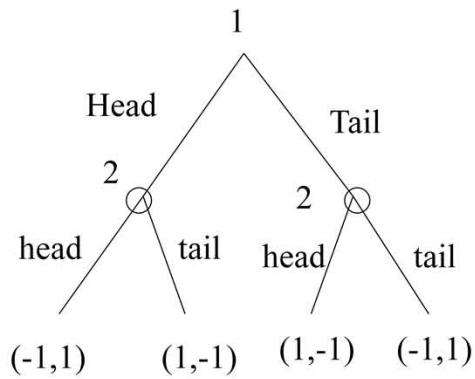
What is wrong?



Strategy

A **strategy** of a player is a **complete contingent-plan**, determining which action he will take at each information set he is to move (including the information sets that will not be reached according to this strategy).

Matching pennies with perfect information



2's Strategies:

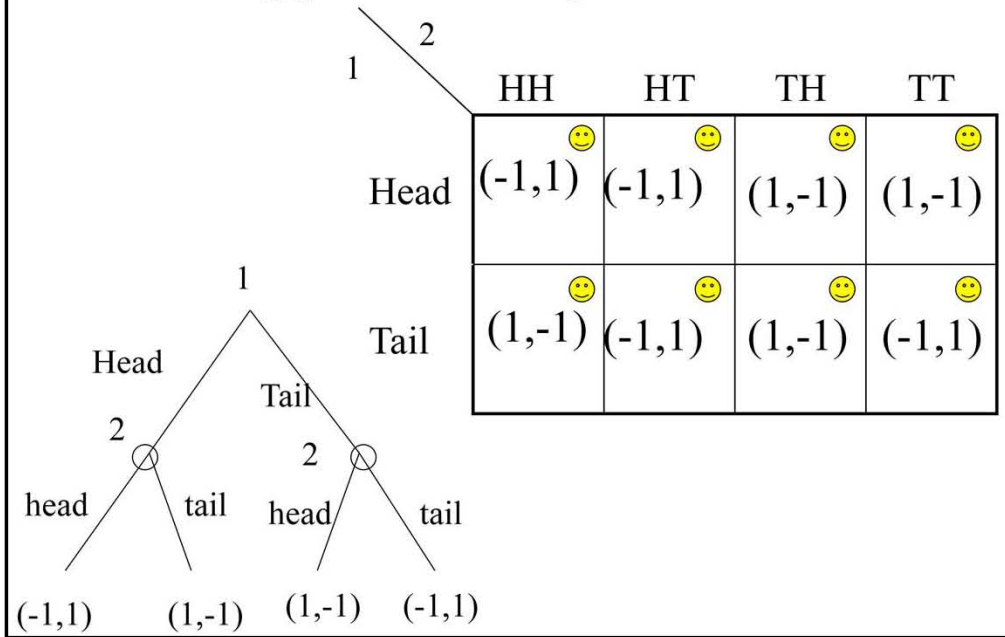
HH = Head if 1 plays Head,
Head if 1 plays Tail;

HT = Head if 1 plays Head,
Tail if 1 plays Tail;

TH = Tail if 1 plays Head,
Head if 1 plays Tail;

TT = Tail if 1 plays Head,
Tail if 1 plays Tail.

Matching pennies with perfect information



Normal-form representation

Definition (Normal form): A game is any list

$$G = (S_1, \dots, S_n; u_1, \dots, u_n)$$

where, for each $i \in N = \{1, 2, \dots, n\}$,

- S_i is the set of all strategies available to i ,
- $u_i : S_1 \times \dots \times S_n \rightarrow \mathfrak{R}$ is the VNM utility function of player i .

Assumption: G is “common knowledge”.

Definition: A player i is rational iff he tries to maximize the expected value of u_i given his beliefs.

Chicken







	
 	 
$(-1, -1)$	$(1, 0)$
$(0, 1)$	$(1/2, 1/2)$

Image by MIT OpenCourseWare.

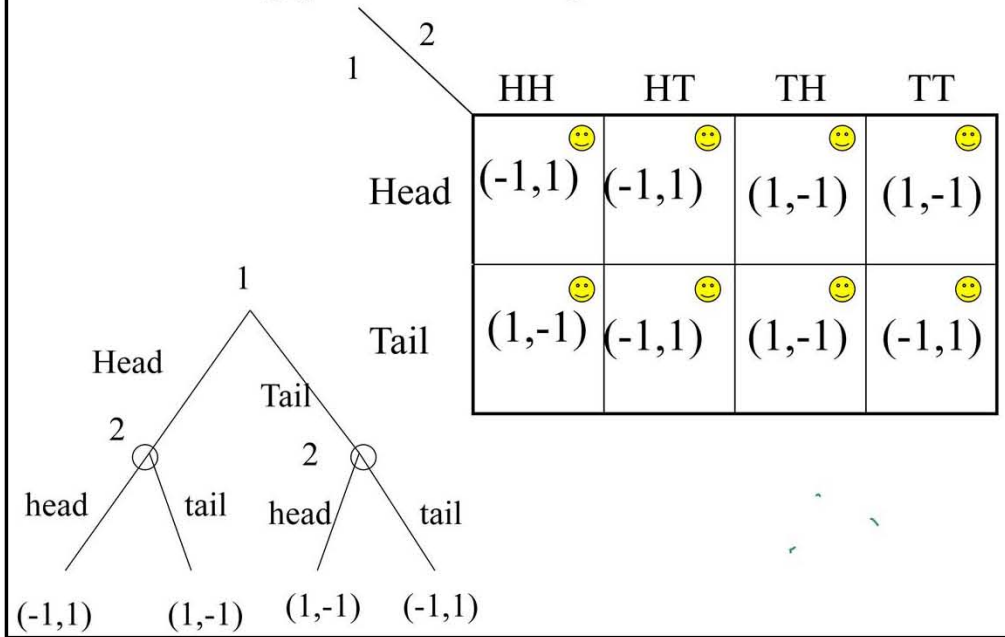
Matching pennies

	Head	Tail
Head	$(-1,1)$	$(1,-1)$
Tail	$(1,-1)$	$(-1,1)$

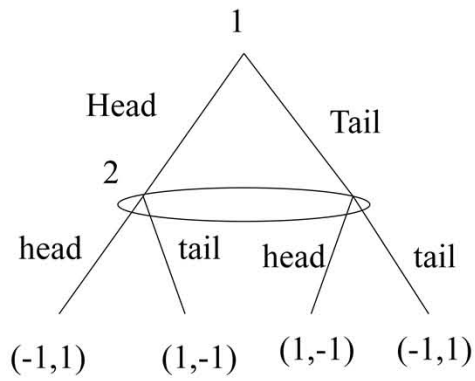
Extensive v. Normal Forms

- Extensive to Normal:
 - Find the set of strategies for each player
 - Every strategy profile s leads to an outcome $z(s)$, a terminal history
 - Utility from s is $u(z(s))$
- Normal to Extensive: many possibilities

Matching pennies with perfect information

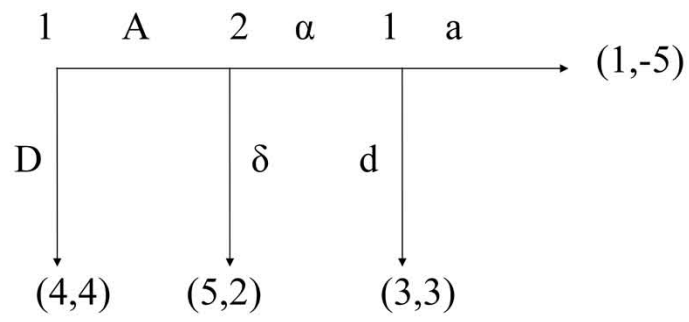


Matching pennies with imperfect information

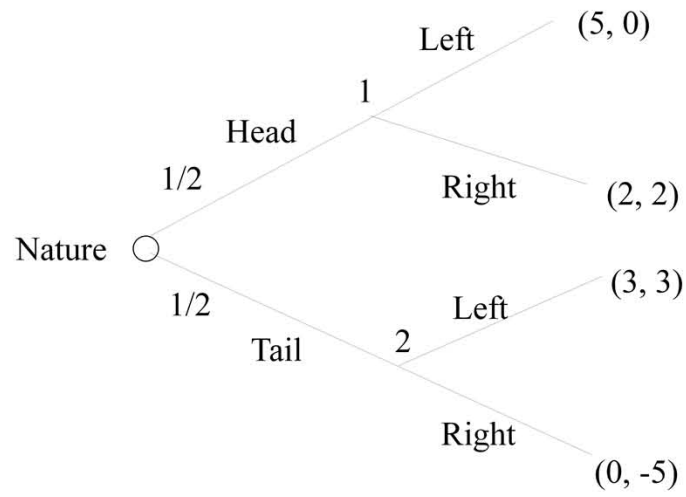


	1	2		
			Head	Tail
Head			$(-1,1)$	$(1,-1)$
Tail			$(1,-1)$	$(-1,1)$

A game



A game with nature



Mixed Strategy

Definition: A **mixed strategy** of a player is a probability distribution over the set of his strategies.

Pure strategies: $S_i = \{s_{i1}, s_{i2}, \dots, s_{ik}\}$

A mixed strategy: $\sigma_i: S_i \rightarrow [0,1]$ s.t.

$$\sigma_i(s_{i1}) + \sigma_i(s_{i2}) + \dots + \sigma_i(s_{ik}) = 1.$$

If the other players play $s_{-i} = (s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_n)$, then the expected utility of playing σ_i is

$$\sigma_i(s_{i1}) u_i(s_{i1}, s_{-i}) + \sigma_i(s_{i2}) u_i(s_{i2}, s_{-i}) + \dots + \sigma_i(s_{ik}) u_i(s_{ik}, s_{-i}).$$

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