Econ 14.04 Fall 2006

Assignment 2: Indirect Utility, Expenditure Functions, and Duality

The due date for this assignment is Friday September 29.
Reading assignment: Sections 1-7.

1. *Deriving the Envelope Thoerem: Consider the more general problem $M(\alpha, \beta) = \max_x f(x, \alpha, \beta)$ subject to $g(x, \alpha, \beta) = 0$. Show that:

$$\frac{dM(\alpha, \beta)}{d\alpha} = \frac{\partial f(x^*, a, b)}{\partial a} + \lambda \frac{\partial g(x^*, a, b)}{\partial a}$$

2. For each of the following, derive $\mathbf{x}(\mathbf{p},m), \mathbf{h}(\mathbf{p},u), v(\mathbf{p},m), \mathbf{h}(\mathbf{p},u)$ using the standard budget constraint $p_1x_1 + p_2x_2 = m$:

   (a) $u(x_1, x_2) = \max(x_1, x_2)$

   (b) $u(x_1, x_2) = \min(x_1, x_2)$

   (c) $u(x_1, x_2) = 2x_1 + x_2$

   (d) $u(x_1, x_2) = x_1^{1/2} x_2^{1/3}$

   (e) $u(x_1, x_2) = \frac{1}{2} \ln x_1 + \frac{1}{3} \ln x_2$

   (f) What happens if we replace u by e^{u^*} in part (e)? Compare this to part (d). Can you work out an easy way to derive the hicksian demand functions of a function when you make monotonic transformations of the original function?

3. *A besotted mathematician named Donic consumes either gin or tonic. His preferences are rare 'cause he thinks in the square in a way that is almost sardonic.

   Specifically, Donic prefers larger drinks to smaller drinks but requires that the square of the amount of lime in a drink equal the sum of the squares of the amounts of gin and tonic. Find a utility function that represents Donic's preferences. Find Donic's Marshalian demand functions for lime, tonic, and gin.

4. (From Midterm 2005) Consider a consumer with a utility function $u(x_1, x_2) = e^{(x_1 + \ln(x_2))^{1/3}}$

   (a) What properties about utility functions will make this problem easier to solve?

   (b) Which of the non negativity input demand constraints will bind for small m?

   (c) Derive for the marshallian (uncompensated) demand functions and the indirect utility function.

   (d) Derive the expenditure function in terms of original utils u.

5. Consider the indirect utility function given by:

$$v(p_1, p_2, m) = \frac{m}{p_1 + p_2}$$

   (a) What are the demand functions

(b) What is the expenditure function?

(c) What is the direct utility function?

6. *Consider the utility function:

$$u(x_1, x_2) = \min(2x_1 + x_2, x_1 + 2x_2)$$

(a) Draw the indiference curve for $u(x_1, x_2) = 20$. Shade the area where $u(x_1, x_2) \geq 20$

(b) For what values of $\frac{p_1}{p_2}$ will the unique optimum be $x_1 = 0$

(c) For what values of $\frac{p_1}{p_2}$ will the unique optimum be $x_2 = 0$

(d) If neither x_1 and x_2 is equal to zero, and the optimum is unique, what must be the value of $\frac{x_1}{x_2}$?

7. Assume that there is a consumer with weakly monotonic, convex preferences and who is a utility maximizer. For each of the following pairs of bundles, specify if bundle 1 is $\succsim$, $\precsim$, or uncomparable to bundle 2.

(a) Suppose you have no data:

1. Bundle 1: $x_1 = 3, x_2 = 3$, Bundle 2: $x_1 = 6, x_2 = 2.5$
2. Bundle 1: $x_1 = 3, x_2 = 3$, Bundle 2: $x_1 = 2.5, x_2 = 2.5$

(b) Suppose that you observe that when $p_1 = 1, p_2 = 1, m = 10$ the consumer chooses $x_1 = 2, x_2 = 8$

1. Bundle 1: $x_1 = 4$, $x_2 = 1$, Bundle 2: $x_1 = 3$, $x_2 = 6$
2. Bundle 1 $x_1 = 6, x_2 = 4$, Bundle 2: $x_1 = 3$, $x_2 = 8$

(c) Suppose that we have two observations. When $p_1 = 1, p_2 = 1, m = 10$ the consumer chooses $x_1 = 2, x_2 = 8$. When $p_1 = 1, p_2 = 3, m = 15$ the consumer chooses $x_1 = 15, x_2 = 0$

1. Bundle 1: $x_1 = 5, x_2 = 2$, Bundle 2: $x_1 = .0, x_2 = 2.5$
2. Bundle 1: $x_1 = 5, x_2 = 2$, Bundle 2: $x_1 = 6.5, x_2 = 0$