12.842 / 12.301 Past and Present Climate
Fall 2008

12.842 and 12.301 Climate Physics and Chemistry

Ocean and Climate

24 Nov 2008 Due: 5 December 2008

1. The pressure field at depth $z = 1000$m in the ocean is found to follow the rule,

$$p = p_c + p_0 \cos\left(\pi x/10^4\right) \cos\left(\pi y/10^4\right)$$

The origin of y is taken to be 30°N and x, y are measured in meters. What are the northward and eastward components of geostrophic velocity at $x = 10^4/2$, $y = 0$? If the fluid density is approximated as uniform, $\rho = \rho_0 = 1.03 \times 10^3 \mathrm{kg/m}^3$, how much water (mass) is moving northward between the seasurface and 1000m, between $x = 0$ and $x = 10^4/2$? For a numerical answer, let $p_0 = 250\mathrm{N/m}^2$.

2. A ship measures the temperature and salinity in the ocean at $x = 0$, and $x = 100$km at a latitude of 40° N. When converted to density, the two profiles are found to be closely approximated as,

$$\rho\left(x = 0, z\right) = 1.03 \times 10^3 \mathrm{kg/m}^3 \left(1 - z/\left(2 \times 10^4\right)\right),$$
$$\rho\left(x = 100\mathrm{km}, z\right) = 1.03 \times 10^3 \mathrm{kg/m}^3 \left[1 - (z + 1 \times 10^{-7} z^2)/\left(2 \times 10^4\right)\right]$$

where z is in meters. Compute and plot the northward velocity as a function of z for $0 \le z \le 3000$m under the assumption that $z = -1500$m is a level of no motion. What is different at 10°N? Take gravity, $g = 10\mathrm{m/s}^2$. What is the surface elevation change between the two locations?

3. A uniform wind blows towards the north such that the windstress on the ocean is $\tau = \tau_0 \left(-1, 1\right)$. Using the equations,

$$-fv = A\frac{\partial^2 u}{\partial z^2},$$
$$fu = A\frac{\partial^2 v}{\partial z^2}$$

which govern the Ekman layer, find u, v as a function of z. (Hint: multiply the second equation by i and add to the first equation. Solve this equation for the complex quantity $u + iv$. Treat f as constant. Consider rotating the x, y axes. Note that the implied density is $\rho_0 = 1$, which for seawater implies cgs units. Alternatively, one can define $A' = A/\rho$.