14.12 Game Theory
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Homework 1

Due on 9/25/2012

1. Consider a homeowner with Von-Neumann and Morgenstern utility function u, where $u(x) = 1 - e^{-x}$ for wealth level x, measured in million US dollars. His entire wealth is his house. The value of a house is 1 (million US dollars), but the house can be destroyed by a flood, reducing its value to 0, with probability $\pi \in (0, 1)$.

   (a) What is the largest premium P is the homeowner is willing to pay for a full insurance? (He pays the premium P and gets back 1 in case of a flood, making his wealth $1 - P$ regardless of the flood.)

   (b) Suppose there is a local insurance company who has insured n houses, all in his neighborhood, for premium P. Suppose also that with probability π there can be flood in the neighborhood destroying all houses (i.e., either all houses are destroyed or none of them is destroyed). Suppose finally that P is small enough that the homeowner has insured is house. Having insured his house, what is the largest Q that he is willing to pay to get the $1/n$ share of the company? (The value of the company is the total premium it collects minus the payments to the insured homeowners in case of a flood.)

   (c) Answer part (b) assuming now that the insurance company is global. It insured n houses in different parts of the world (all outside of his neighborhood), so that the destruction of houses by flood are all independent (i.e., the probability of flood in one house is π independent of how many other houses has been flooded).

   (d) Assume that n is large enough so that $\sum_{k=0}^{n} C_{n,k} e^{k/n} \pi^k (1-\pi)^{n-k} \cong e^{\pi + \pi(1-\pi)/(2n)}$, discuss your answers to above questions (briefly). [Here, $C_{n,k}$ denotes the number of k combinations out of n, and the sum is one minus the expected payoff from the loss due to the payments to the flooded houses.]

2. Consider the game in which the following are commonly known. First, Ann chooses between actions a and b. Then, with probability 1/3, Bob observes which action Ann has chosen and with probability 2/3 he does not observe the action she has chosen. In all cases (regardless of whether he has observed Ann chose a, or he has observed Ann chose b, or he has not observed any action), Bob chooses between actions α and β. The payoff of each player is 1 after (a, α) and (b, β) and 0 otherwise.

   (a) Write the above game in extensive form.

   (b) Write the above game in normal form.

3. Consider the following variation of the above game. First, Ann chooses between actions a and b. Then, Bob decides whether to observe the chosen action of Ann or not, by choosing between the actions Open and Shut, respectively. In all cases, Bob then

chooses between actions α and β. The payoff of Ann is 1 after (a, α) and (b, β) and 0 otherwise, regardless of whether Bob chooses Open or Shut. The payoff of Bob is equal to the payoff of Ann if he has chosen Shut, and his payoff is equal to the payoff of Ann minus 1/2 if he has chosen Open.

(a) Write the above game in extensive form.

(b) Write the above game in normal form.

4. Federal government is planning to build an interstate highway between two states, named A and B. The highway costs $C > 0$ to the government, and the value of the highway to the states A and B are $v_A \geq 0$ and $v_B \geq 0$, respectively. Simultaneously, each state $i \in \{A, B\}$ is to bid $b_i \in [0, \infty)$. If $b_A + b_B \geq C$ the highway is constructed. For any distinct $i, j \in \{A, B\}$, state i pays $C - b_j$ to the federal government if $b_j < C \leq b_A + b_B$. (There is no payment otherwise.) The payoff of a state is the value of the highway to the state minus its own payment to the government if the highway is built, and 0 otherwise. (You can focus on the case $v_A + v_B < C$.)

(a) Write this in the normal form.

(b) Check if there is a dominant strategy equilibrium, and compute it if there is one.

14.12 Economic Applications of Game Theory
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