PS5 Solutions

1. (a) Setting up the problem:

$$\max_p pD(p) - c(D(p))$$

Taking the FOC we have:

$$D(p) + pD'(p) - c'(D(p))D'(p) = 0$$

Rearranging yields:

$$p - c'(D(p)) = -\frac{D(p)}{D'(p)}$$

Dividing both sides by p and noting $D(p) = q$ yields:

$$\frac{p - c'(q)}{p} = -\frac{D(p)}{D'(p)p} = \frac{1}{\varepsilon}$$

(b) Setting up the maximization problem:

$$\max_q P(q)q - c(q)$$

Taking the FOC with respect to q yields:

$$P(q) + P'(q)q - c'(q) = 0$$

Rearranging yields:

$$P(q) - c'(q) = -P'(q)q$$

Division by $P(q)$ and noting $P(q) = p$ yields:

$$\frac{p - c'(q)}{p} = -\frac{P'(q)q}{P(q)} = \frac{1}{\varepsilon}$$

2. (a) $\varepsilon = -\frac{\frac{1}{q}}{\frac{P'(q)}{P(q)}} = -\frac{1}{q}\frac{(1-q)}{-1} = \frac{1-q}{q}$ thus:

   1. $\varepsilon|_{(p,q)=(0,1)} = 0$
   2. $\varepsilon|_{(p,q)=(0.5,0.5)} = 1$
   3. $\varepsilon|_{(p,q)=(1,0)} = \infty$

(b) We know that at the optimum:

$$\frac{p - c'(q)}{p} = \frac{1}{\varepsilon}$$

Since $c'(q) \geq 0$, $\frac{1}{\varepsilon} \leq 1 \rightarrow \varepsilon \geq 1$. Thus since ε is increasing in decreasing in q and $\varepsilon|_{(p,q)=(0.5,0.5)} = 1 \rightarrow q < .5$.

(c) Inelastic demand means that the quantity demanded does not change much with price. This would meand that ε is close to zero. However, we see by question (1) that a monopolist would never stop at a point where demand is inelastic because they could raise the price, lower their cost, and increase profit.

3. (a) The monopolist's program is

$$\max_p pD(p) - D(p)$$

or

$$\max_p \begin{array}{cc} 100 - \frac{100}{p} & p \leq 20 \\ 0 & p > 20 \end{array}$$

Since profits are increasing in p$\rightarrow p = 20, q = 5, \pi = 80$

(b) The CE maxiumizes social surplus thus, D(p)=MC$\rightarrow p = 1, q = 100, \pi = 0$

(c) p=1,q$\in$ $[0,100]$. Since profits are zero, the firm is indifferent in how much it produces. We assume that the government can give a small ε of money to induce the firm to produce its full amount q=100

4. (a) Demand is $P(q) = A - Bq$, setting up the maximization problem we have:

$$\max(A - Bq)q - cq - t$$

The FOC is:

$$q = \frac{a - c - t}{2b}$$

Thus $\frac{\Delta q}{\Delta t} = \frac{-1}{2b}$. $P(q) = A - Bq$ so $\frac{\Delta p}{\Delta t} = -B\frac{\Delta q}{\Delta t} = \frac{-1}{2}$. Thus a change in taxes of \$6 will lead to a \$3 increase in price to consumers.

(b) If the monoplolist has constant leaasticity of substitution:

$$\frac{p - c}{p} = \frac{1}{3}$$

at all points. Thus:

$$p - \frac{1}{3}p = c \rightarrow p = \frac{c}{1 - \frac{1}{3}}$$

If c increases to c + t:

$$p = \frac{c + t}{1 - \frac{1}{3}} \rightarrow \frac{\Delta p}{\Delta t} = \frac{3}{2}$$

So an increase of taxes of \$6 will yield an increase of \$9 in prices to consumers.

5. (a) Given a price p, the high type agents solve:

$$\begin{aligned} & \max 4x_1 - \frac{x_1^2}{2} + x_2 \\ st \quad : \quad & px_1 + x_2 \leq \omega \end{aligned}$$

The FOC conditions are:

$$\begin{aligned} 4 - x_1 &= \lambda p \\ 1 &= \lambda \end{aligned}$$

Thus the demand is:

$$q_1(p) = \begin{matrix} 4 - p & p < 4 \\ 0 & otherwise \end{matrix}$$

Similarly the low type problem has::

$$q_2(p) = \begin{matrix} 2 - p & p < 4 \\ 0 & otherwise \end{matrix}$$

If the monopolist serves the whole market he solves:

$$\max_p [N(4 - P) + N(2 - P)][P - C]$$

FOC:

$$\begin{aligned} N(4 - P) + N(2 - P) - [2N][P - C] &= 0 \\ 2N + 2N + C2N &= 2NP \\ \frac{1}{2} + 1 + \frac{C}{2} &= P \end{aligned}$$

If $C > 1$, $P > 2$ and this equation won't be true since $M(2-P)$ will be negatiove..If the monopolist only serves the top of the market the FOC is::

$$\begin{aligned} N[4-P] - N[P-C] &= 0 \\ 2 + \frac{C}{2} &= P \end{aligned}$$

The monopolists profit in serving both markets is:

$$\left[\frac{3+C}{2}\right] N\,[3-C] = \frac{9-C^2}{2} N$$

The monopolist profits for serving only the high market is:

$$\left[\frac{4+C}{2}\right] N[\left[\frac{4-C}{2}\right] = \frac{16-C^2}{4}$$

The monopolist will serve the high market as long as the profit from the high market is higher than serving both markets:

$$\pi_{High} \geq \pi_{Both}$$

iff:

$$\frac{16-C^2}{4} N \geq \frac{18-2C^2}{4} N \rightarrow C^2 \geq 2 \rightarrow C \geq 2^{\frac{1}{2}}$$

The low market is only available when $C \leq 1$ however, so this is the switch point.

(b) When we have only type A agents, we offer a single bundle that maximizes the total surplus and then uses the fixed fee to take it. The agents outside option is buying only x_2 yielding a utility of 100. The monopolist thus maximizes:

$$Max_{P,K,x_1(p,k)}\ [P-C] + K$$

subject to the agent maximizing:

$$\begin{aligned} & Max_{x_1,x_2}\ 4x_1 - \frac{x_1^2}{2} + x_2 \\ st \quad : \quad & px_1 + x_2 = 100 - K \end{aligned}$$

$$4x_1 - \frac{x_1^2}{2} + x_2 \geq 100$$

Solving the agents problem:

$$\begin{aligned} x_1 &= 4 - p \\ x_2 &= 100 - K - p(4-p) \end{aligned}$$

Plugging these into the IR constraint:

$$\begin{aligned} 4(4-p) - \frac{(4-p)^2}{2} + 100 - K - p(4-p) &\geq 100 \\ \frac{(4-p)^2}{2} &\geq K \end{aligned}$$

Thus demand for the high types is:

$$x_1^A(p,k) = \begin{matrix} 4-p & k \leq \frac{(4-p)^2}{2} \\ 0 & otherwise \end{matrix}$$

Demand for the low types is similarly:

$$x_1^A(p,k) = \begin{matrix} 2-p & k \leq \frac{(2-p)^2}{2} \\ 0 & otherwise \end{matrix}$$

Total Demand is:

$$x_1(p,k) = \begin{array}{cc} N(4-p)+N(2-p) & k \leq \frac{(2-p)^2}{2} \\ N(4-p) & \frac{(2-p)^2}{2} \leq k \leq \frac{(4-p)^2}{2} \\ 0 & otherwise \end{array}$$

(c) Set $p = c$ and $k = \frac{(4-c)^2}{2}$. The profit will be $N(4-c)^2/2$.

(d) Requiring both types of agents to consumer requires that$k \leq \frac{(2-p)^2}{2}$. we solve the problem:

$$\max_p N(4-p)(p-C) + N(2-p)(p-C) + 2Nk$$
$$st \quad : \quad k = \frac{(2-p)^2}{2}$$

The FOC of this is:

$$N(4-P) + N(2-P) - [2N][P-C] - 2N(2-p) = 0$$

Summing up yields:

$$6N - 2NP - 2NP - 2NC - 4N + 2NP = 0$$

Solving:

$$P = 1 - C$$

6. (a) Solving the FOC yields:

$$[1-\pi(x)]p - \pi'(x)[px+F] = 0$$

The entry restriction is that an entrant makes no profit:

$$[1-\pi(x)]px - \pi(x)F = 0$$

Rearranging this yields:

$$\begin{aligned} px - \pi(x)[px+F] &= 0 \\ [px+F] &= \frac{px}{\pi(x)} \end{aligned}$$

Plugging this into the FOC yields:

$$[1-\pi(x^*)]p - \pi'(x)\frac{px}{\pi(x)} = 0$$

Rearranging yields:

$$x^* = \frac{[1-\pi(x^*)]\pi(x^*)}{\pi'(x^*)}$$

This is not dependent on p or F.

(b) Plugging this into the the entry restriction gives us p :

$$\begin{aligned} [1-\pi(x)]px - \pi(x)F &= 0 \\ p^* &= \frac{\pi(x^*)}{[1-\pi(x^*)]x^*}F \end{aligned}$$

(c) The game company is going to be constrained by the price that bootleggers can enter. Thus:

$$\frac{\pi(x^*)}{[1-\pi(x^*)]x^*}FD(p^*) \geq K$$

Rearranging yields:

$$\pi F \geq [1-\pi(x^*)]\frac{x^*}{D(p^*)}K$$

(d) Notice that the LHS of the previous equation is based on the fine and the probability of being caught. In china, the probability of being caught is low, thus F must be high for development to exist. In the US π is higher which reduces the required level of F.