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**GREG HUTKO:** Today we're going to do Fall 2010, P Set Six, Problem Number Four. And for this problem we're going to shift away from what we've usually been talking about, where we're dealing with a straight equilibrium, setting the demand curve equal to the supply curve.

And now we're going to think about what happens when the supplier has market power. When they're the only competitor in the market, and they can decide how much quantity they want to produce. And they don't have to worry about other producers coming in and producing.

Problem Number Four-- I'll read through Part A-- states, "A monopolist firm faces the following cost curve. The cost equal  $Q^2 + 15$ , where  $Q$  is the output produced, the demand for its product is given by  $P = 24 - Q$ . We need to calculate the non-price discriminating consumer surplus, the producer surplus, and the deadweight loss associated with the monopoly."

Now, what this problem's really going to look like-- we can think about it starting with this graph. Is, instead of producing output to the point of the equilibrium right here, the supplier can actually make more money by saying I'm only going to produce to a point right here, so this is what we're looking for. We're wondering how much is the supplier actually going to constrict the supply in the market.

And when they constrict this supply, what happens is this small triangle right here becomes the deadweight loss. This is potential surplus that would have existed when this whole big triangle was the consumer surplus plus producer surplus. So now nobody's getting that triangle.

But the producer surplus is much bigger than it would have been when it was just the space below the price level to the supply curve. So the producers are better off. The consumers are going to be worse off. And society as a whole-- adding together the producers' and the consumers' surplus-- is going to be worse off.

Now, the way the producers actually make their decision on how much to produce is, when they're moving this line back and forth deciding how much they want to constrict the quantity

that they're going to supply, and when they're supplying more, the quantity they're supplying is going to be increasing. But as they supply more-- since the demand curve is downward sloping-- the price is going to be going down.

And now the way the producer actually makes their production decision is to say, OK, I know I'm going to lose some money if I'm producing more, because the price is going to be falling. What I want to know is I want to produce as much as I can so that, at the margin, the cost of producing that one additional unit is the same as the revenue that I'm going to be taking in for that additional unit. The point where I'm producing, and the additional cost of that unit is more than the additional money that I'm taking in, I'm going to stop assuming that there's no competition.

So the monopolist firm is going to set the marginal cost equal to the marginal revenue. So we have a total cost function, so calculating the marginal cost is pretty straightforward. I'm just going to take the derivative with respect to  $Q$ , and the marginal cost for a monopolist firm is going to be  $2Q$ .

Now, it's tempting when we look at this revenue function-- revenue just being the total quantity I'm producing times the price I'm receiving. It's tempting to just take the derivative here with respect to  $Q$ , and say that marginal revenue is going to be equal to price. But that's not what the monopolist does. Because in a competitive situation, we were setting marginal cost equal to  $P$ .

In the monopolist situation, the monopolist is going to look at this  $P$  right here, and they're going to say I know how the consumers are going to respond based on my decision to produce. So I'm going to replace this  $P$  with the demand curve,  $24 - Q$ . So I can plan how much I'm producing based on what I know the consumer's response to my production choice is going to be.

So instead of taking the derivative of this function, I'm going to plug-in  $24 - Q$  and we're going to find the marginal revenue using this function. When we do this, we find that the marginal revenue is equal to  $24 - 2Q$ . And all we have to do now is we have to set the marginal revenue and the marginal cost equal, and we can find the quantity that's going to be produced at the monopolist outcome.

Solving for  $Q$ , you find that the quantity is going to be equal to 6. And then we can solve for the

price just by going back to the demand curve that's given on our graph. The price here in the monopolist case is going to be 18.

So now we can come back to our graph. We know that the monopolist level of output is going to be 6, so we can label this 0.6. We know the price that's going to be charged-- which is not the intersection with the supply curve, it's going to be the intersection with the demand curve-- this price is going to be 18. And on our graph it's going to also be useful to label two more points. That'll just make it easier for us to calculate consumer surplus, producer surplus, and deadweight loss.

We're going to want to label this point right here so the equilibrium quantity is 8-- and you'll see in a second why I'm labeling that. And you're also going to want to label where, when the quantity is 6, the intersection with the supply curve. So when the quantity is 6, you know that the marginal cost curve is given here. So that means the intersection right here is going to be 12. And this is going to make our calculations of the area of PS, the area of CS, and the area of DWL just a little bit easier.

Now, to calculate consumer surplus, I'm just going to multiply the height of this triangle right here by the length of the triangle, and I'm going to take one half of that. So consumer surplus in this situation is going to equal 18. And now we're going to do the same thing for producer surplus. We're going to add the area of this rectangle to the area of this triangle at the bottom as well.

So the first term here that's given is the area of the rectangle, and the term here is the area of the triangle. Adding these together, we're going to find that the producer surplus is 72.

And now, to calculate the deadweight loss you really have two options. One, you could find the total producer and consumer surplus at equilibrium-- so the area of this large triangle right here. And you could subtract out the new consumer surplus and the new producer surplus, and you'll be left with only the deadweight loss.

For our purposes, it's going to be a little bit easier to just take the height of the triangle and the length of the base, and to multiply through. When we do that, we're going to find that the deadweight loss is going to be equal to 6.

And so you can see that the producer surplus is pretty high in this situation. And so the government's going to come in in our next problem, and they're going to say we have an

intervention that might be able to correct this problem that we see in the market. Part B says, "How does charging the monopolist a specific tax of \$8 per unit affect the monopoly optimum, and the welfare of consumers, the monopoly, and society, where society's welfare or surplus includes the tax revenue?"

So, what's basically happening in this new case is we're going to start off with the same sort of problem where the monopolist gets to decide how much they're going to output. And we're interested in the marginal cost and the marginal revenue.

Now, the marginal revenue is going to be represented by the same equation, and we're going to substitute in for price the demand curve again. And when we solve through, substituting in for the demand curve and taking the derivative, we're going to find that the marginal revenue is again going to be equal to  $24 - 2Q$ .

So the marginal revenue hasn't changed at all. What is going to change is going to be the total cost curve for the monopolist. So this was the cost curve that we started off with, but now for each unit  $Q$  that the monopolist produces, it's going to be taxed at a rate of  $t$ . So we can add in the cost of the tax.

And in the next step, I'm going to take the derivative with respect to  $Q$ , and I'm going to substitute in for  $t$  the price of the tax, or 8. So now our new marginal cost is equal to  $2Q + 8$ .

And to solve for our new equilibrium, we're just going to set marginal cost and marginal revenue equal. And when we do that, we're going to find that  $Q$  is equal to 4. And plugging into the demand curve, you're going to find that the price is equal to 20.

Now in this problem, you could go through and you could go ahead and you could calculate the consumer surplus, the producer surplus, the deadweight loss and the tax revenue, and you could figure out quantitatively how much they've changed. But we're just going to draw a new graph, and we're going to look at the changes in consumer surplus, producer surplus, and deadweight loss, and make a qualitative assessment of how those quantities have changed.

So on a new axes, I'm going to draw the old supply curve and the demand curve. Now, what's essentially happening is, since the suppliers know for each unit they're producing they're going to have to pay a tax of 8, the supply curve is essentially shifting up by 8 units. And I'm representing the new supply curve with an  $s$  prime.

So now, instead of the suppliers making their monopolist decision based on this supply curve and the demand curve, they're now making it over here. And so what's going to happen is they're going to have some new monopolist output. And in this case, we know the new monopolist output is 4. We know that the new monopolist price is going to be 20.

And so we can see on this graph that producer surplus is going to be represented by this four-sided figure right here. We know that the tax revenue is going to be-- since the distance from this point to this point is 8, from 0 to 4-- this is going to represent the tax, this box right here. And we know that the consumer surplus is going to be this small triangle up top.

Meanwhile, the dead weight loss is anything that's not represented that would have been in our original producer surplus, plus consumer surplus. So the deadweight loss is this large triangle over here.

So basically what's happened is, compared to our initial case, the government's come in and for society-- since there's less being produced-- we've basically shifted the monopolist quantity over to the left. This means that, overall, the deadweight loss, this triangle over here, has increased in size. So if the deadweight loss is increasing, we can say that society is going to be worse off in this situation.

In another case, it's also clear to see that the consumer surplus, since the consumers are paying a higher price for a lower quantity, we can also say that the consumer surplus is going to decrease. So we can safely say that the consumers are going to be worse off as well.

And then the last interpretation is knowing that in the first case the producers were allowed to make their production decision just given the demand curve and their original supply curve. Now their production decision also has to take into account the government taking away some of their profits. If the government is taking some away some of their profits, the producers are necessarily going to have less surplus. So the producers are going to be worse off as well.

So overall, the only person who might possibly benefit from this policy would be the government. But overall, the producers, the consumers, and society are going to be worse off.

Now, the last part of this problem is part C. And instead of implementing a tax on the per unit production decision for the producers, now the government's going to consider a different tax policy. Part C says "How does imposing a tax on profits-- profit after tax equals  $1 - t$ -- affect the monopoly optimum, and the welfare of consumers, the monopoly, and society?"

Now basically, what's happening in this situation is the government's going to come in. And they're going to say all right, after you've made your decision on how much to produce, we're going to take a set percentage of the producer's surplus. So if you get a producer's surplus of this amount, then a certain chunk of it is going to go to Uncle Sam at a tax percentage which we can say is just  $x$  percent.

Now that percentage of tax, it doesn't change the fact that the producers want to have as much surplus as possible. Just because they're going to lose, say, 10% of it because of Uncle Sam, it's not actually going to affect the fact that they want their producer surplus to be as big as possible.

So what happens in this situation is that this after profit tax will not affect the equilibrium at all. And we're going to be left with the same consumer surplus. Producer surplus is going to be lower because of the tax, but overall, society is going to be left with the same social welfare.

So really what this problem is looking at through its three parts, we look at the monopolist situation, and we look at how the government can try to adjust with tax policy what's happening in the market. And what we saw in our second scenario is that when they charge a per unit tax on the producers, societal welfare is going to go down. But in the third case, when they're just taking a set percentage from the producer surplus, the overall welfare for the society is going to stay the same.

So bundled in this problem we had the monopolist situation, setting marginal cost equal to marginal revenue. And we also looked at tax implications on a per unit basis, and on a profit basis with a set tax after the production decision is made.

I hope you found this problem helpful.