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GREG HUTKO: Welcome back to the 14.01 problem-solving videos. Today I'm going to be working on Fall 2010 PSET 8, Problem Number 2. And we've seen the case with the monopolist where we've only had one producer in a market, and we've seen how that affects consumer surplus, produce surplus, and deadweight loss. Now we're going to look at the case where we have two firms competing. And specifically, for the first part, we're going to have the Cournot equilibrium, where they're going to be competing not by setting the price of their product. Instead they're going to compete by setting how much they're going to produce. Let's go ahead and read part A of this problem.

Consider a market in which two firms produce a homogeneous product. Market demand is given by quantity equals 200 minus p . The cost functions for Firm A and Firm B, the total cost for A equals $5q_A$ and the total cost for B is going to equal $\frac{1}{2} q_B^2$, respectively. Find the Cournot equilibrium quantity supplied by each firm. We're going to graph our results using reaction functions and we're going to find the market price, and then calculate the profits for each firm.

Now in the duopoly model, both firms in the Cournot equilibrium are going to set their quantities at the same time. But what they're going to do is they're going to know what each other's revenues and costs look like. So they can know, I know that if I make this move and set this quantity, I know that my competition is going to have this reaction. Since they can plan for each other's reaction, they can decide how much quantity they're going to produce at the same time and they're going to reach an equilibrium.

What this looks like on our reaction curves, up here we're going to have q_A . And down here is going to be q_B . We're going to graph the reaction function for Firm A first. And we're now going to graph the reaction function for Firm B.

Now all this tells us is if we're at this point, if we're Firm A and we decide to produce this much, then we know that Firm B's best reaction is going to produce this quantity here, the intersection of how much I'm producing with their reaction curve. Similarly, if Firm B is going to

decide to produce this much, then my best reaction going over to Firm A's reaction function straight across, is going to be to produce this much.

Now since they're both choosing at the same time, neither of the firms can decide to declare a larger amount. And they have to kind of use their intuition to determine what they're going to produce. So since they're choosing at the same time, they have to plan for each other's reactions. They're going to produce at this point where the two reaction curves meet. And that's what we're going to be calculating. We're going to be calculating the quantity that Firm A produces and the quantity that Firm B produces when the reaction curves are set equal to each other.

Now to get the reaction curves, we're going to start off with our revenue function for both Firm A and Firm B. And revenue is just going to be the price times the quantity that either Firm A or Firm B is producing.

Now instead of saying that we are just going to take the marginal revenue according to this P times Q , we're going to plug-in for P from the demand curve 200 minus a disaggregated quantity where we disaggregate into the amount Firm A is producing and the amount Firm B is producing. We're going to plug this in to the revenue function.

And so just like the monopolist did where they're maximizing by setting marginal revenue equal to marginal cost, Firm A and Firm B we're going to do the same thing. We're going to set marginal revenue equal to marginal cost. And then we're going to solve through for q_A in terms of the quantity that the other firm is producing. That's why it's considered a reaction curve because it's in terms of what the other firm is producing.

So solving through for Firm A's marginal revenue, we're going to find that marginal revenue for Firm A is equal to 200 minus $2q_A$ minus q_B . And it makes sense that the more that they're producing, Firm A and Firm B, the lower the revenue that they're going to be taking in.

Now we're going to also calculate the marginal cost for Firm A by taking the derivative with respect to the total cost function. We're going to find that the marginal cost is going to be equal to 5 .

Now we're just going to set the marginal cost and the marginal revenue for Firm A equal. And we're going to solve through for q_A .

When we do that, we're going to find that q_A is equal to $97.5 - 0.5 q_B$. And we're going to repeat this exact same process for Firm B. But when we do it for Firm B, instead of taking the derivative with respect to q_A , we're going to take the derivative with respect to q_B .

So the marginal revenue for Firm B is going to be equal to $200 - q_A - 2 q_B$. And the marginal cost is just going to be equal to q_B . Again, we're going to set marginal cost and marginal revenue equal to each other. And now we're going to solve through for q_B . And when we do this, we're going to have Firm B's reaction curve.

And now what we're going to do since we have these two reaction curves, we have a reaction for Firm A and a reaction for Firm B, all we're going to do is we're going to plug-in for this q_B , q_B 's reaction curve. And when we do that, when we plug-in $66.67 - 0.33 q_A$, we can solve through for just q_A .

Doing this we're going to find that Firm A is going to produce approximately 77 units. And then taking this 77 and plugging it in to Firm B's reaction function, we can solve for Firm B's production amount. And we're going to find that Firm B is going to produce approximately 41 units.

Now to find the equilibrium price, we're just going to come back up here to our disaggregated demand function, and we're going to plug-in for q_A and q_B . And we can solve through for the price being equal to 82.

Now what we've just done for this problem is we solved on our graph for the intersection of the two reaction functions. We found that q_B is going to be equal to 41 and we found that q_A is going to be equal to 77. So we just calculated the intersection point for the Cournot equilibrium.

Now the last part of this problem asks us to calculate the profits for both the firms. The profits for Firm A are just going to be price times the quantity that A is producing minus the cost as a function of q_A . So we're just going to take the total revenues minus the total costs. For Firm A, we're going to find that the total profits are going to be about \$5,929. And doing the same process for Firm B, we can find that the profits for Firm B are going to be equal to approximately \$2,521.

Now part B of this problem is going to ask us instead of having this Cournot equilibrium where neither firm can go ahead and produce a higher quantity or move first in the market, we're

going to look at something different than the Cournot equilibrium. We're going to look at the case where one of the firms gets to decide how much they're going to produce before the other firm. And if you get to decide first, you get to produce a higher quantity and get more of the profits.

Part B says, now suppose that Firm A chooses how much to produce before firm B does. In this case, Firm A is a Stackelberg leader and B a follower. We're going to calculate the quantities, the market price, and the profit for each firm.

Now coming over to this side of the board, we see that I'm going to keep Firm B's reaction function the same. So Firm B is going to be reacting in the same way to Firm A's decision. Only now, the only difference is when we calculate marginal revenue equal the marginal cost for Firm A, instead of just saying the q_B is going to be random, we can't account for it. We're going to plug-in, we're going to take into account Firm B's reaction when we're maximizing or taking the derivative with respect to q_A . So instead of having q_B in here, I'm going to plug-in this reaction function.

So in this case, the equation that I'm going to be maximizing is going to be this one right here. I'm going to take the derivative with respect to q_A to find the marginal revenue for A.

And again, I'm just going to set this equal to the marginal cost, which we found earlier is equal to 5. And when we set these equal, we can solve through for the quantity that Firm A is going to produce. And we're going to find just like we predicted that the leader is going to produce more. In this case, Firm A has increased their production to 96.25.

And then plugging in this quantity in to Firm B's reaction function, we can find that Firm B in this case, is going to produce 34.6.

Now in this case, we can again calculate the price by taking the demand function that we have. We can take the demand function that we're given in the problem. Plugging in for q_A and q_B , we find that the new price in the Stackelberg problem is going to be 69.15. And again, we can calculate the profits going through the same process of doing total revenue minus total cost. And we're going to have that the profit for Firm A is going to be equal to about 6,174. And the profit for Firm B is going to be equal to about 1,794.

And so what we can do here is we can compare the profits that we had in the Stackelberg case to the profits that we had at the start of our problem. So before we can see that Firm A

was not as profitable when they had to choose their quantity at the same time as Firm B. We can see that their profits have increased. But we can see that Firm B, their profits have actually decreased because they're a follower in the Stackelberg model.

Now the last thing, and the thought I want to leave you with is, how do we actually interpret this when we look at the reaction functions on our graph? We're no longer at the point where we're setting the two reaction functions equal. What's happening now is we're way up here and q_A is choosing their production way up here. And q_B is forced to react by choosing their production right here. And what happens is since they both increased their production or since q_A has increased their production and q_B has decreased their production, but since production has increased overall, the price has dropped compared to when they were at the Cournot equilibrium. So total profits have actually dropped as well.

So really what the first two parts of these problems were having us look at, they were looking at two different situations of duopoly where we have two competitors in the market. The first one they were choosing their outputs at the same time. And in the second problem, one of the firms had the advantage of getting to choose a higher quantity and making a credible threat that they were going to make that quantity to begin with.

For the last parts of these problems, you're going to go ahead and you can look at what the implications are when we think about what the total quantity is produced in a competitive market aggregating the supplies of these two firms. And then you can compare the output in the three different scenarios. But for now, I'm going to leave you here. Go ahead and finish the rest of the problem, and I hope you found this part helpful.