14.12 Game Theory
10/16/2008

Prof. Casey Rothschild

Instructions. This is an open book exam; you can use any written material. You may use a calculator. You may not use a computer or any electronic device with wireless communication capacity. You have one hour and 20 minutes. Each question is 25 points, and the breakdown of points within each question is specified below. Good luck!

1. A normal form Game is depicted below. Player 1 chooses the row (T or B), Player 2 chooses the column (L,M, or R), and Player 3 chooses the matrix (W,X,Y, or Z).

W

	L	M	R
T	1,1,10	1,0,10	5,-5,0
B	0,1,10	0,0,-10	5,-5,0

X

	L	M	R
T	1,1,-10	1,0,10	5,-5,0
B	0,1,10	0,0,10	5,-5,0

	L	M	R
T	20,15,2	12,20,2	5,-5,40
B	16,20,2	20,15,2	5,-5,40

Y

	L	M	R
T	8,8,-20	8,8,-20	5,40,-20
B	8,8,-20	8,8,-20	5,40,-20

Z

   (a) (5 points) Write a strategic form game tree for this game, and indicate the payoffs on any two terminal nodes of your choice. You *don't* need to write the payoffs at any other terminal nodes.

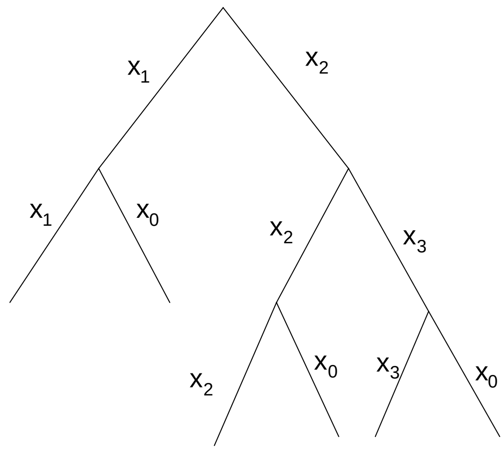
   (b) (5 points) Find all pure strategy Nash Equilibria for this game.

   (c) (5 points) Find a mixed strategy equilibrium of the two-player game (between Players 1 and 2) that results if Player 3 is forced to play Y.

   (d) (10 points) Find all of the rationalizable strategies in the full **3 player** game. Show your reasoning.

2. **“Quickies” Part (a) Required. CHOOSE 1 or (b) or (c).**

(a) (**REQUIRED;** 15 points) If Bob, Sue and Mary are rational voters with strict preferences given in the table to the right, with top being better, and all this is common knowledge, what outcome do you expect the binary agenda at left to produce?



Bob	Sue	May
X_1	X_0	X_3
X_3	X_2	X_0
X_2	X_1	X_1
X_0	X_3	X_2

Strictly Worse

(b) (**CHOOSE (b) OR (c);**10 points) What, if anything, is wrong with the following pattern of choices? (If you don’t have a calulator and want to know: $5/6 = 0.8\overline{3}$.)

- Choice 1: $0.5[\$100] + 0.5[\$0] = p \succ q = 0.6[\$80] + 0.4[\$0]$.
- Choice 2: $1[\$80] = r \succ s = (5/6)[\$100] + (1/6)[\$0]$.(10 points)

(c) **(If you already answered (b) don’t do this – we won’t grade it!)** Consider a Judicial Settlement problem:

- At each date $t = 1, 2, ...n$ the **Plaintiff** makes a settlement offer s_t. The **Defendent** can either accept or reject each offer. (Note that the same player is making offers each period.)
- If the Plaintiff accepts at date t, the “game” ends with the Defendent paying s_t to the Plaintiff, and the Defendent and Plaintiff paying tc_D and tc_P to their respective lawyers.
- If the Plaintiff rejects at all dates, the case goes to court. The Plaintiff will lose and have to pay J to the Defendent. The Plaintif and Defendent will also have to pay lawyer’s fees $(n + 1)c_P$ and $(n + 1)c_D$ respectively.

If it is common knowledge that Plaintiff and Defendent are sequentially rational, how much will the settlement be, and at what date will it take place? (You **don’t** have to show the backward induction reasoning explicitly. Just give the answer and 1 or two sentences of intuition.)

3. In this question you are asked to compute the rationalizable strategies in a linear Bertrand-duopoly with discrete prices and fixed "startup" costs. We consider a world where the prices must be an odd multiple of 10 cents, i.e.,

$$P = \{0.1, 0.3, 0.5, ..., 0.1 + 0.2n, ...\}$$

is the set of feasible prices. For each price p, the demand is:

$$Q(p) = \max\{1 - p, 0\}.$$

We have two firms $N = \{1, 2\}$, each with 0 marginal cost, but each with a fixed "start-up" cost k. That is, if the firm produces a positive amount, it must bear the cost k. *If it produces 0, it does not have to pay k.* Simultaneously, each firm sets a price $p_i \in P$. Observing prices p_1 and p_2, consumers buy from the firm with the lowest price. When prices are equal, they divide the demand equally between the two firms. Each firm i wishes to maximize its profit.

$$\pi_i(p_1, p_2) = \left\{ \begin{array}{cc} p_i Q(p_i) - k & \text{if } p_i < p_j \text{ and } Q(p_i) > 0 \\ p_i Q(p_i)/2 - k & p_i = p_j \text{ and } Q(p_i) > 0 \\ 0 & \text{otherwise} \end{array} \right\}.$$

(a) If $k = 0.1$:

1. (5 points) Show that $p_i = 0.1$ is strictly dominated.
2. (5 points) Show that there are prices greater than the monopoly price ($p = 0.5$) that are *not* strictly dominated.
3. (15 points) Iteratively eliminate all strictly dominated strategies to find the set of rationalizable strategies. Explain your reasoning.

4. There are three "dates", $t = 1, 2, 3$, and two players: Government and Worker.

- At $t = 1$, Worker expends effort to build $K \in [0, \infty)$ units of capital.
- At $t = 2$, Government sets tax rates $\tau_K \in [0, 1]$ and $\tau_e \in [0, 1]$ on capital-holdings and on labor income.
- At $t = 3$, Worker chooses effort $e_2 \in [0, \infty)$ to produce output Ke_2.
  The payoffs of Government and Worker are:

$$U_G = \tau_K K + \tau_e K e_2$$

and

$$U_W = (1 - \tau_e)Ke_2 + (1 - \tau_K)K - K^2/2 - e_2^2/2.$$

(a) (20 points) Solve the game by backwards induction.

(b) (5 points) Now suppose before the game is played, Government can "delegate" its job to an independent IRS Agent at period $t = 0$. At $t = 0$, the Government will offer a fraction $\beta_K \in [0, 1]$ of its capital tax revenue and a fraction $\beta_e \in [0, 1]$ of its labor tax revenue to the Agent. The Agent can either Accept or Reject. If the Agent Accepts, she will take the place of the Government in setting tax rates $\tau_K \in [0, 1]$ and $\tau_e \in [0, 1]$ at $t = 2$. If the Agent Rejects, the game procedes as before. The Agent has payoff:

$$U_A = \left\{ \begin{array}{c} \beta_K \tau_K K + \beta_e \tau_e K e_2 - \varepsilon \text{ if accept} \\ 0 \text{ if reject.} \end{array} \right\},$$

where ε is a very small but positive "acceptance" cost. The Government's payoff will be:

$$U_G = \left\{ \begin{array}{c} (1 - \beta_K)\tau_K K + (1 - \beta_e)\tau_e K e_2 \text{ if Agent Accepts} \\ \tau_K K + \tau_e K e_2 \text{ otherwise} \end{array} \right\}.$$

Assume that an Agent who accepts will choose the smalest tax rate(s) consistent with sequential rationality. Find an equilbrium of the game using backward induction, and briefly comment on it.