

1) Part a (i).

3rd Degree Price Discrimination

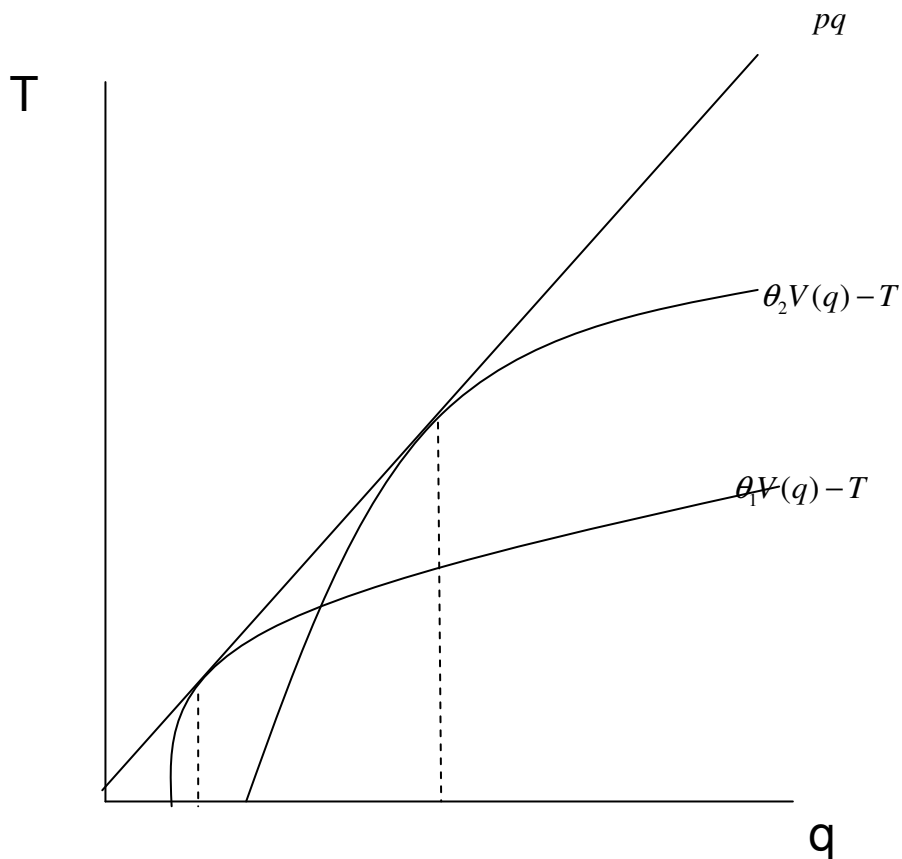
$$P_L = P_H, K_L = K_H = 0$$

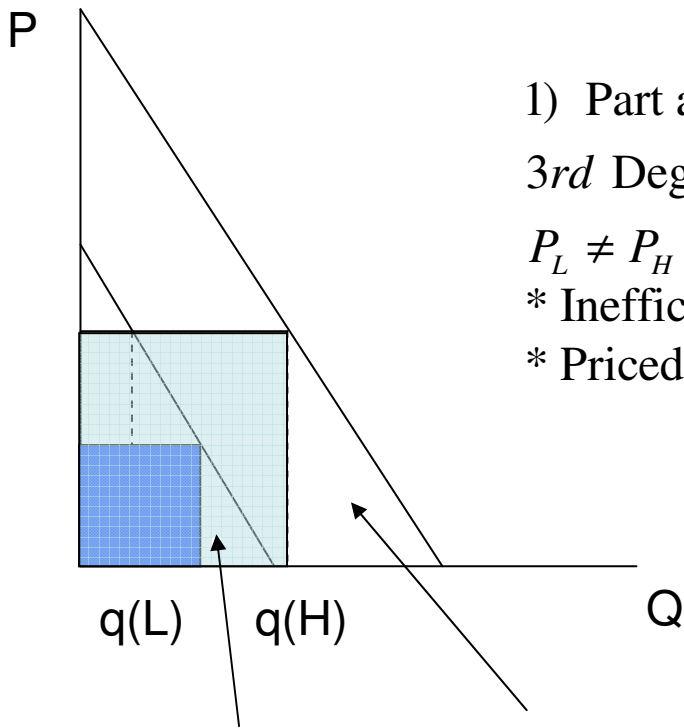
* Inefficiency in both markets

* Price between the monopoly price w/ Seperate Markets

Deadweight
Loss in Low Market

Deadweight
Loss in Low Market





1) Part a (ii).

3rd Degree Price Discrimination

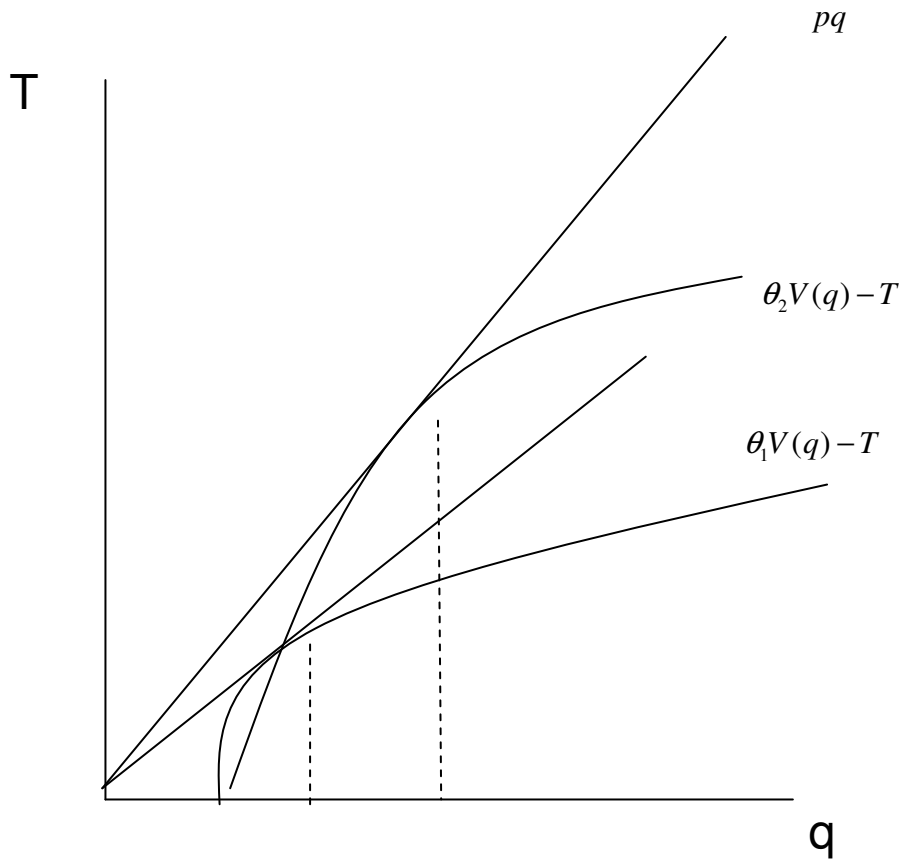
$$P_L \neq P_H, K_L = K_H = 0$$

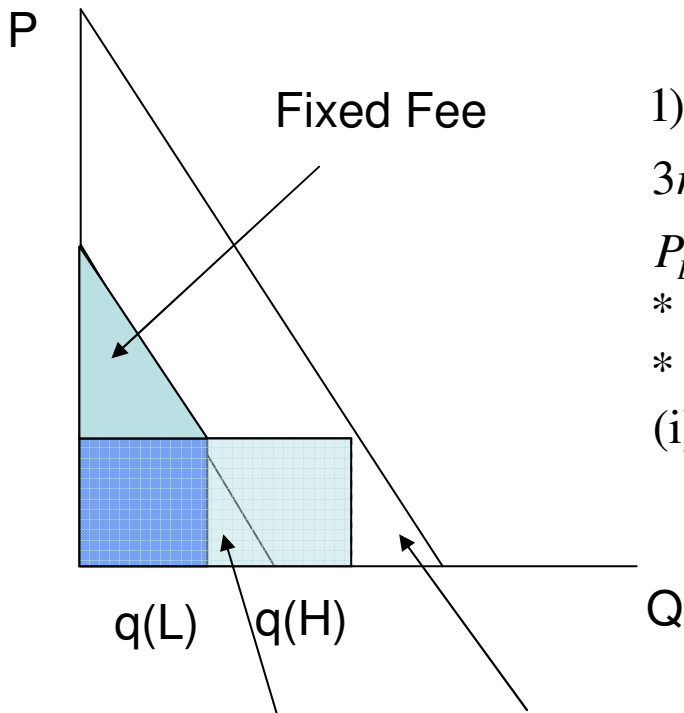
* Inefficiency in both markets

* Priced as independent Monopolies

Deadweight
Loss in Low Market

Deadweight
Loss in Low Market





1) Part a (iii).

3rd Degree Price Discrimination

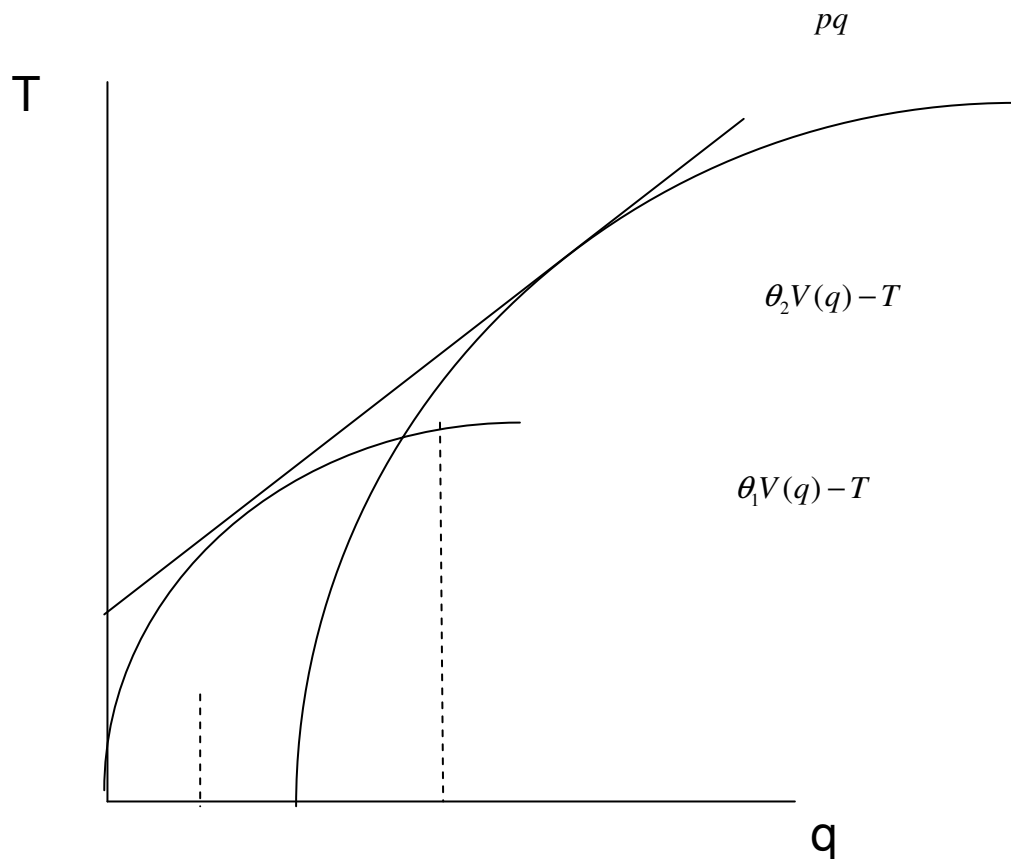
$$P_L = P_H, K_L = K_H$$

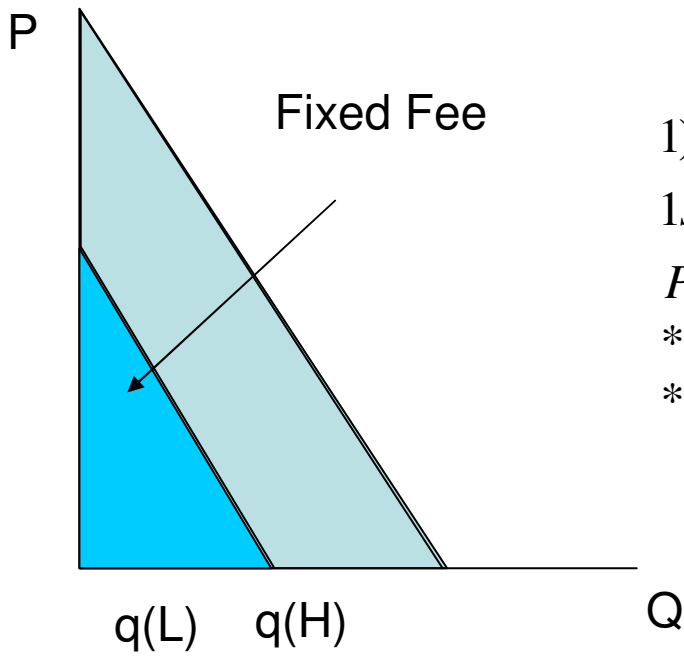
* Inefficiency in both markets

* Price Decreases from part (i) due to value of fixed fees

Deadweight
Loss in Low Market

Deadweight
Loss in Low Market





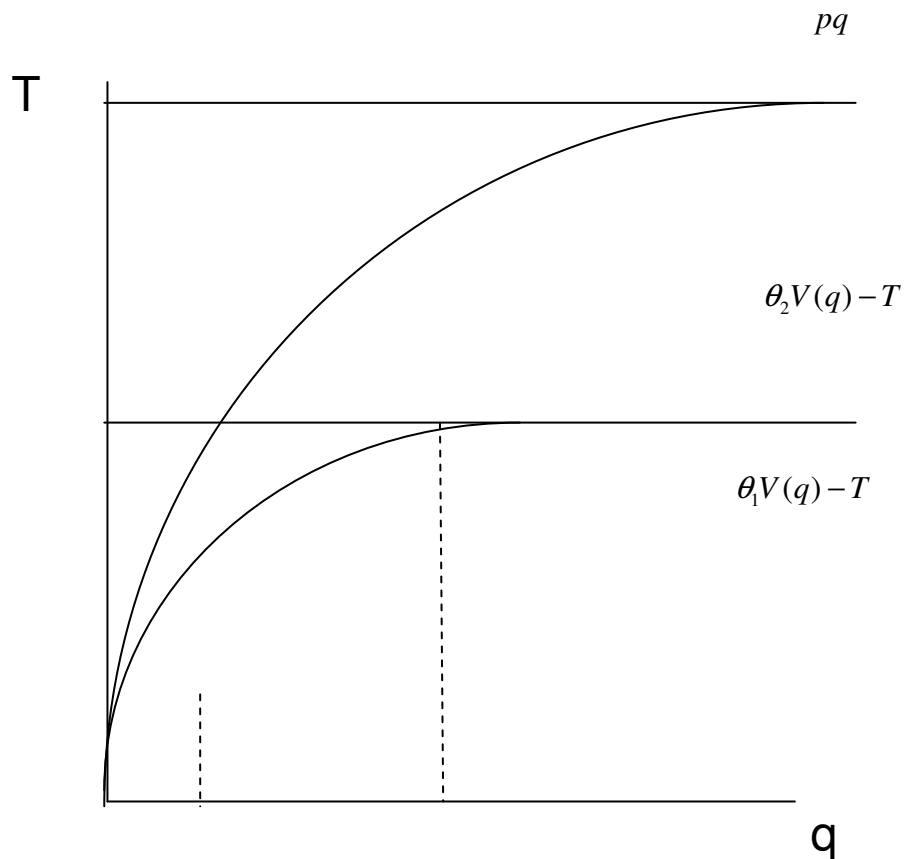
1) Part a (iii).

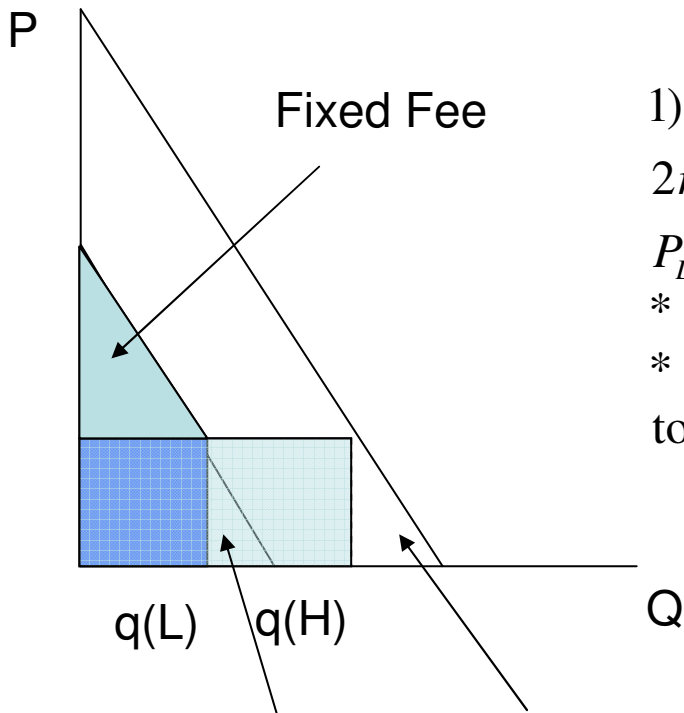
1st Degree Price Discrimination

$$P_L \neq P_H, K_L \neq K_H$$

* Efficiency in both markets

* Price = Marginal Cost (zero)





1) Part b (i).

2rd Degree Price Discrimination

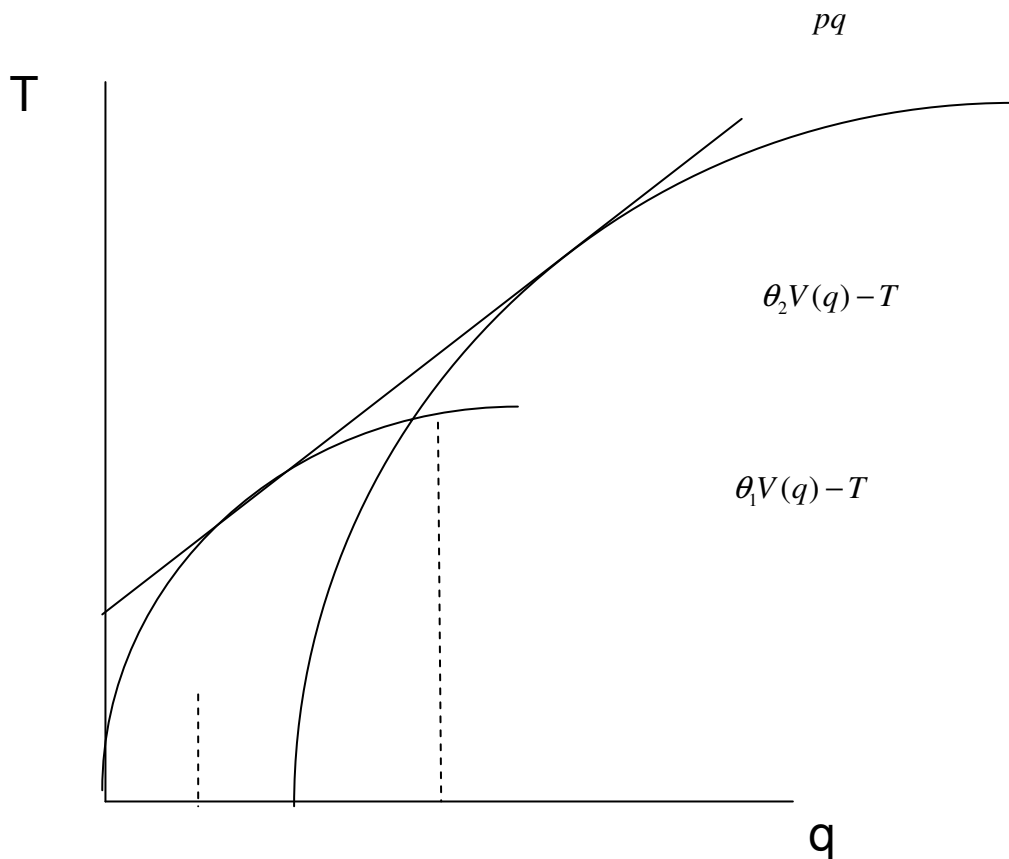
$$P_L = P_H, K_L = K_H$$

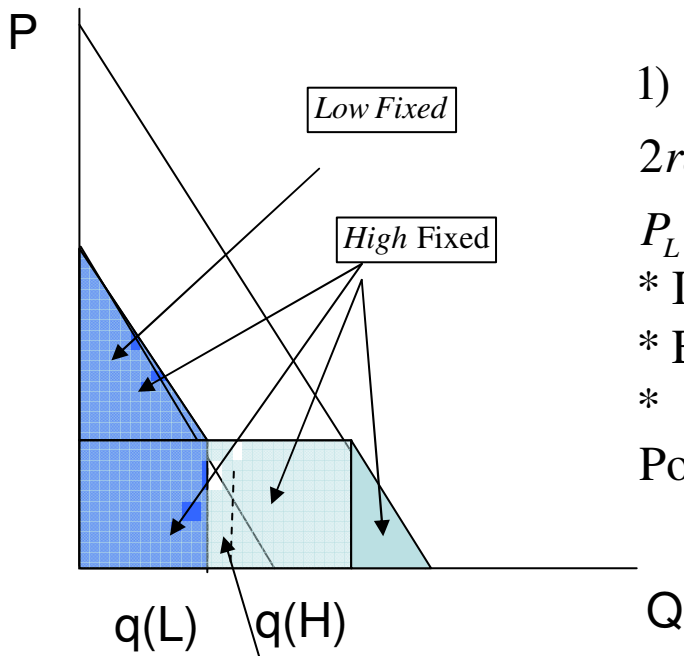
* Inefficiency in both markets

* Note that this is identical to part a (iii)

Deadweight
Loss in Low Market

Deadweight
Loss in Low Market





1) Part b (ii).

2nd Degree Price Discrimination

$$P_L \neq P_H, K_L \neq K_H$$

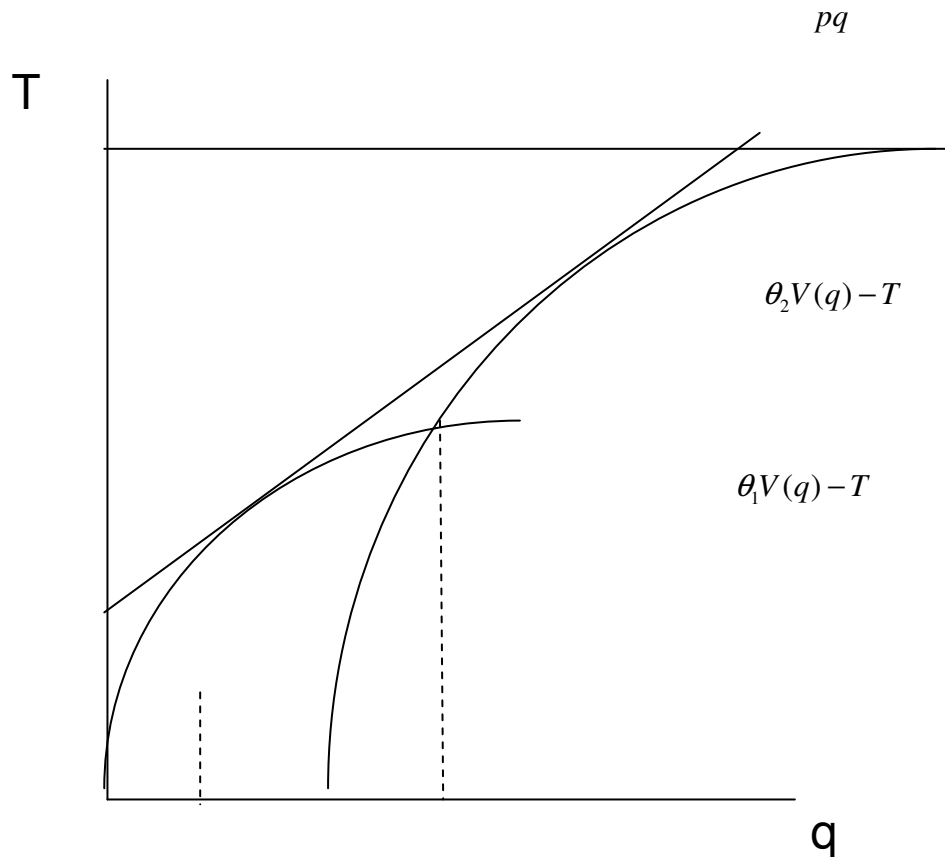
* Inefficiency in low market

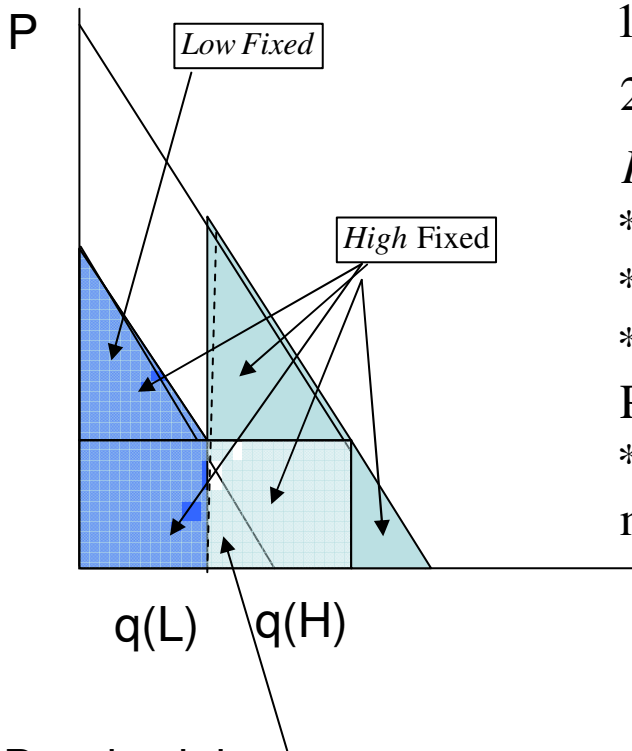
* Efficiency in high market

* IC constraint gives High market

Positive Rents

Deadweight
Loss in Low Market





1) Part b (iii).

2rd Degree Price Discrimination

$P_L \neq P_H, K_L \neq K_H, Q_L$ constrained

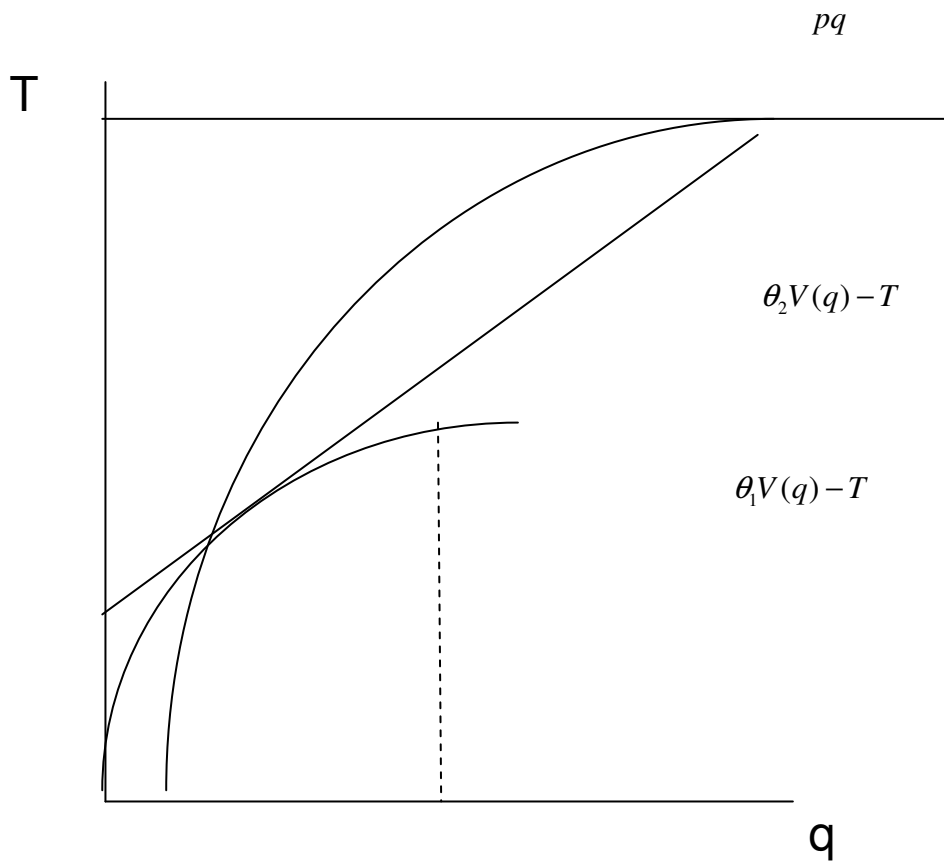
* Inefficiency in low market

* Efficiency in high market

* IC constraint gives High market Positive Rents

* Quality constraints increases monopolists Rents

Deadweight
Loss in Low Market



Economics 14.04
 Problem Set 6 - Due Nov 22nd in class

1. See additional Solutions
2. (a) If the monopolist serves both markets, it would solve:

$$\max_{r_h, r_l} .25(r_h - c_h) + .75(r_l - c_l)$$

$$\begin{aligned} ST \quad U_h - r_h &\geq U_h - r_l \\ U_h &\geq r_h \\ U_l &\geq r_l \end{aligned}$$

in this case, $r_h = r_l = 3.00$ and his profits would be $\pi(r_h = r_l = 3) = \$2$

If the monopolist serves only the high market it solves:

$$\begin{aligned} \max_{r_h, r_l} .25(r_h - c_h) \\ U_h &\geq r_h \end{aligned}$$

and would thus set $r_h = 11$. In this case, $\pi(r_h = r_l = 11) = .25 * 10 = 2.50$. Thus the monopolist will serve only the high market.

- i. The monopolist solves:

$$\max_{r_h, r_l, s_h, s_l} .25(r_h - c_h(s_h)) + .75(r_l - c_l(s_l))$$

$$\begin{aligned} ST \quad U_h(1, s_h) - r_h &\geq U_h(1, s_l) - r_l \\ U_l(1, s_l) - r_l &\geq U_l(1, s_h) - r_h \\ U_h(1, s_h) &\geq r_h \\ U_l(1, s_l) &\geq r_l \end{aligned}$$

Notice that if $U_l(1, s_l) = r_l$ and $U_h(1, s_h) - r_h = U_h(1, s_l) - r_l \rightarrow U_h(1, s_h) > r_h$ thus the IR constraint for the high type won't bind. Substitution of IC_H and IR_L into IC_L yields:

$$0 \geq U_l(1, s_h) - U_h(1, s_h) + U_h(1, s_l) - U_l(1, s_l)$$

Rearranging we have:

$$U_h(1, s_h) - U_h(1, s_l) \geq U_l(1, s_h) - U_l(1, s_l)$$

Dividing both sides by $s_h - s_l$ (a negative number) we have:

$$\frac{U_h(1, s_h) - U_h(1, s_l)}{s_h - s_l} \leq \frac{U_l(1, s_h) - U_l(1, s_l)}{s_h - s_l}$$

But this is just a slope measure and since $\frac{\partial U_h}{\partial s} < \frac{\partial U_l}{\partial s}$ by assumption this is always met. thus we are left with the following problem:

$$\begin{aligned} & \max_{r_h, r_l, s_h, s_l} .25(r_h - c_h(s_h)) + .75(r_l - c_l(s_l)) \\ ST : & (1) U_h(1, s_h) - r_h \geq U_h(1, s_l) - r_l \\ & (2) U_l(1, s_l) \geq r_l \end{aligned}$$

Substitution of the utility functions in (1) and (2) gives us:

$$\begin{aligned} 11 - 2s_h - r_h & \geq 11 - 2s_l - r_l \\ 3 - .25s_l & = r_l \end{aligned}$$

Substitution of (2) into (1) yields:

$$\begin{aligned} & \max_{r_h, r_l, s_h, s_l} .25(r_h - c_h(s_h)) + .75(r_l - c_l(s_l)) \\ ST \quad r_h & = \min(1.75s_l + 3 - 2s_h, 11 - 2s_h) \\ r_l & = 3 - .25s_l \end{aligned}$$

ii. Substitution in for r_h and r_l yields:

$$\max_{r_h, r_l, s_h, s_l} .25(1.75s_l + 3 - 1 - 2s_h) + .75(3 - .25s_l - 1)$$

Notice that this function is decreasing in s_h , thus $s_h = 0$. Likewise, the function is increasing in s_l thus we want to increase s_l up to the point where the IC_H no longer binding (ie the point where the low market does not affect the high market). This will occur when $11 = 1.75s_l + 3$. At this point $s_l = \frac{8}{1.75}$ or $s_l \approx 4.57$

iii. We know that in part i, that a monopolist wants to increase differentiation up to the point where $s_l = 4.57$. Thus if the monopolist offers both objects he will offer the high types a product with $s_h = 0$ and the low types a product with $s_l = 3$. Now however, the monopolist must lower the amount it charges the high type to maintain incentive compatibility. At $s = 3$, he would set $r_h = 8.25$ and his profits would be $.25 * 7.25 + .75 * 1.25 = 2.8125$. Everyone is at least as well off - the high type get a reduction in price, the low types get to buy the product, and the monopolist makes more profit.

3. (a) $\frac{K}{N}$ is the probability of any agent receiving the drug. Thus $E(u) = \text{prob}(\text{drug}) * U(\text{drug}) * \#\text{RealPatients} = 60 \frac{K}{N}$
- (b) i. The only time p is affected is when $w = 4$, thus $w=0, w=4$ is optimal

- ii. Define $U(w, p)$ as the utility an agent gets with a wait time w imposed and a probability of receiving a drug p based on that wait time. Suppose that $w = 4$. We have two cases. Suppose that $K > 6$. When the agent screens, every patient gets a dose. Thus :

$$E(U(w, 1)) = 10 - 4w = 2$$

Suppose that $K < 6$. Then

$$E(U(w, \frac{K}{6})) = 10\frac{K}{6} - 4w = 10\frac{K}{6} - 8$$

We can write these two cases as:

$$E(U(4, \min(\frac{K}{6}, 1))) = 10\min(\frac{K}{6}, 1) - 8$$

When $w = 0$ all the agents stay. Thus

$$E(U(0, \frac{K}{N})) = 10\frac{K}{N}$$

- iii. when $\frac{k}{n} = \frac{1}{6}$,

$$\begin{aligned} E(U(4, 1)) &= 2 \\ E(U(0, \frac{K}{N})) &= \frac{10}{6} \end{aligned}$$

4 q)

Since $E(U(0, \frac{K}{N})) < E(U(4, 1))$, the hospital screens.

Chris's better than set is to the North West. Tatiana better than set is to the South East. Thus the area to the north east would make the both better off than autarky

(Graph on Next Page)

- (b) Consider any other point inside the region in which both people would trade that is not on the boundary. Notice that there is always a point to the north west will be better for both participants. Thus the pareto optimal set will be on the boundary.
- (c) Uniqueness of a price is guaranteed when the pareto optimal point is on the interior of the Edgeworth box. In this problem, all outcomes are boundary condiditons and thus there are a lot of price vectors that will lead to a situation where markets clear.
- (d) The social planner maximizes:

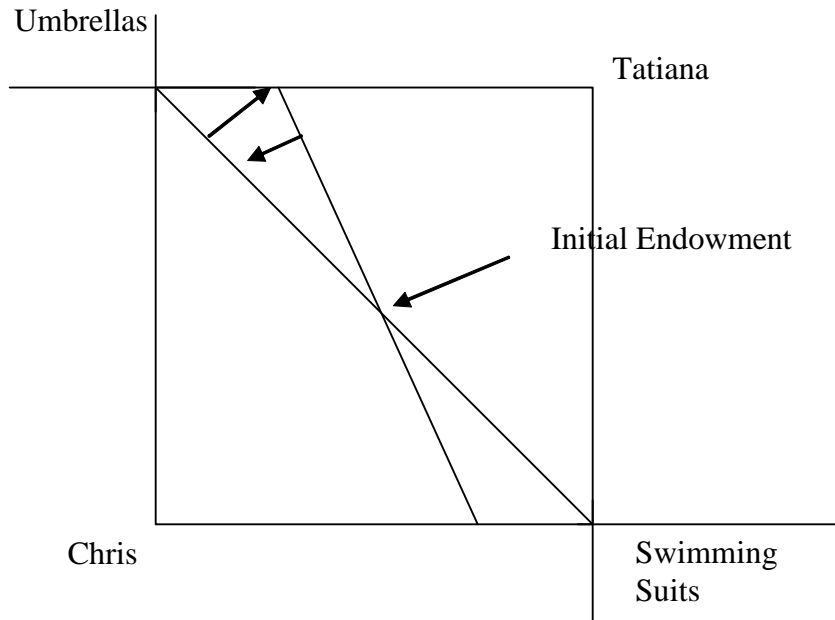
$$\max_{u_c, s_c, u_t, s_t} \lambda(U_t(u_t, s_t)) + (1 - \lambda)(U_c(u_c, s_c))$$

$$ST \ u_c + u_t = 2, s_c + s_t = 2$$

expanding out the utility functions we have:

$$\max_{u_c, s_c, u_t, s_t} \lambda(.25u_t + .75s_t) + (1 - \lambda)(.5u_c + .5s_c)$$

4. (a)



$$ST \ u_c = 2 - u_t, s_c = 2 - s_t$$

Substitution yields:

$$\max_{u_t, s_t} \lambda(.25u_t + .75s_t) + (1 - \lambda)(1 - .5u_t + 1 - .5s_t)$$

$$ST \ u_t \in (0, 2), s_c \in (0, 2)$$

or

$$\max_{u_t, s_t} (.75\lambda - .5)u_t + (1.25\lambda - .5)s_t + 2(1 - \lambda)$$

This is a linear function and thus the social planner will have three possible allocations depending on λ . When $\lambda > \frac{2}{3}$ the social planner allocates everything to Tatiana. Between $\frac{2}{3}$ and $\frac{2}{5}$ he allocates swimming suits to Tatiana and umbrellas to Chris. With $\lambda < \frac{2}{5}$ he allocates everything to Chris.

(a) Agent 1 maximizes:

$$\max_{x_1, x_2} (x_1 x_2)^{1/2}$$

$$ST : p_1 x_1 + p_2 x_2 = p_1 \bar{x}_1 + p_2 \bar{x}_2$$

FOC yield:

$$\frac{x_2}{x_1} = \frac{p_1}{p_2} \rightarrow p_1 x_1 = p_2 x_2$$

$$D_{x_1}^1(p_1, p_2, \bar{x}_1, \bar{x}_2) = \frac{1}{2p_1}[p_1\bar{x}_1^1 + p_2\bar{x}_2^1]$$

$$D_{x_2}^1(p_1, p_2, \bar{x}_1, \bar{x}_2) = \frac{1}{2p_2}[p_1\bar{x}_1^1 + p_2\bar{x}_2^1]$$

Likewise:

$$D_{x_1}^2(p_1, p_2, \bar{x}_1, \bar{x}_2) = \frac{1}{2p_1}[p_1\bar{x}_1^2 + p_2\bar{x}_2^2]$$

$$D_{x_2}^2(p_1, p_2, \bar{x}_1, \bar{x}_2) = \frac{1}{2p_2}[p_1\bar{x}_1^2 + p_2\bar{x}_2^2]$$

Total demand for good 1 at prices p_1, p_2 is thus:

$$D_{x_1}^1(p_1, p_2, \bar{x}_1, \bar{x}_2) + D_{x_1}^2(p_1, p_2, \bar{x}_1, \bar{x}_2)$$

$$= \frac{1}{2p_1}[p_1\bar{x}_1^1 + p_2\bar{x}_2^1] + \frac{1}{2p_1}[p_1\bar{x}_1^2 + p_2\bar{x}_2^2]$$

The Walrasian Equilibrium requires total demand to equal to demand:

$$\frac{X_1}{2} + \frac{p_2}{2p_1}X_2 = X_1 \rightarrow \frac{p_2}{p_1} = \frac{X_1}{X_2}$$

where $X_i = \bar{x}_i^1 + \bar{x}_i^2$. Likewise, in the other market:

$$\frac{p_1}{p_2} \frac{X_1}{2} + \frac{X_2}{2} = X_2$$

agent 1 is endowed with $\bar{x}_1 = 1, \bar{x}_2 = 0$ and agent 2 is endowed with $\bar{x}_1 = 0, \bar{x}_2 = 2$. Thus if we set $p_1 = 1, p_2 = \frac{X_1}{X_2}p_1 = \frac{1}{2}$. As expected, when one market clears the other clears. Notice also that price is also only a function of the total amount of allocation in the economy, not who actually owns the resources. This is a characteristic of Cobb Douglas utility functions in a general equilibrium.

- (b) The contract curve is all allocations that are pareto efficient. This occurs when the indifference curves of the two firms are equal to one another and total demand equals total supply. We just saw that the competitive equilibrium solution is just in terms of the total amount of goods. We also know that the consumption of each agent is a ratio of the value of the total endowment.

Starting with agent 1, his demand is:

$$D_{x_1}^1(p_1, p_2, \bar{x}_1, \bar{x}_2) = \frac{1}{2}[\bar{x}_1^1 + \frac{p_2}{p_1}\bar{x}_2^1] = \frac{1}{2}[\frac{X_2\bar{x}_1^1 + X_1\bar{x}_2^1}{X_2}]$$

$$D_{x_2}^1(p_1, p_2, \bar{x}_1, \bar{x}_2) = \frac{1}{2}[\frac{p_1}{p_2}\bar{x}_1^1 + \bar{x}_2^1] = \frac{1}{2}[\frac{X_2\bar{x}_1^1 + X_1\bar{x}_2^1}{X_1}]$$

Thus the ratio of consumption is just:

$$\frac{D_{x_1}^1(p_1, p_2, \bar{x}_1, \bar{x}_2)}{D_{x_2}^1(p_1, p_2, \bar{x}_1, \bar{x}_2)} = \frac{X_1}{X_2}$$

Thus the contract curve is a straight line starting from agent 1's origin and going to the other persons origin. This has a slope of 2.

- (c) We know from the change in x_2 with a change in x_1 is just equal to the negation of the ratio of marginal utilities. Starting from a utility function:

$$u(x_1, x_2(x_1))$$

Taking the FOC wrt x_1 :

$$\frac{\partial U}{\partial x_1} + \frac{\partial U}{\partial x_2} \frac{dx_2}{dx_1} = 0$$

This implies that

$$\frac{dx_2}{dx_1} = -\frac{\frac{\partial U}{\partial x_1}}{\frac{\partial U}{\partial x_2}} = -\frac{x_2}{x_1} = -2$$

The prices that clear the market are any set of prices that $p_2 = \frac{1}{2}p_1$, thus $\frac{dp_2}{dp_1} = \frac{1}{2}$. These two vectors are orthogonal since.

$$[1, -2] \cdot [1, \frac{1}{2}] = 0$$