Lecture 13
Infinitely Repeated Games

14.12 Game Theory
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Road Map

1. Definitions
2. Single-deviation principle
3. Examples

Infinitely repeated Games with observable actions

- $T = \{0,1,2,\ldots,t,\ldots\}$
- G = "stage game" = a finite game
- At each t in T, G is played, and players remember which actions taken before t;
- Payoffs = Discounted sum of payoffs in the stage game.
- Call this game $G(T)$.

Infinitely-repeated PD

	C	D
C	5,5	0,6
D	6,0	1,1

Some Strategies:

Grim Strategy: Play C at t=0; thereafter play C iff D has never played before.

Tit for Tat: Start with C; thereafter, play what the other player played in the previous round.

Naively Cooperate: always play C.

C=> 5+5dVd

C=> 6+dVD

C <=> d>1/5.

Definitions

The *Present Value* of a given payoff stream $\pi = (\pi_0, \pi_1, \ldots, \pi_t, \ldots)$ is

$$PV(\pi;\delta) = \pi_0 + \delta\pi_1 + \ldots + \delta^t\pi_t + \ldots$$

The *Average Value* of a given payoff stream π is

$$(1-\delta)PV(\pi;\delta) = (1-\delta)(\pi_0 + \delta\pi_1 + \ldots + \delta^t\pi_t + \ldots)$$

The *Present Value* of a given payoff stream π *at* t is

$$PV_t(\pi;\delta) = \pi_t + \delta\pi_{t+1} + \ldots + \delta^s\pi_{t+s} + \ldots$$

A *history* is a sequence of past observed plays

e.g. (C,D), (C,C), (D,D), (D,D) (C,C)

Recall: Single-Deviation Principle

- $s = (s_1, s_2, \ldots, s_n)$ is a SPE
- ⇔ it passes the following test
- for each information set, where a player *i* moves,
  - fix the other players' strategies as in *s*,
  - fix the moves of *i* at other information sets as in *s*;
  - then *i* cannot improve her conditional payoff at the information set by deviating from s_i at the information set only.

Single-Deviation Principle: Reduced Game

- $s = (s_1, s_2, \ldots, s_n)$, date t, and history h fixed
- **Reduced Game**: For each terminal node a of the stage game at t,
  - assume that s is played from t+1 on given (h,a)
  - write PV(h,a,s,t+1) for present value at t+1
  - Define utility of each player i at the terminal node a as

$$u_i(a) + \delta \, \mathrm{PV}(h,a,s,t+1)$$

- **Single-Deviation Principle:** s is SPE ⇔ for every h and t, s gives a SPE in the reduced game

Reduced Game for (Grim,Grim)

With previous defection:

	C	D
C	$5 + \delta/(1-\delta)$ $5 + \delta/(1-\delta)$	$0 + \delta/(1-\delta)$ $6 + \delta/(1-\delta)$
D	$6 + \delta/(1-\delta)$ $0 + \delta/(1-\delta)$	$1 + \delta/(1-\delta)$ $1 + \delta/(1-\delta)$

Without previous defection:

	C	D
C	$5 + 5\delta/(1-\delta)$ $5 + 5\delta/(1-\delta)$	$0 + \delta/(1-\delta)$ $6 + \delta/(1-\delta)$
D	$6 + \delta/(1-\delta)$ $0 + \delta/(1-\delta)$	$1 + \delta/(1-\delta)$ $1 + \delta/(1-\delta)$

C=> 5+5dVd

C=> 6+dVD

C <=> d>1/5.

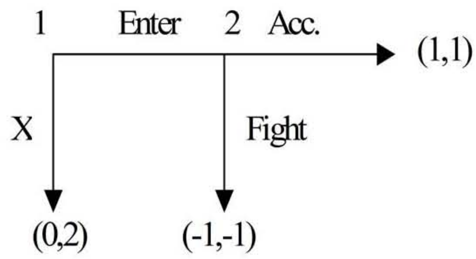
Is (Tit-for-tat,Tit-for-tat) a SPE?

- **Tit-for-Tat:** Start with C; thereafter, play what the other player played in the previous round.
- No!
- Consider (C,C) at *t*-1 and Player 1.
  - C => $5/(1-\delta)$
  - D => $6/(1-\delta^2)$
  - No Deviation ⇔ $\delta \geq 1/5$.
- Consider (C,D) at *t*- and Player 1.
  - C => $5/(1-\delta)$
  - D => $6/(1-\delta^2)$
  - No Deviation ⇔ $\delta \leq 1/5$.
- Not SPE if $\delta \neq 1/5$.

Modified Tit-for-Tat

Start with C; if any player plays D when the previous play is (C,C), play D in the next period, then switch back to C.

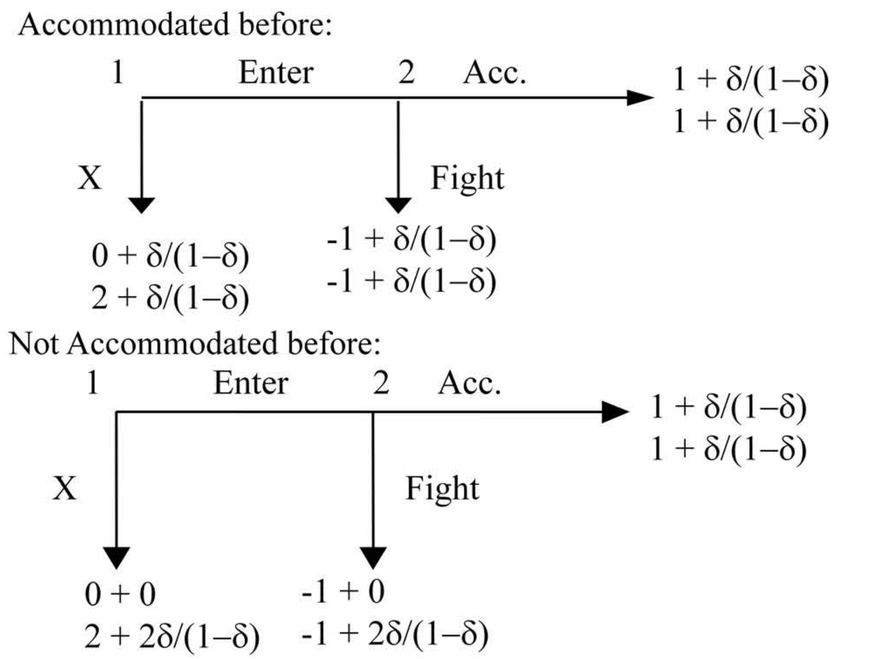
Infinite-period entry deterrence



Strategy of Entrant:
Enter iff
Accomodated before.

Strategy of Incumbent:
Accommodate iff
accomodated before.

Reduced Games



14.12 Economic Applications of Game Theory
Fall 2012