Atmospheric Convection

A Primer

Homogeneous Compressible Gases

Buoyancy and Entropy

Specific Volume: $\alpha = {}^{1}\!/_{\rho}$

Specific Entropy: s

$$Maxwell: \left(\frac{\partial \alpha}{\partial s}\right)_p = \left(\frac{\partial T}{\partial p}\right)_s$$

$$\alpha = \alpha(p,s)$$

$$\left(\delta\alpha\right)_p = \left(\frac{\partial \alpha}{\partial s}\right)_p \delta s = \left(\frac{\partial T}{\partial p}\right)_s \delta s$$

$$B = g\frac{\left(\delta\alpha\right)_p}{\alpha} = \frac{g}{\alpha}\left(\frac{\partial T}{\partial p}\right)_s \delta s = -\left(\frac{\partial T}{\partial z}\right)_s \delta s \equiv \Gamma\,\delta s$$

The adiabatic lapse rate:

$$\textit{First Law of Thermodynamics}:$$

$$\dot{Q} = T\frac{ds_{rev}}{dt} = c_v\frac{dT}{dt} + p\frac{d\alpha}{dt}$$

$$= c_v\frac{dT}{dt} + \frac{d\left(\alpha p\right)}{dt} - \alpha\frac{dp}{dt}$$

$$= \left(c_v + R\right)\frac{dT}{dt} - \alpha\frac{dp}{dt}$$

$$= c_p\frac{dT}{dt} - \alpha\frac{dp}{dt}$$

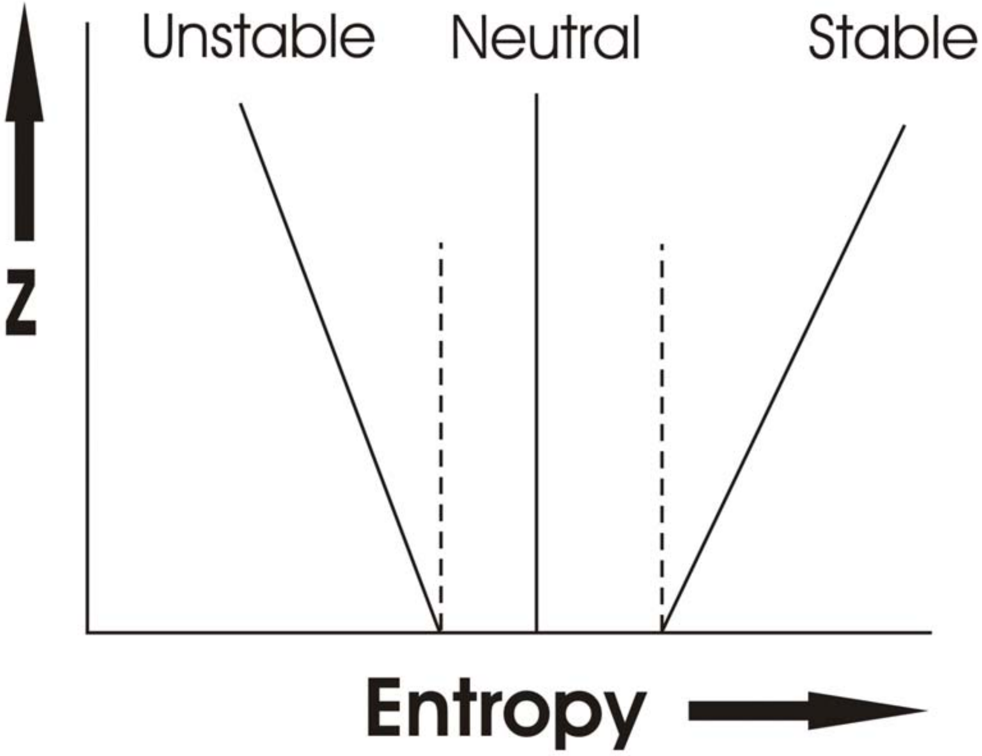
$$\textit{Adiabatic}: \quad c_p dT - \alpha dp = 0$$

$$\textit{Hydrostatic}: \quad c_p dT + g dz = 0$$

$$\rightarrow \quad \left({}^{dT}\!\big/_{dz}\right)_s = -{}^{g}\!\big/_{c_p} \equiv -\Gamma_d$$

$$\Gamma = {}^{g}\!/_{c_p}$$

Earth's atmosphere: $\Gamma = {}^{1\,K}\!/_{100\,m}$



Model Aircraft Measurements (Renno and Williams, 1995)

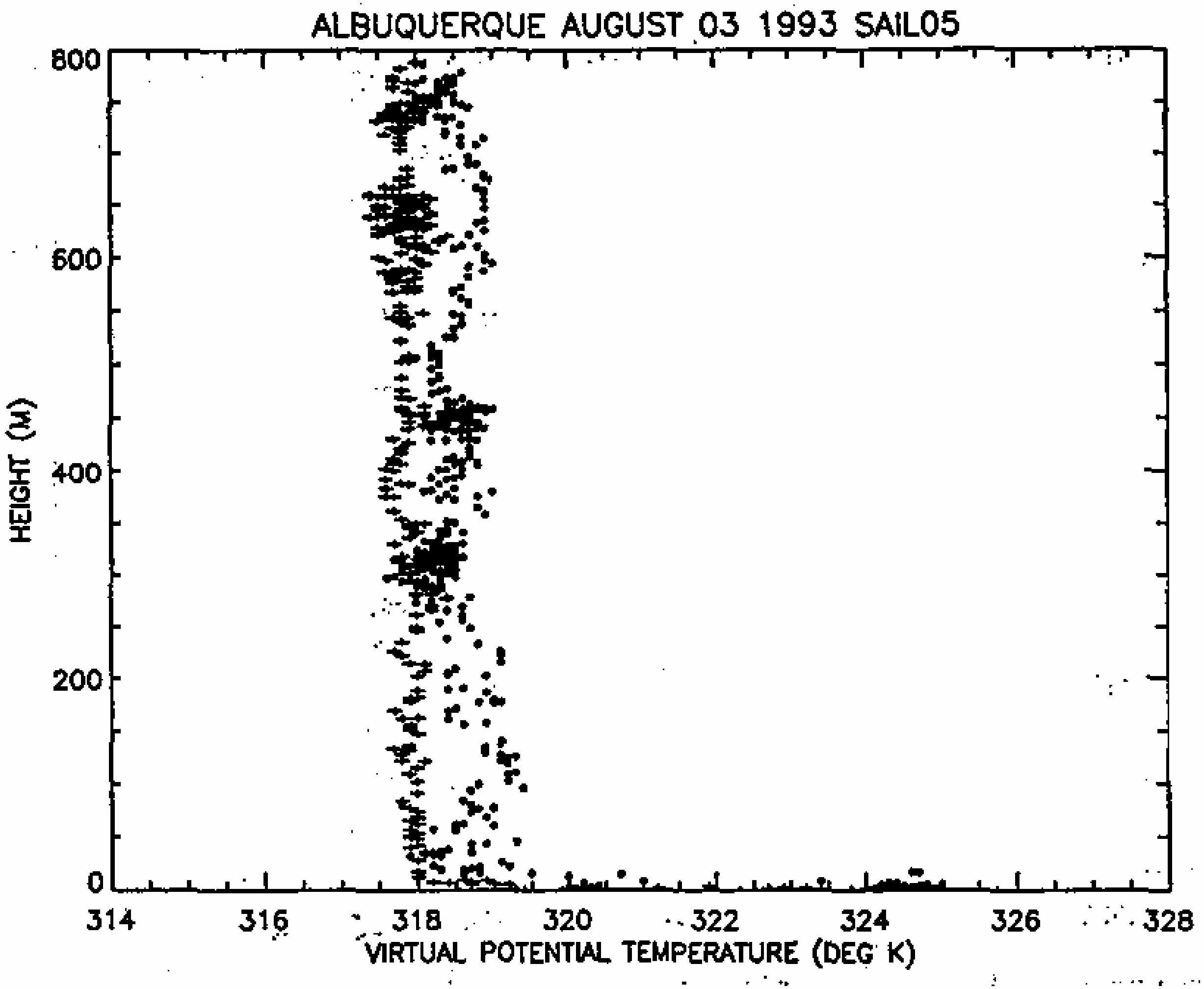
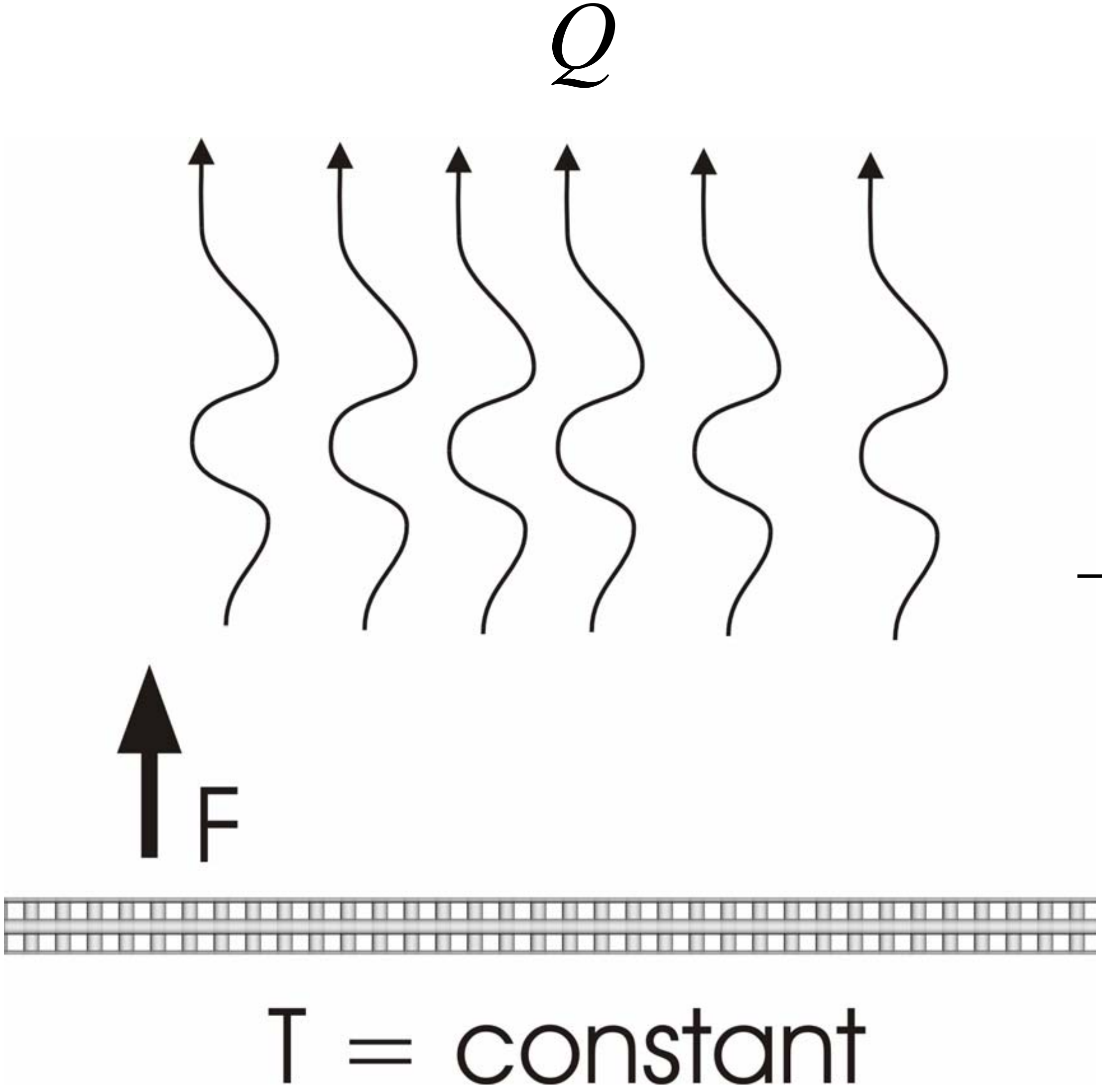


Image courtesy of American Meteorological Society.

Prandtl's (1925) problem



$$\int_0^\infty \dot{Q}\,dz = F$$

$$F \sim L^2 t^{-3}$$

$$\rightarrow w \sim \left(Fz\right)^{1/3}$$

$$B \sim \left(F^2/z\right)^{1/3}$$

Above a thin boundary layer, most atmospheric convection involves phase change of water:

Moist Convection

Moist Convection

- Significant heating owing to phase changes of water
- Redistribution of water vapor – most important greenhouse gas
- Significant contributor to stratiform cloudiness – albedo and longwave trapping

Water Variables

Mass concentration of water vapor (*specific humidity*):

$$q \equiv M_{H_2O} \big/ M_{air}$$

Vapor pressure (partial pressure of water vapor): e

Saturation vapor pressure: e^*

C-C: $$e^* = 6.112\, hPa\, e^{\frac{17.67(T-273)}{T+30}}$$

Relative Humidity: $$\mathcal{H} \equiv e \big/ e^*$$

The Saturation Specific Humidity

Ideal Gas Law:

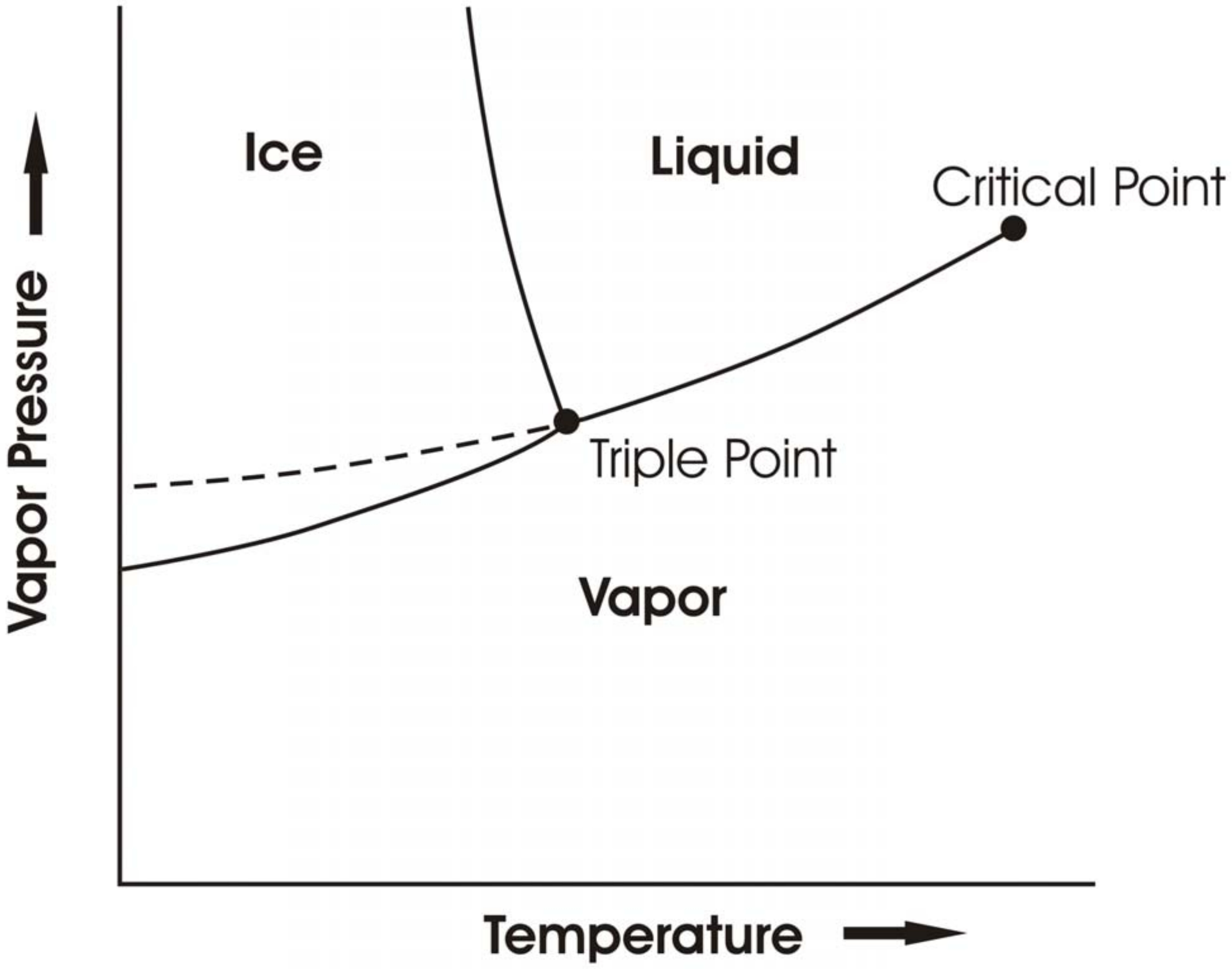
$$p = \rho \frac{R^* T}{\bar{m}}$$

$$e = \rho_v \frac{R^* T}{m_v}$$

$$q = \rho_v / \rho = \frac{m_v}{\bar{m}} \frac{e}{p}$$

$$q^* = \frac{m_v}{\bar{m}} \frac{e^*}{p}$$

Phase Equilibria



Bringing Air to Saturation

$$e = qp\left(\bar{m} / m_v \right)$$

$$e^* = e^*(T)$$

1. Increase *q (or p)*

2. Decrease *e* (T)*

When Saturation Occurs…

- Heterogeneous Nucleation
- Supersaturations very small in atmosphere
- Drop size distribution sensitive to size distribution of cloud condensation nuclei

ICE NUCLEATION PROBLEMATIC

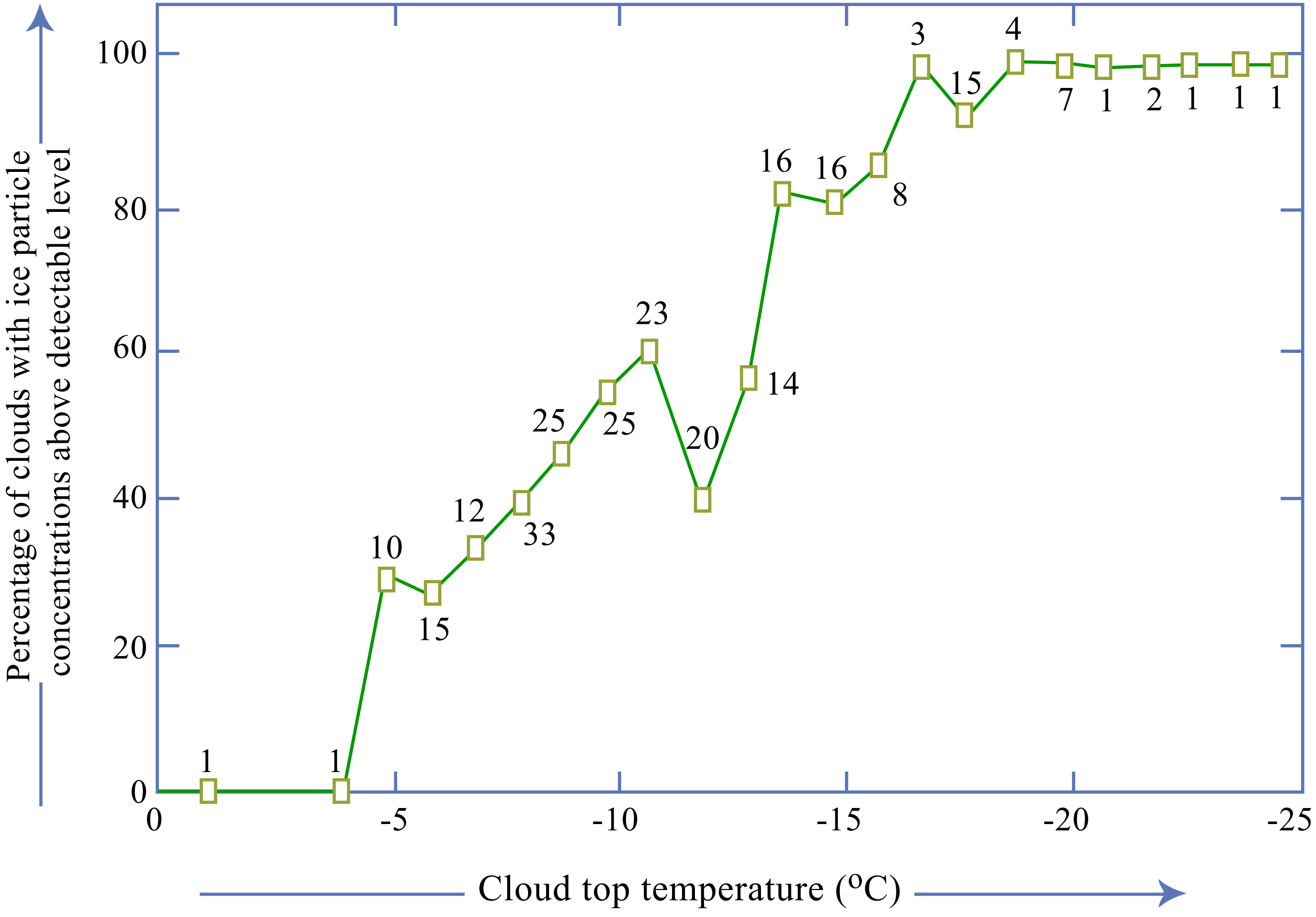


Figure by MIT OCW.

Precipitation Formation:

- Stochastic coalescence (sensitive to drop size distributions)
- Bergeron-Findeisen Process
- Strongly nonlinear function of cloud water concentration
- Time scale of precipitation formation ~10-30 minutes

Stability

No simple criterion based on entropy:

$$s_d = c_p \ln\left(\frac{T}{T_0}\right) - R_d \ln\left(\frac{p}{p_0}\right)$$

$$\alpha = \alpha\left(s_d, p\right)$$

$$s = c_p \ln\left(\frac{T}{T_0}\right) - R_d \ln\left(\frac{p}{p_0}\right) + L_v \frac{q}{T} - qR_v \ln\left(\mathcal{H}\right)$$

$$\alpha = \alpha\left(s, p, q_t\right)$$

Virtual Temperature and Density Temperature

Assume all condensed water falls at terminal velocity

$$\alpha = \frac{V_a + V_c}{M_d + M_v + M_c}$$

$$pV = nR^*T$$

$$V_a = \frac{R^*T}{p}\left(\frac{M_d}{m_d} + \frac{M_v}{m_v}\right),$$

$$\overline{m_d} \equiv \frac{1}{\frac{1}{M_d}\sum_i \frac{M_i}{m_i}}$$

$$\rightarrow \quad V_a = \frac{R_d T}{p}\left(M_d + {}^{M_v}\!/_{\varepsilon}\right),$$

where $$\varepsilon \equiv {}^{m_v}\!/_{m_d} \cong 0.622$$

$$R_d \equiv {}^{R^*}\!/_{m_d}$$

$$\alpha = \frac{V_a + V_c}{M_d + M_v + M_c} = \frac{R_d T}{p}\left(1 - q_t + {q}/{\varepsilon}\right)\left(1 + \frac{q_c}{1 - q_c}\frac{\rho_a}{\rho_c}\right)$$

$$\cong \frac{R_d T}{p}\left(1 - q_t + {q}/{\varepsilon}\right)$$

$$q_t \equiv \frac{M_v + M_c}{M}, \qquad q \equiv \frac{M_v}{M}$$

Density temperature:

$$T_\rho \equiv T\left(1 - q_t + {q}/{\varepsilon}\right)$$

$$\alpha = \frac{R_d T_\rho}{p}$$

Trick:

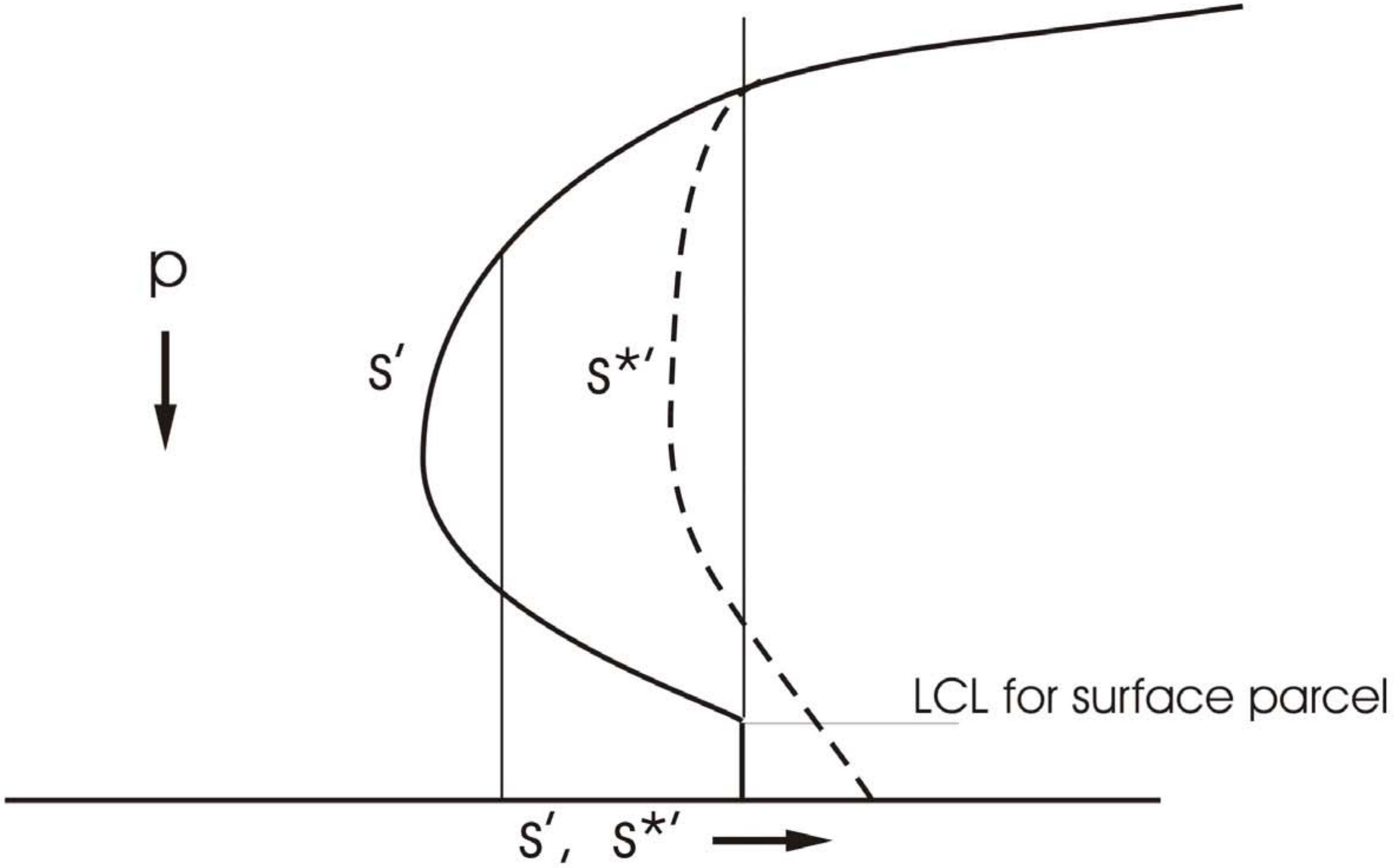
Define a *saturation entropy, s** :

$$s^* \equiv s\left(T, p, q^*\right)$$

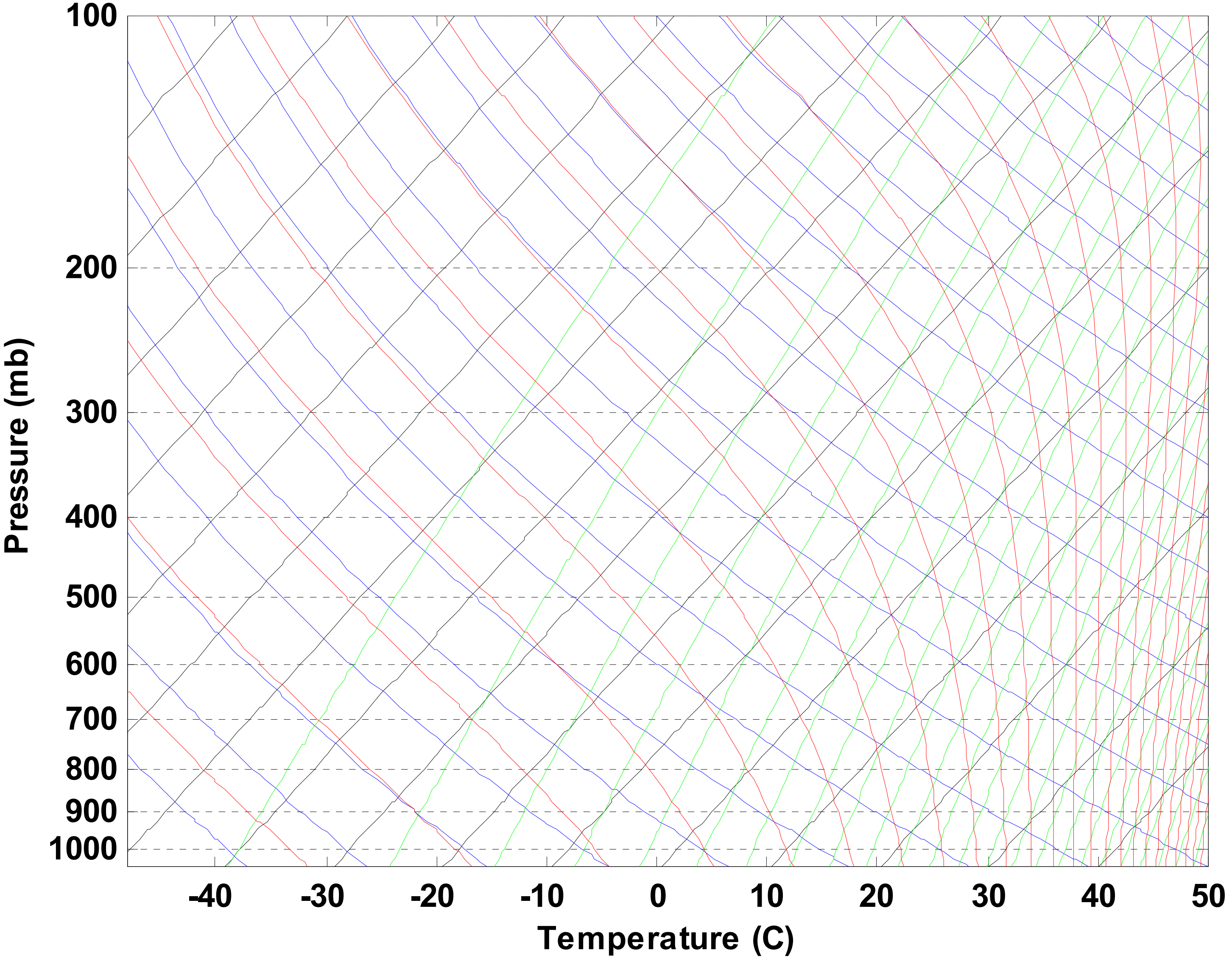
$$\alpha = \alpha\left(s^*, p, q_t\right)$$

We can add an arbitrary function of q_t to s^* such that

$$\alpha \cong \alpha\left(s^{*\prime}, p\right)$$


p
s′
s*′
LCL for surface parcel
s′, s*′

Stability Assessment using Tephigrams:

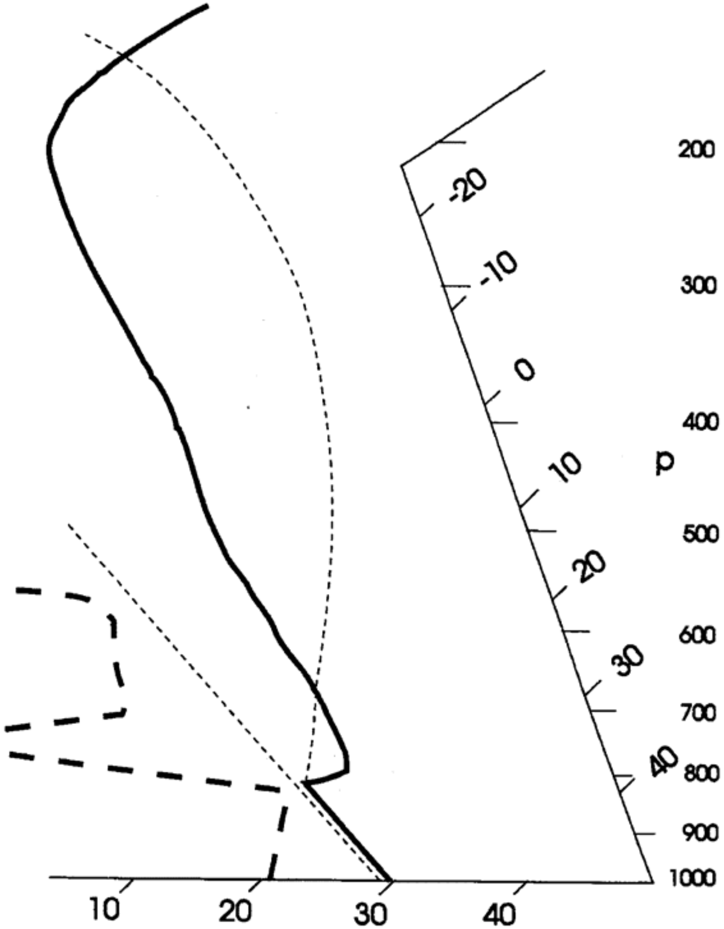


Stability Assessment using Tephigrams:

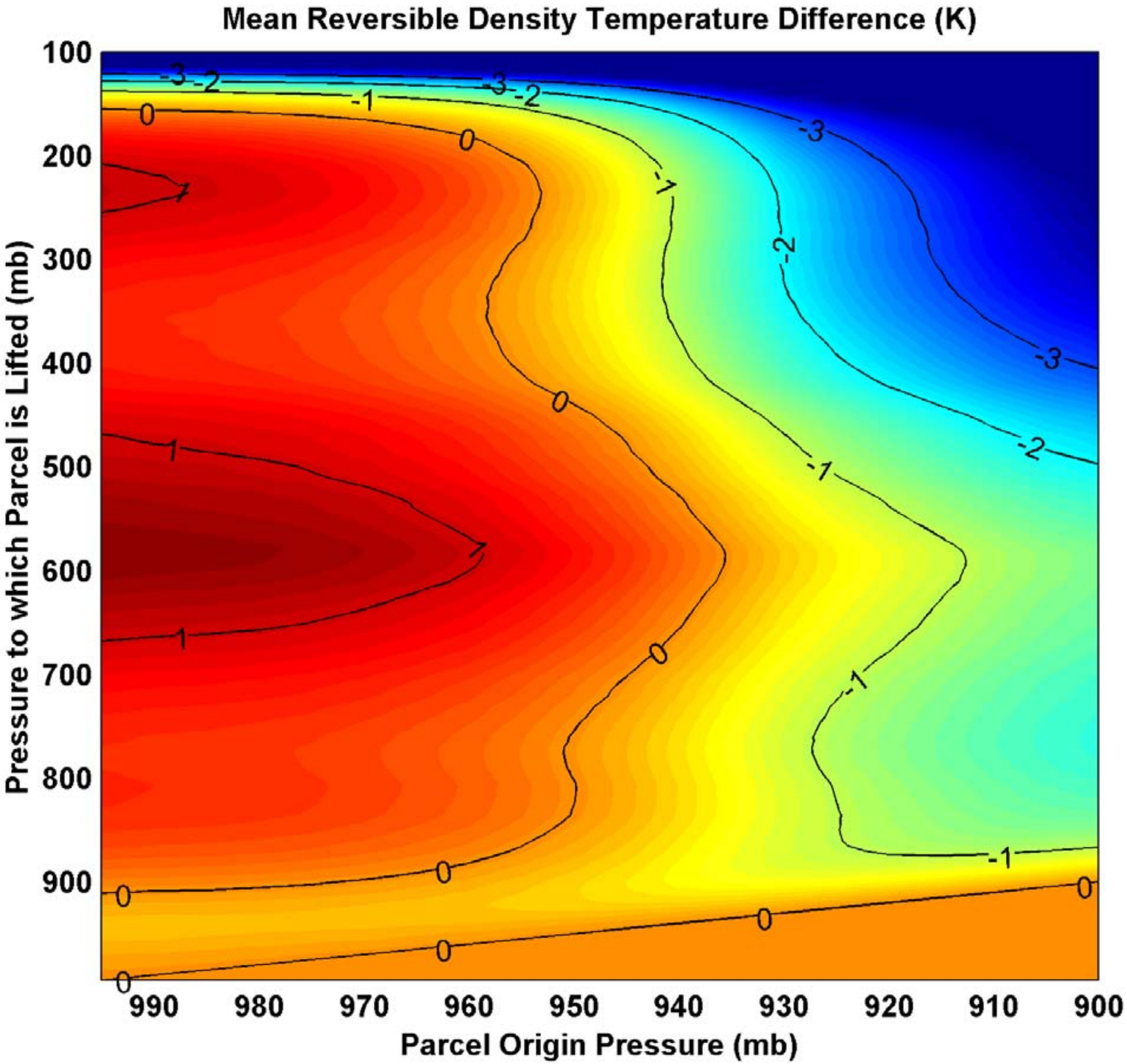
Convective Available Potential Energy
(CAPE):

$$CAPE_i \equiv \int_{p_n}^{p_i} \left(\alpha_p - \alpha_e\right) dp$$

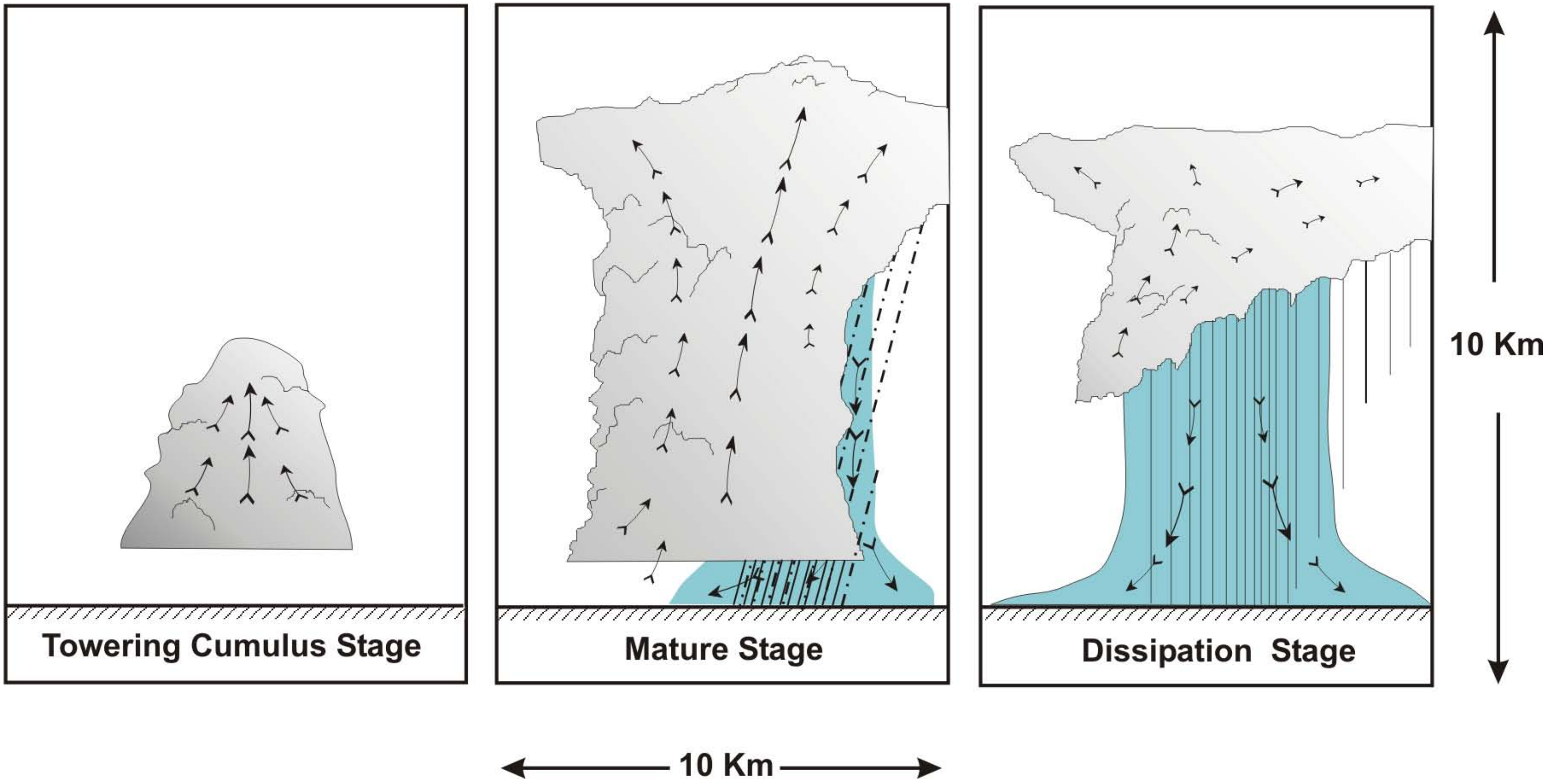
$$= \int_{p}^{p_i} R_d \left(T_{\rho_p} - T_{\rho_e}\right) d \ln\left(p\right)$$



Other Stability Diagrams:



“Air-Mass” Showers:



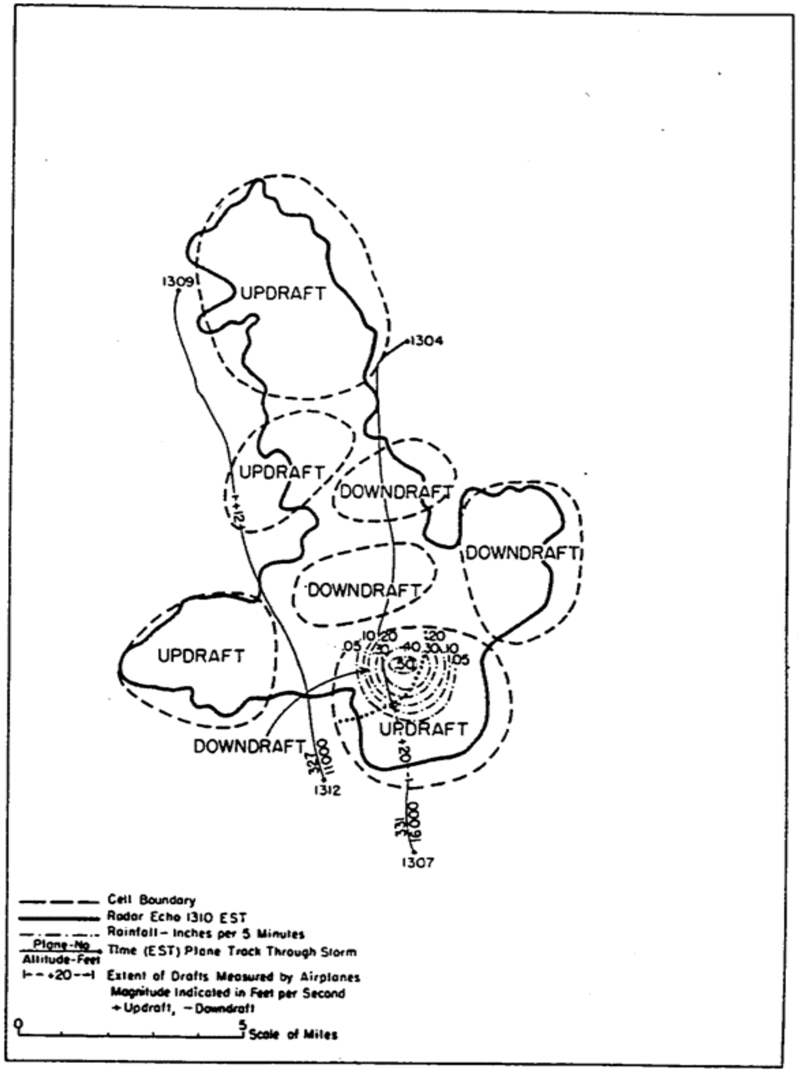


FIG. 15. Radar echo, plane paths, measured draft data, and cell outlines, 1310 EST 9 July 1946.

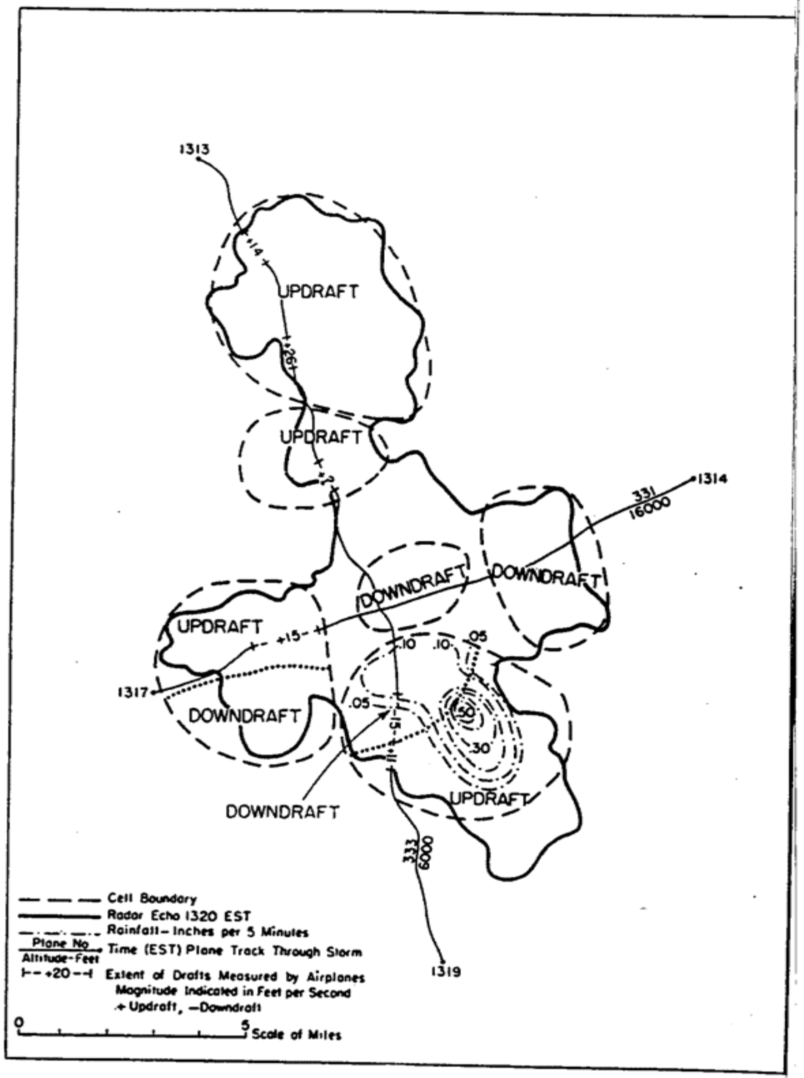
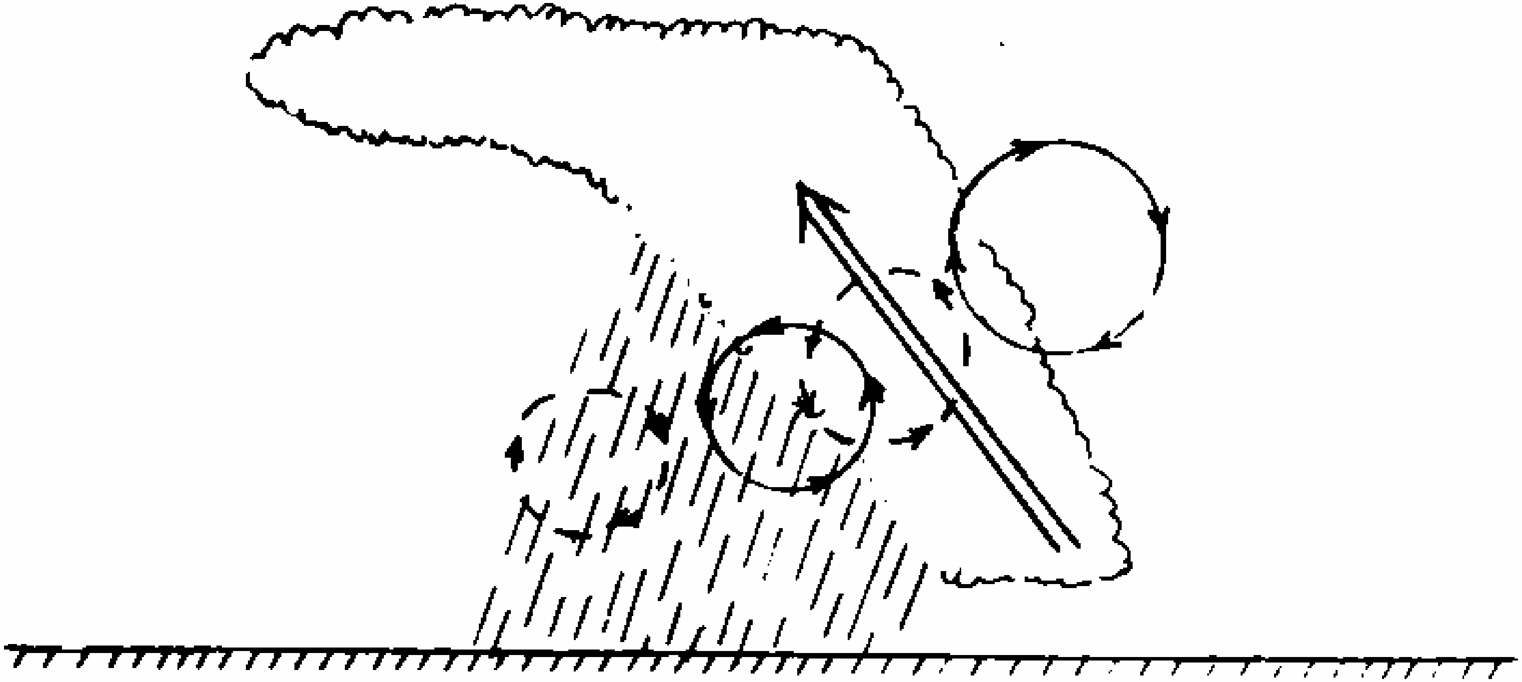


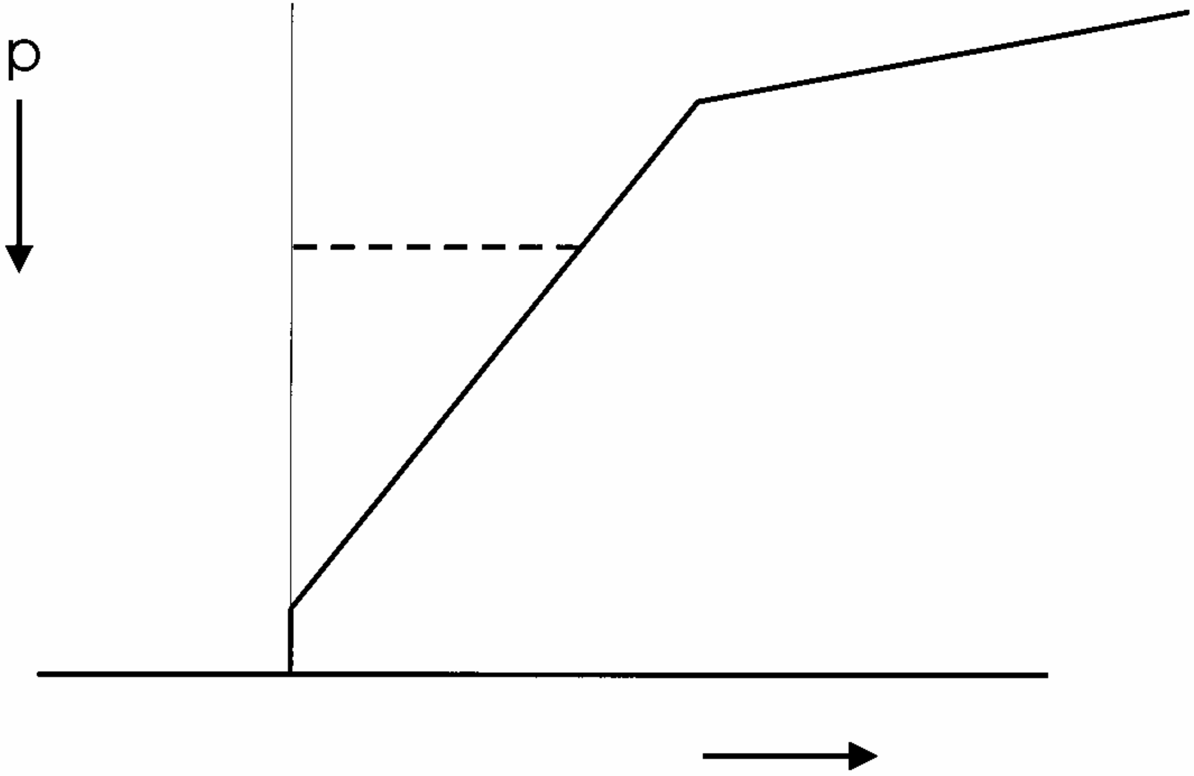
FIG. 16. Radar echo, plane paths, measured draft data, and cell outlines, 1320 EST 9 July 1946.

Image courtesy of American Meteorological Society.

Precipitation Effects:



Buoyancy Reversal:



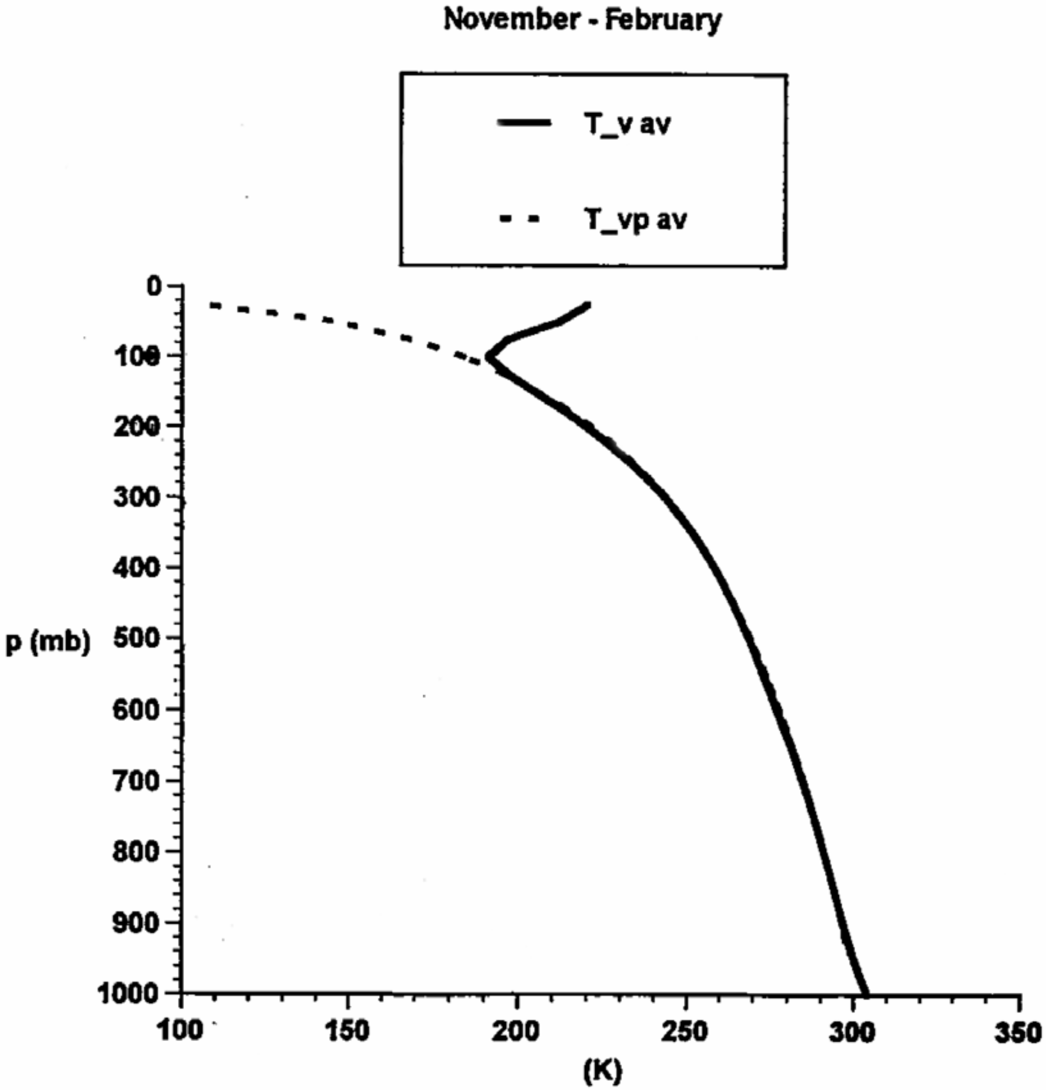
$$h_{lv} = C_p T_v + gz - L_v \ell$$

Summary of Differences Between Dry and Moist Convection:

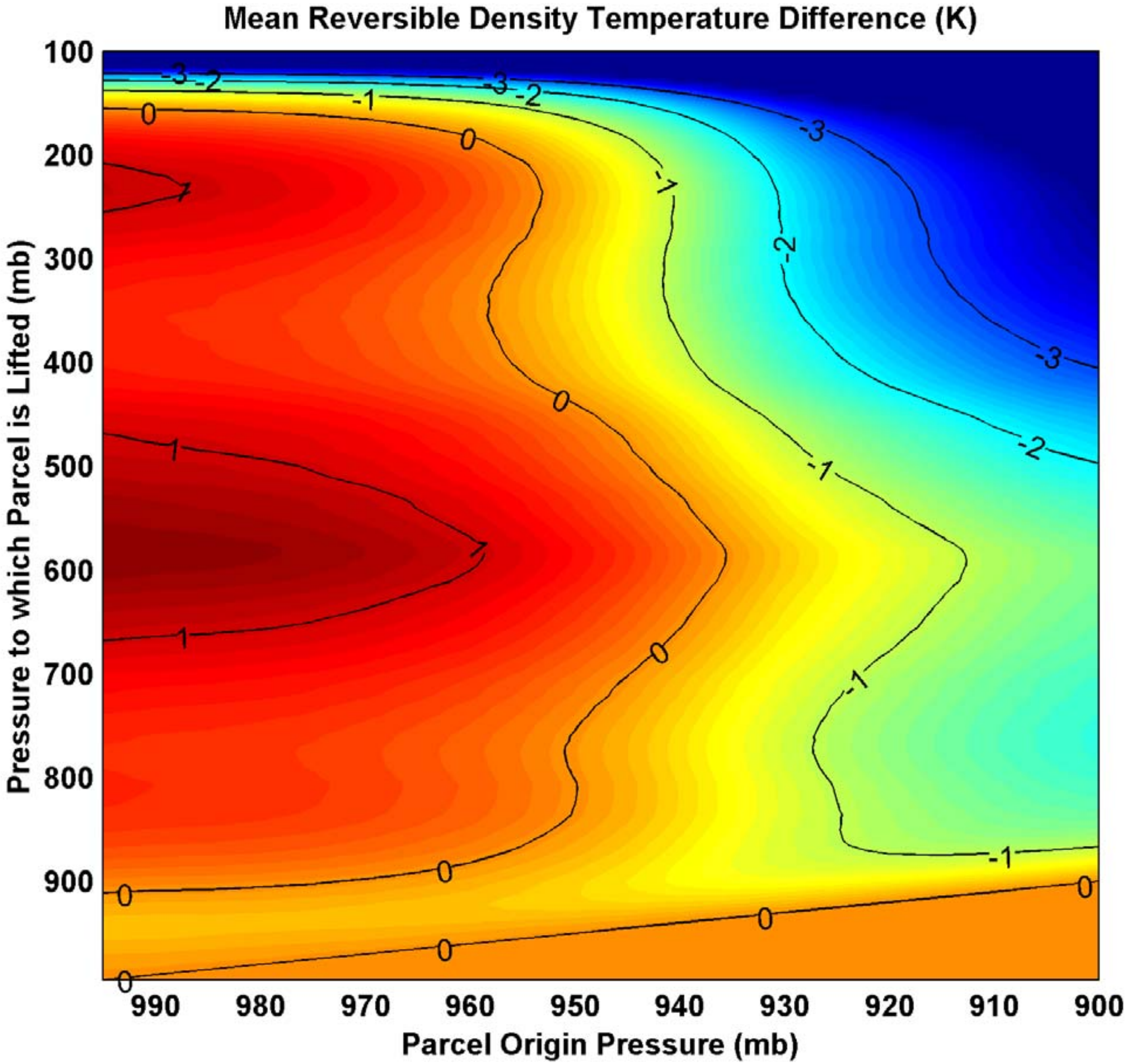
- Possibility of metastable states
- Strong asymmetry between cloudy and clear regions
- Typically, only thin layers near surface are unstable to upward displacements
- Mixing can cause buoyancy reversal
- Large potential for evaporatively cooled downdrafts
- Separation of buoyancy from displacement can lead to propagating convection

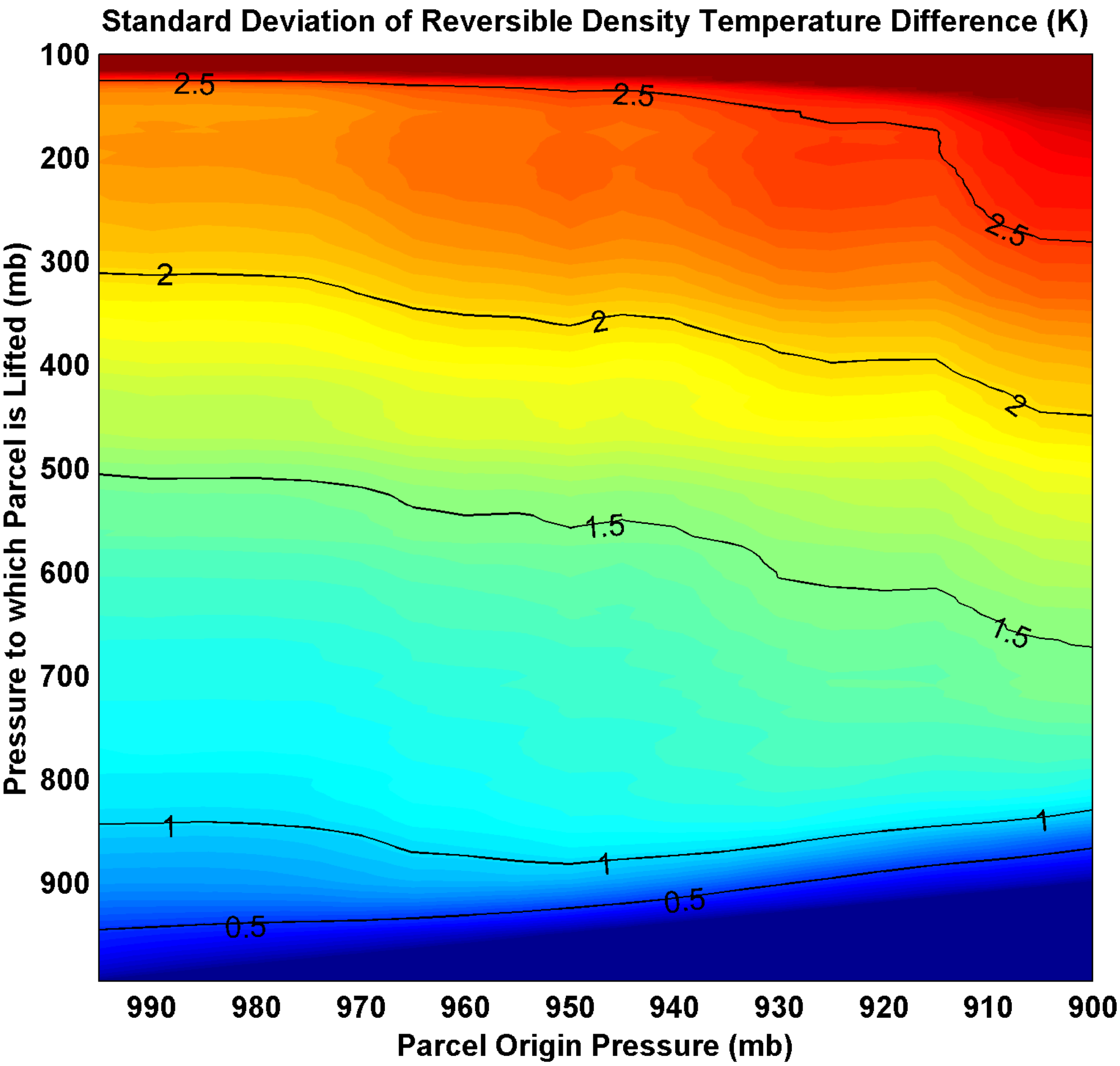
- Buoyancy of unsaturated downdrafts depends on supply of precipitation
- Entropy produced mostly through mixing, not dissipation
- Internal waves can co-exist with unstable convection

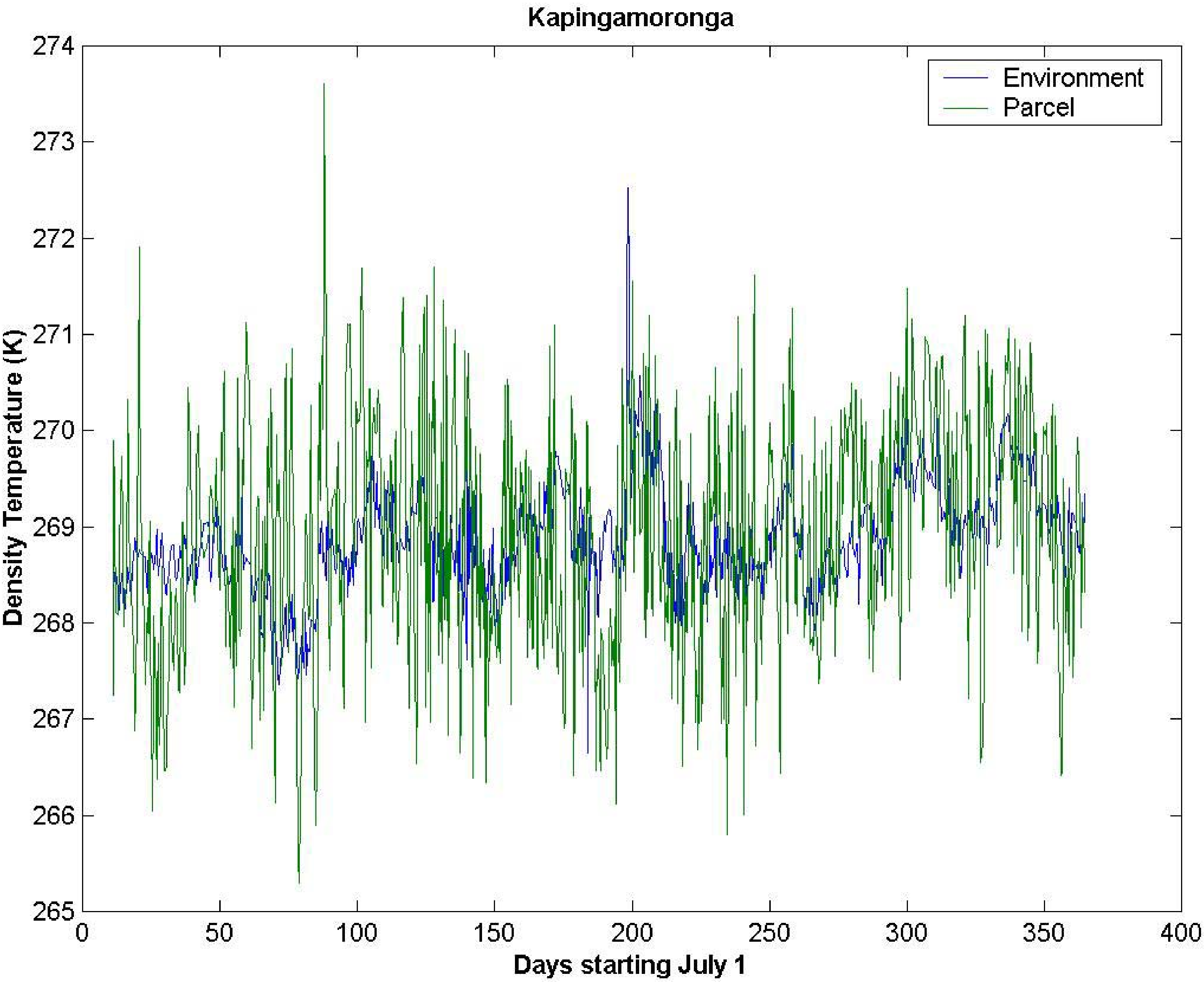
Tropical Soundings

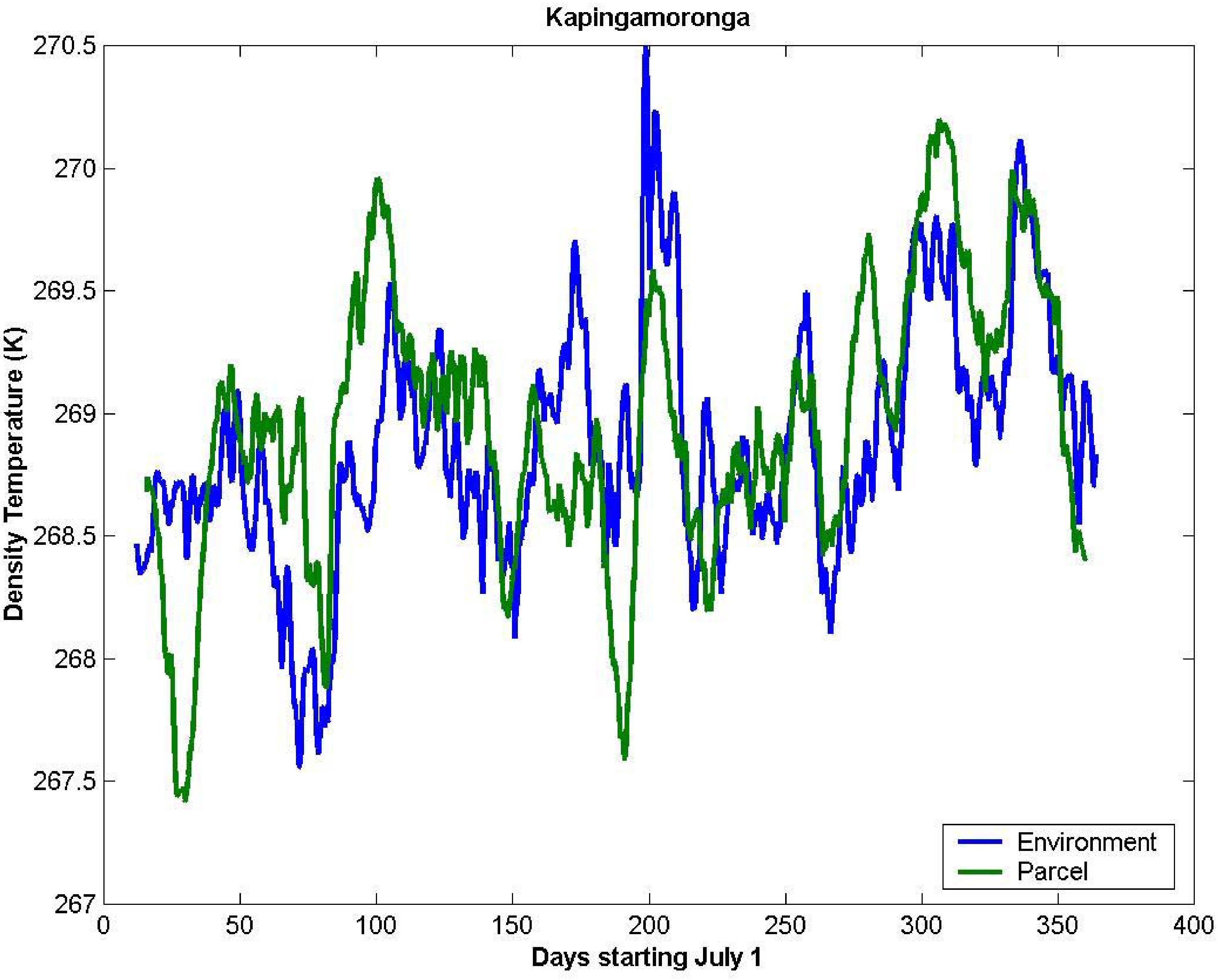


Annual Mean Kapingamoronga:

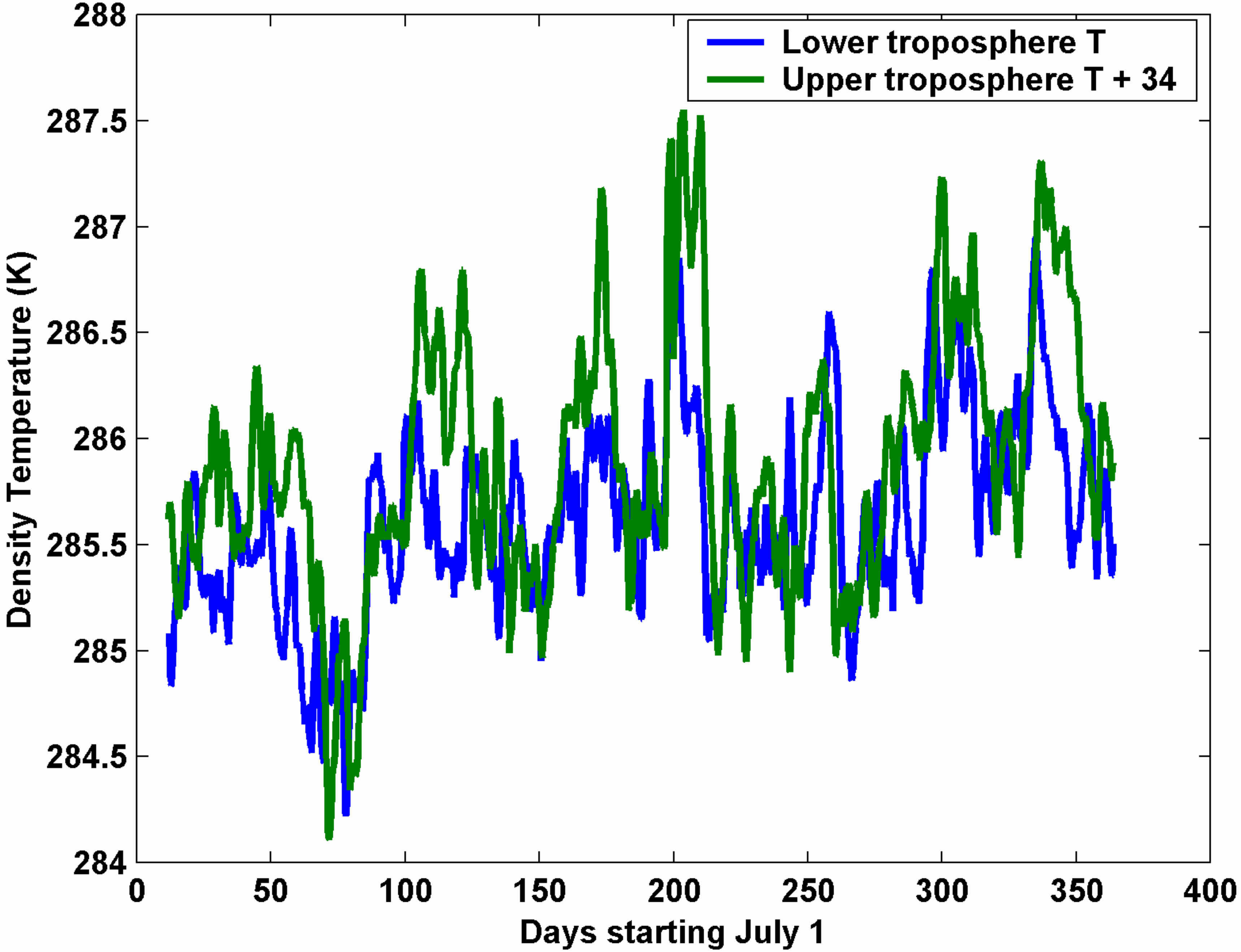


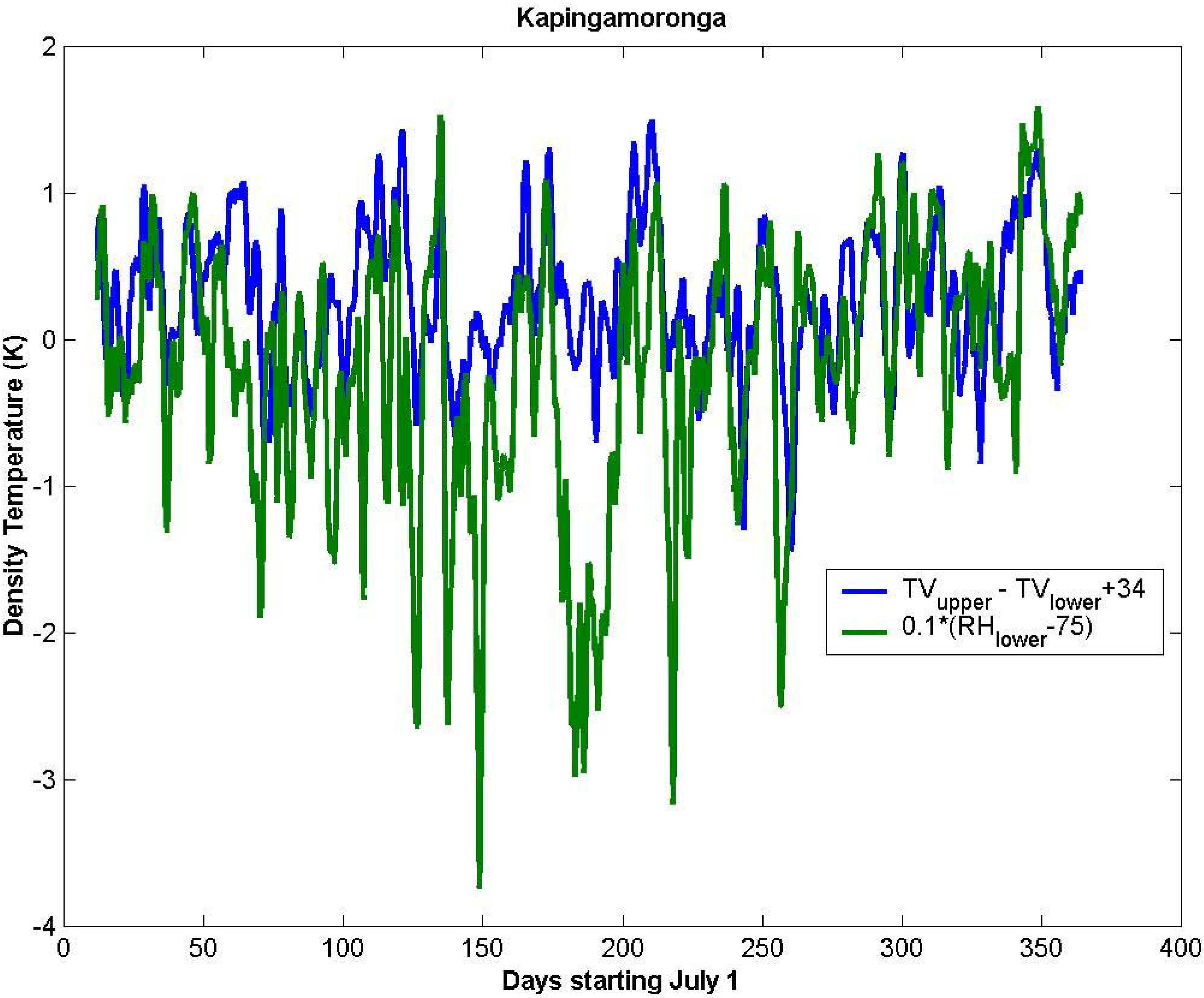

Standard Deviation of Reversible Density Temperature Difference (K)
Pressure to which Parcel is Lifted (mb)
100
200
300
400
500
600
700
800
900
990
980
970
960
950
940
930
920
910
900
Parcel Origin Pressure (mb)
2.5
2.5
2.5
2
2
2
1.5
1.5
1.5
1
1
1
0.5
0.5

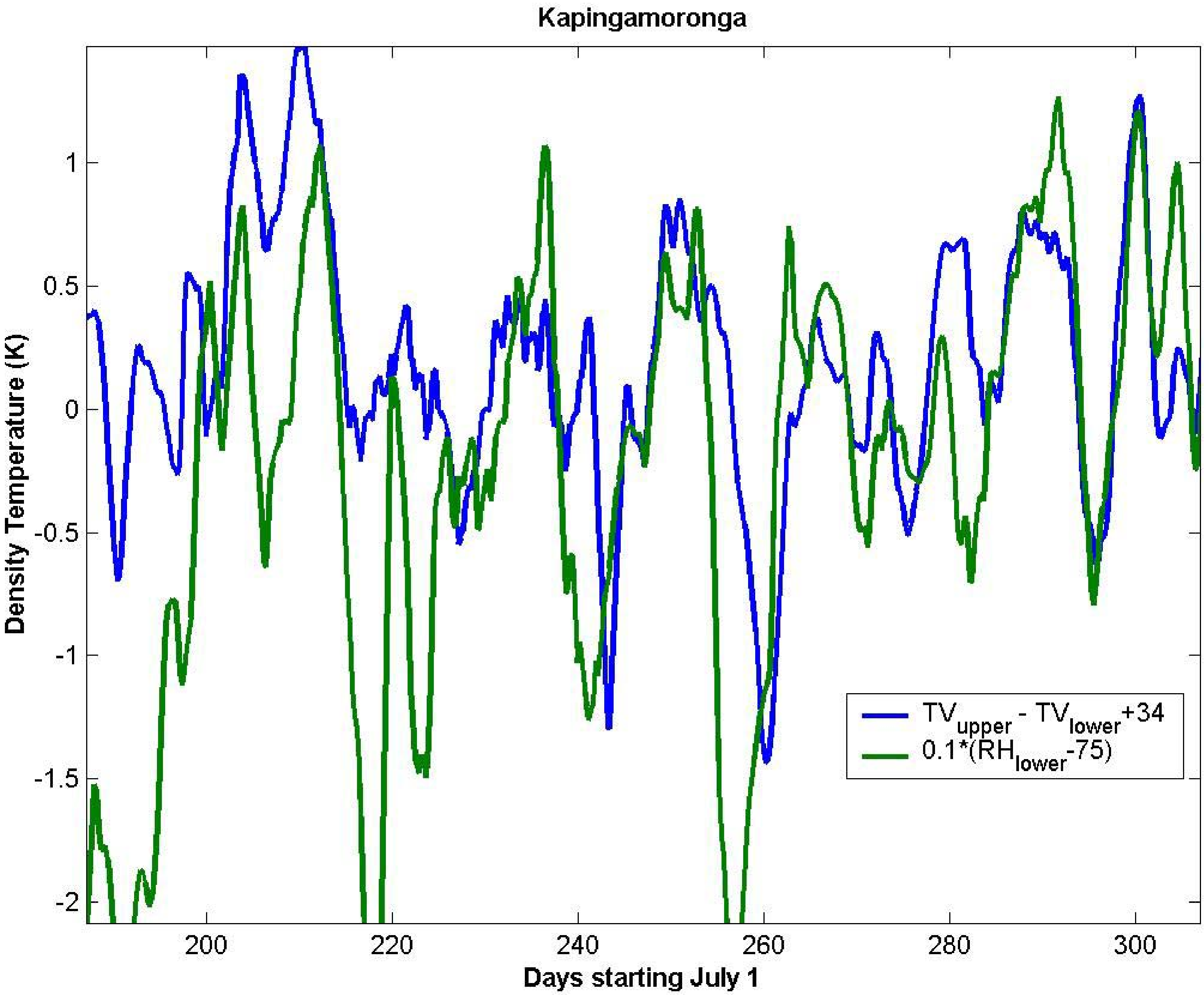

Kapingamoronga
Environment
Parcel
Density Temperature (K)
274
273
272
271
270
269
268
267
266
265
0
50
100
150
200
250
300
350
400
Days starting July 1

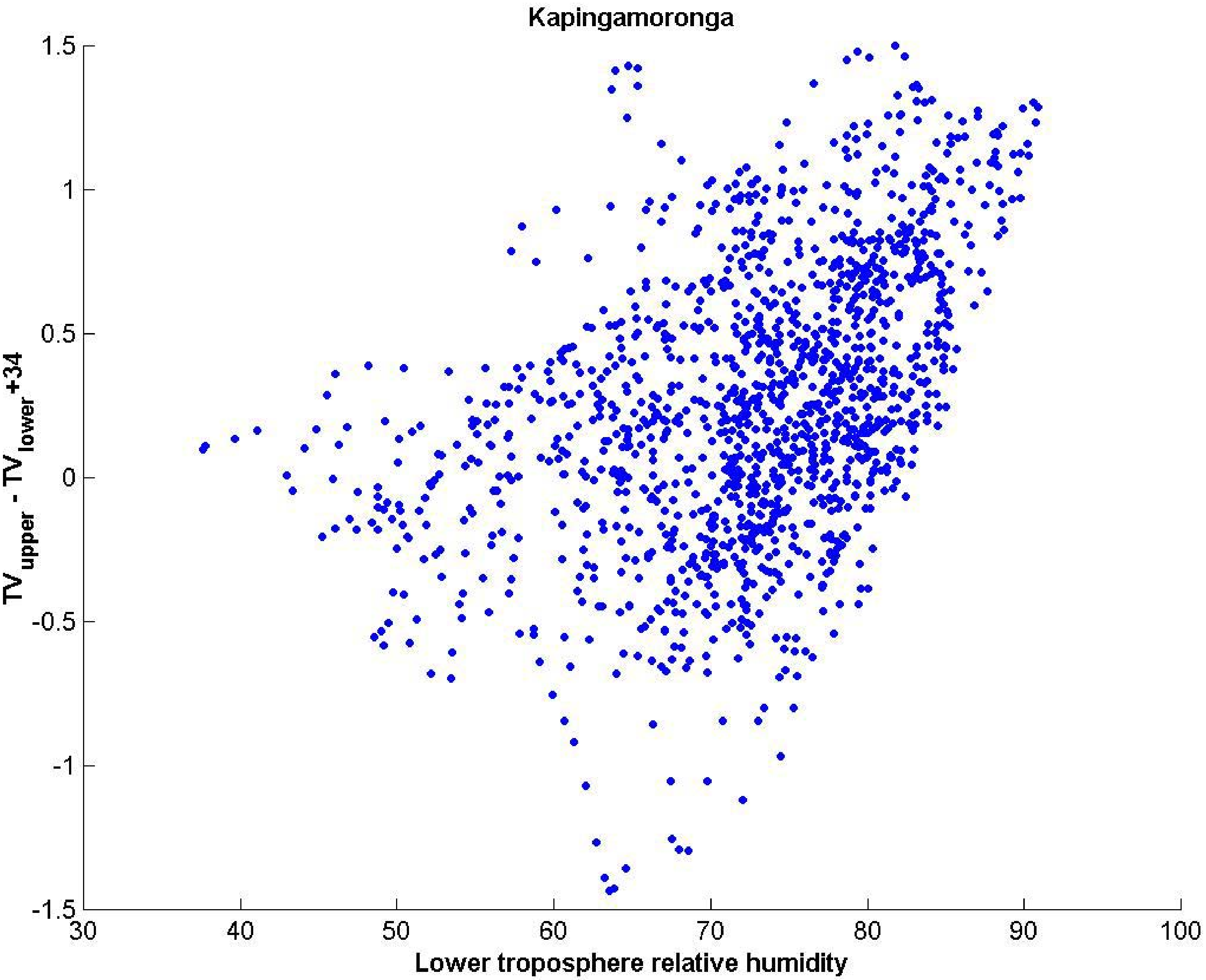

Kapingamoronga
Density Temperature (K)
270.5
270
269.5
269
268.5
268
267.5
267
0
50
100
150
200
250
300
350
400
Days starting July 1
Environment
Parcel

Kapingamoronga



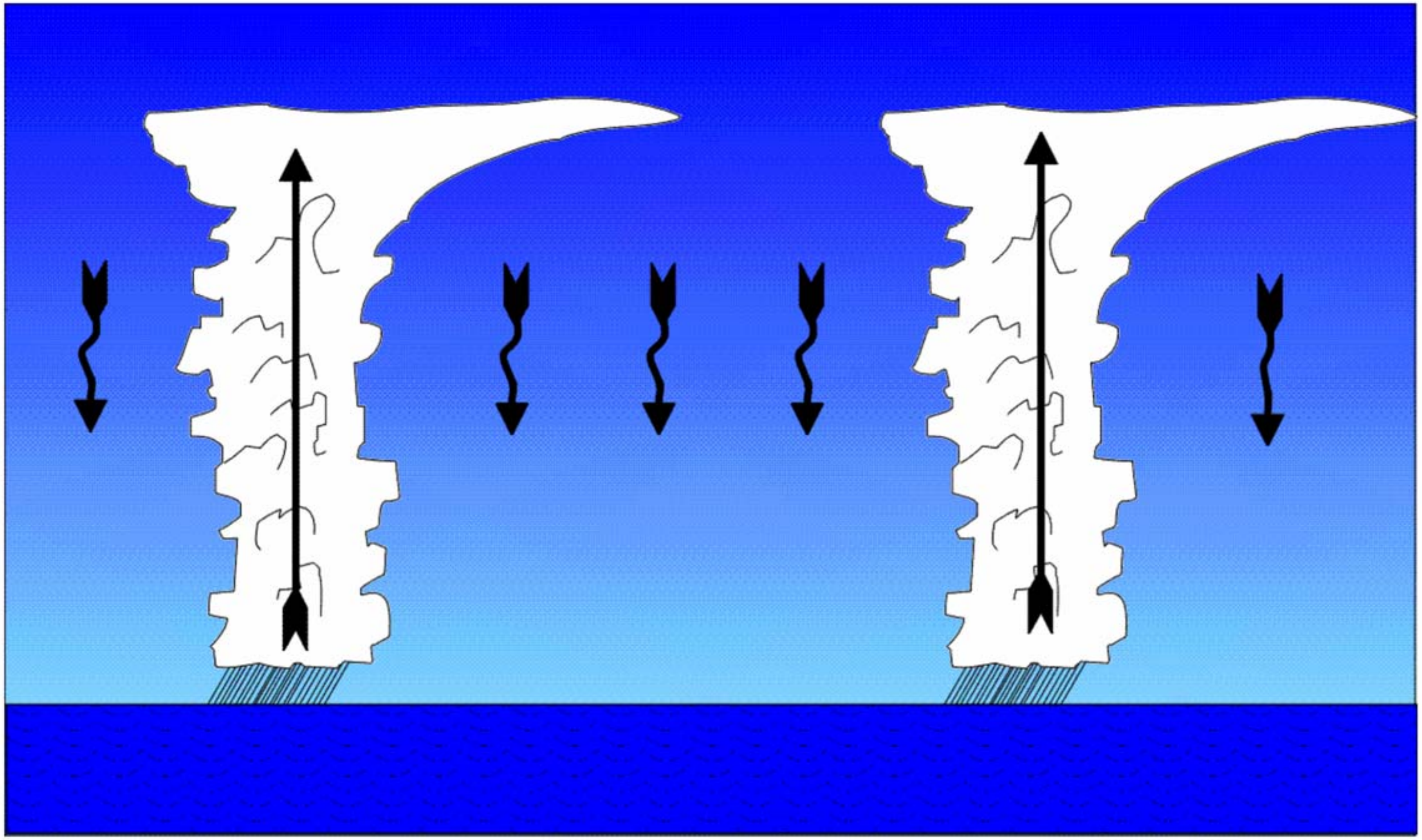

Kapingamoronga
Density Temperature (K)
Days starting July 1
2
1
0
-1
-2
-3
-4
0
50
100
150
200
250
300
350
400
TV_upper - TV_lower+34
0.1*(RH_lower-75)


Kapingamoronga
Density Temperature (K)
Days starting July 1
1
0.5
0
-0.5
-1
-1.5
-2
200
220
240
260
280
300
TV_{upper} - TV_{lower}+34
0.1*(RH_{lower}-75)


Kapingamoronga
1.5
1
0.5
0
-0.5
-1
-1.5
30
40
50
60
70
80
90
100
Lower troposphere relative humidity
TVupper - TVlower +34

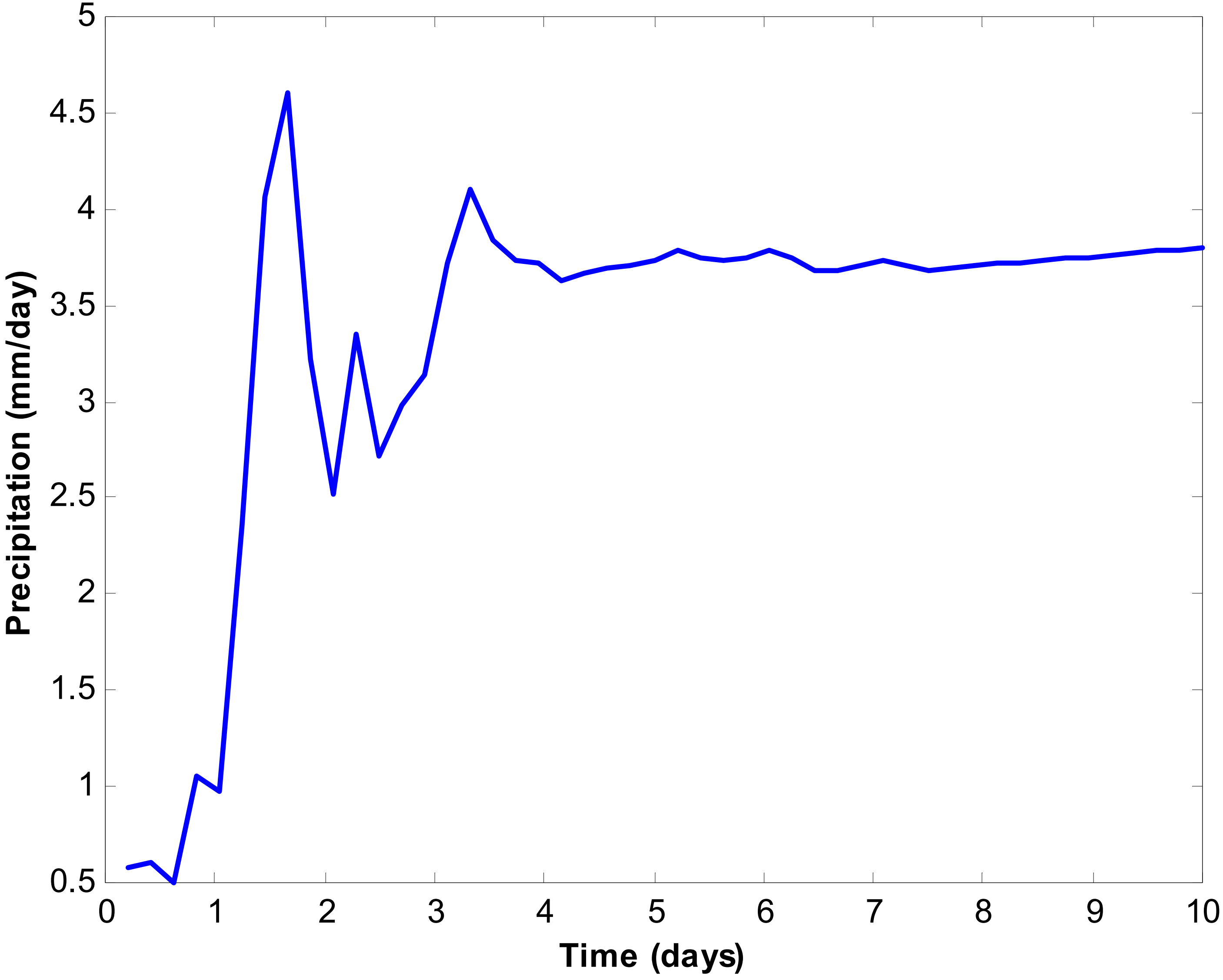
Radiative-Moist Convective Equilibrium

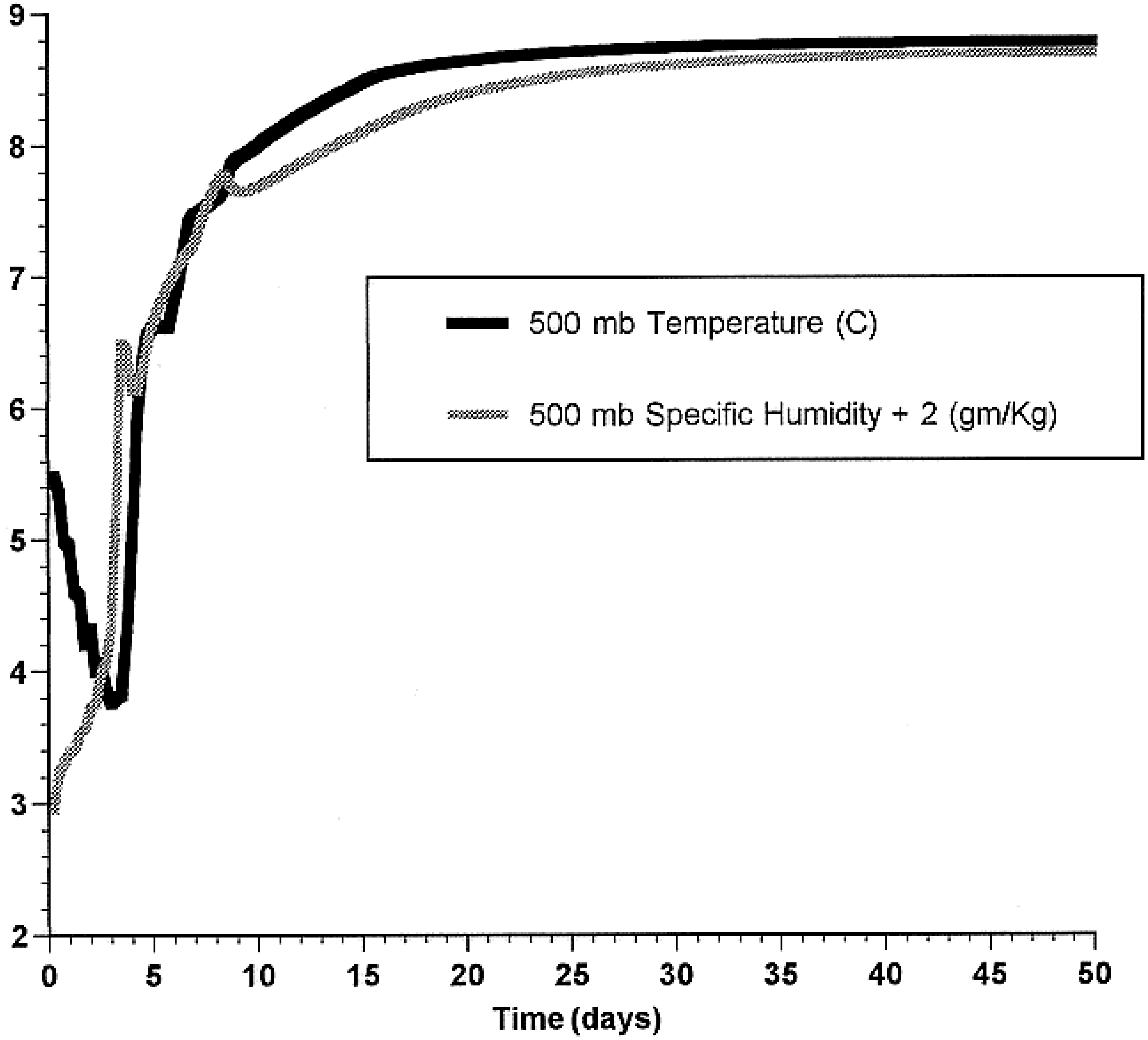
Precipitating Convection favors Widely Spaced Clouds (Bjerknes, 1938)



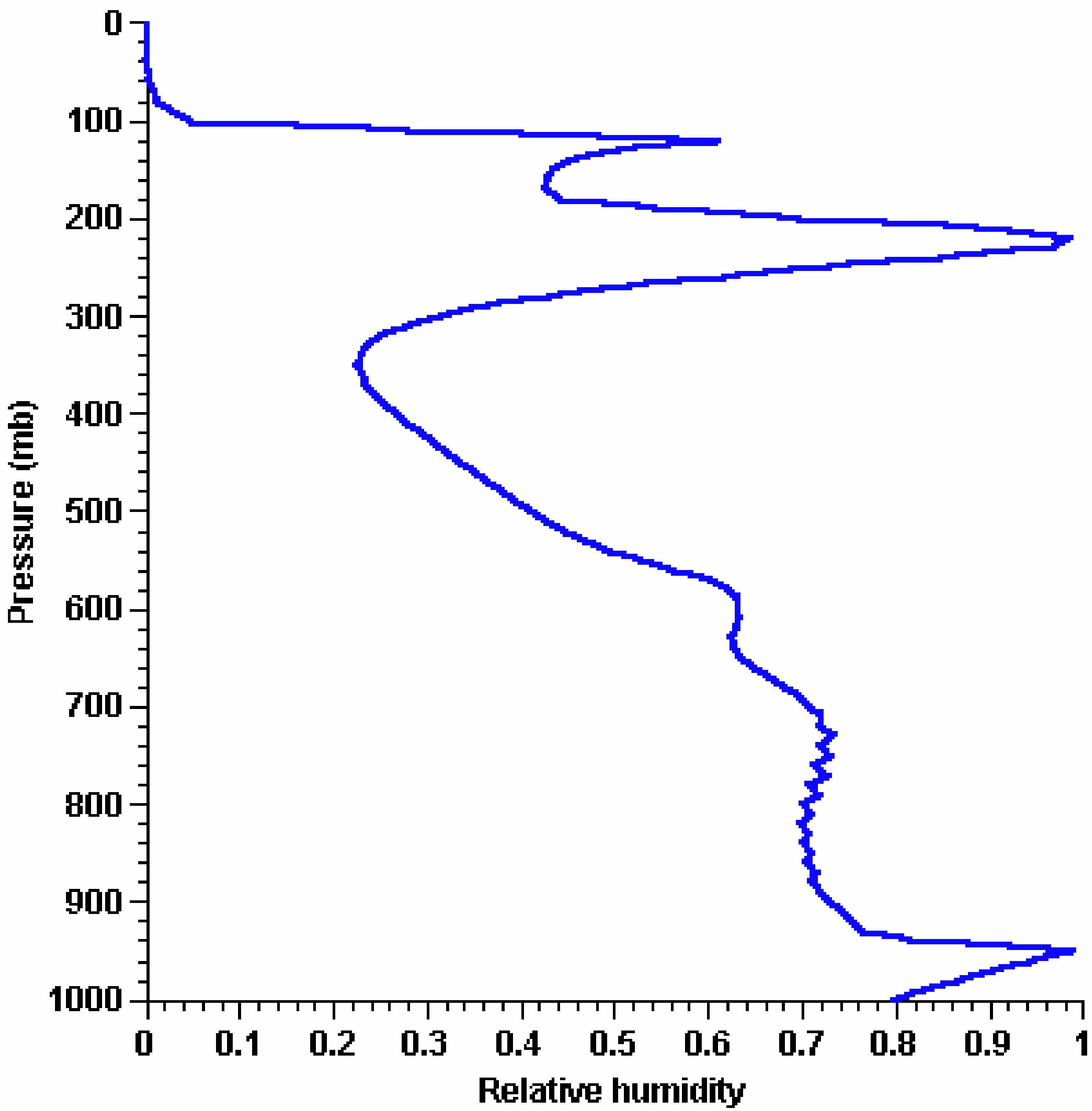
Properties:

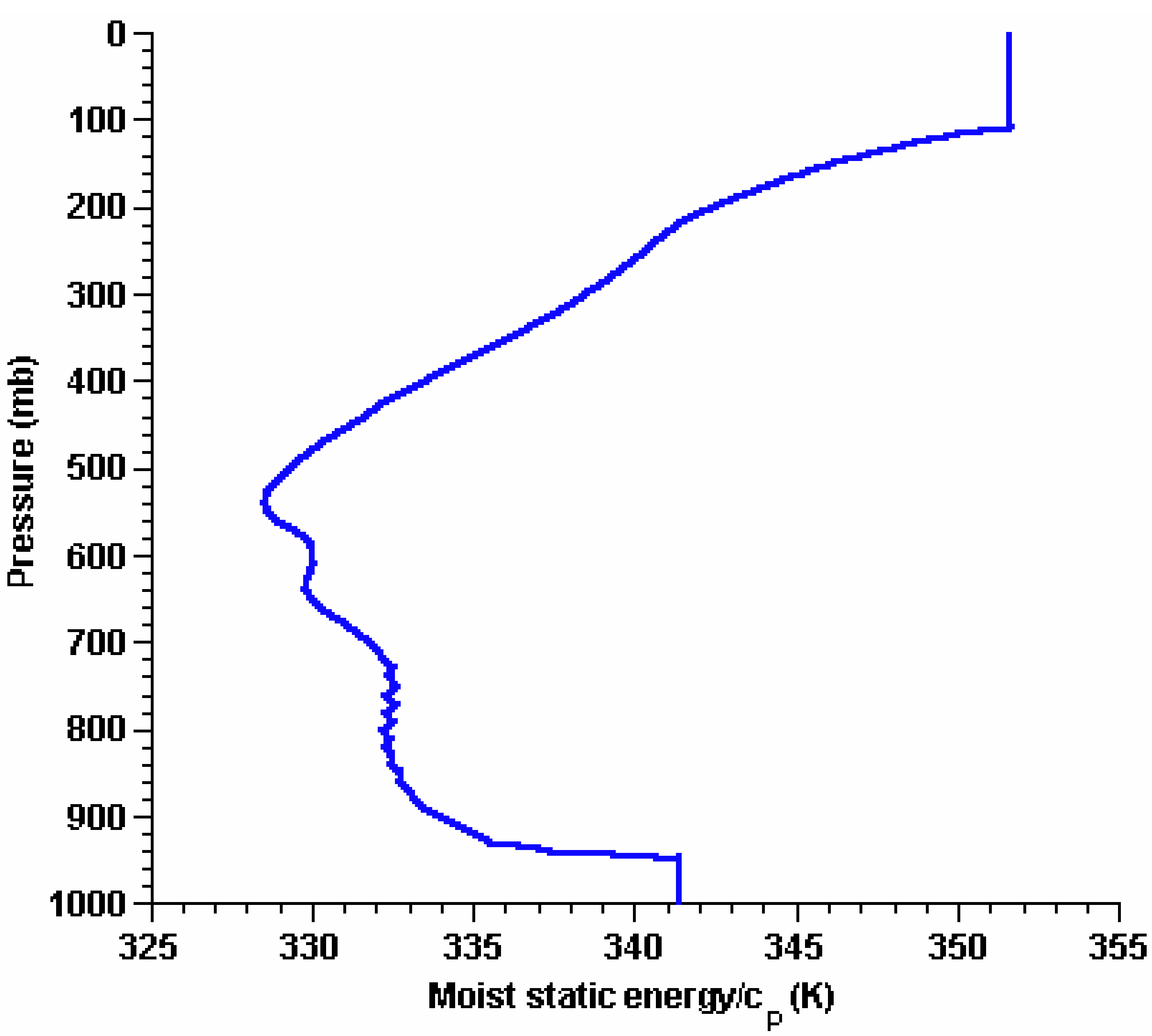
- Convective updrafts widely spaced
- Surface enthalpy flux equal to vertically integrated radiative cooling
- $M \frac{c_p T}{\theta} \frac{\partial \theta}{\partial z} = -\dot{Q}$
- Precipitation = Evaporation = Radiative Cooling
- Radiation and convection *highly* interactive

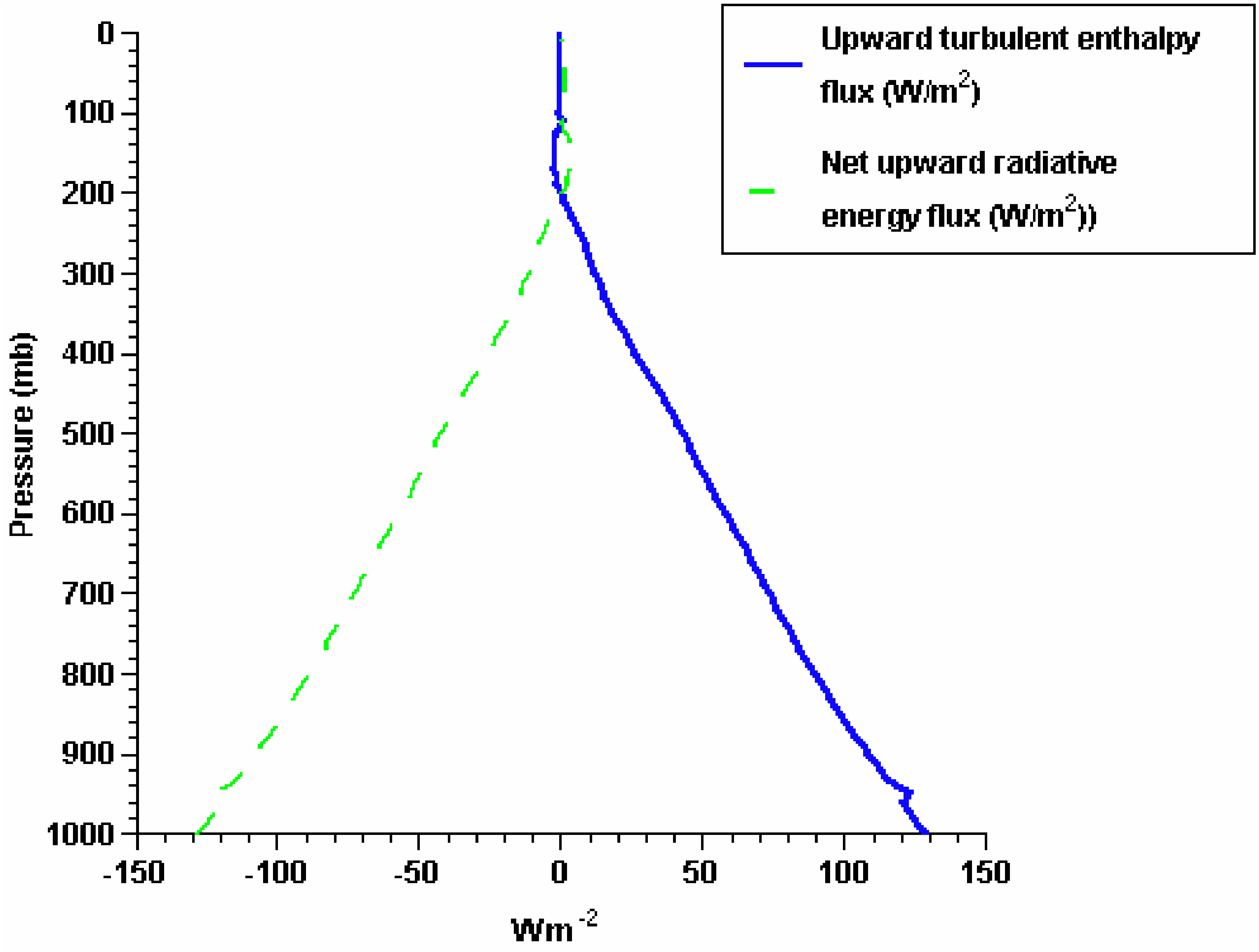

5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
Precipitation (mm/day)
0
1
2
3
4
5
6
7
8
9
10
Time (days)



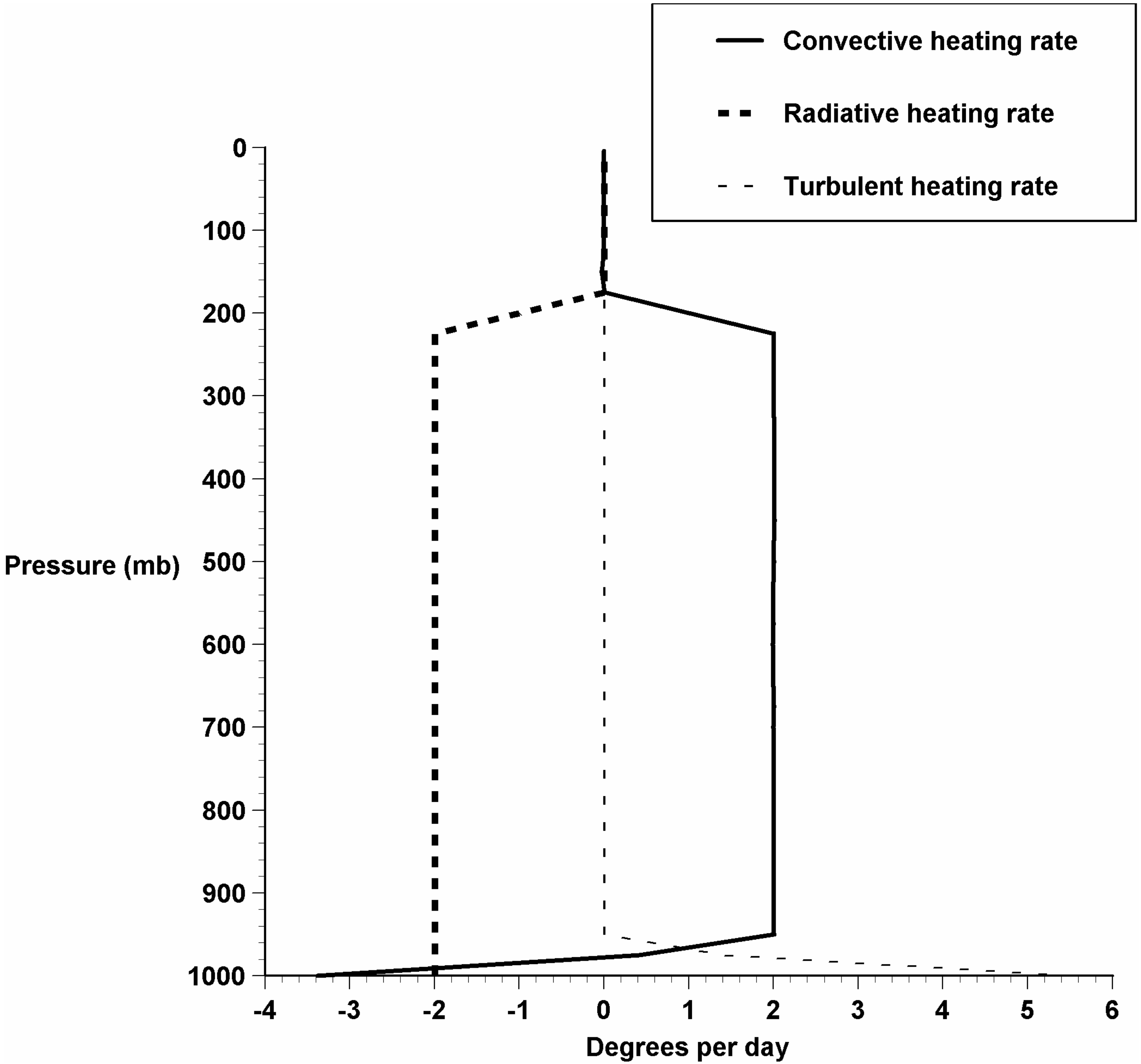
9
8
7
6
5
4
3
2
0
5
10
15
20
25
30
35
40
45
50
Time (days)
500 mb Temperature (C)
500 mb Specific Humidity + 2 (gm/Kg)

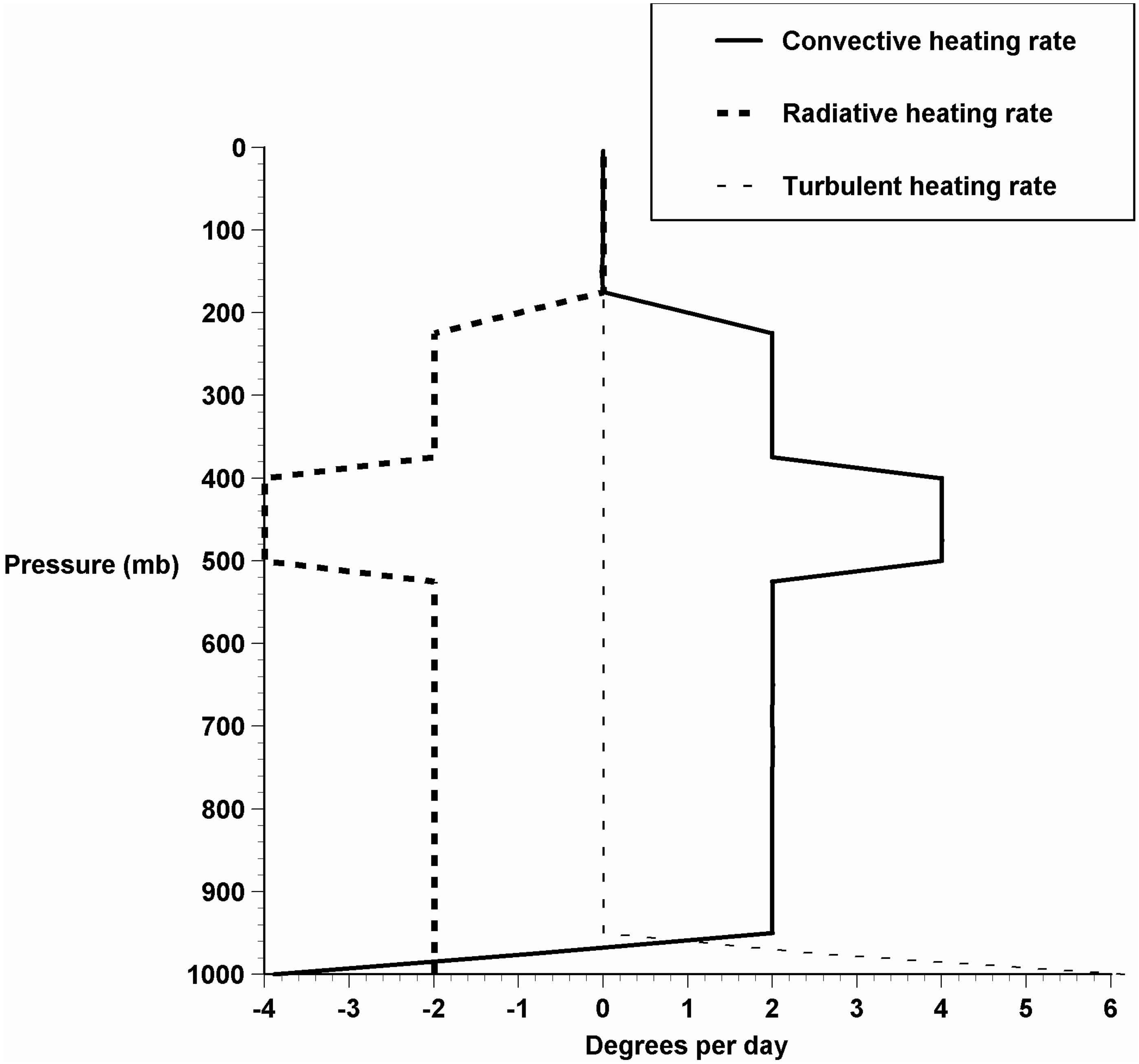

Pressure (mb)
0
100
200
300
400
500
600
700
800
900
1000
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Relative humidity

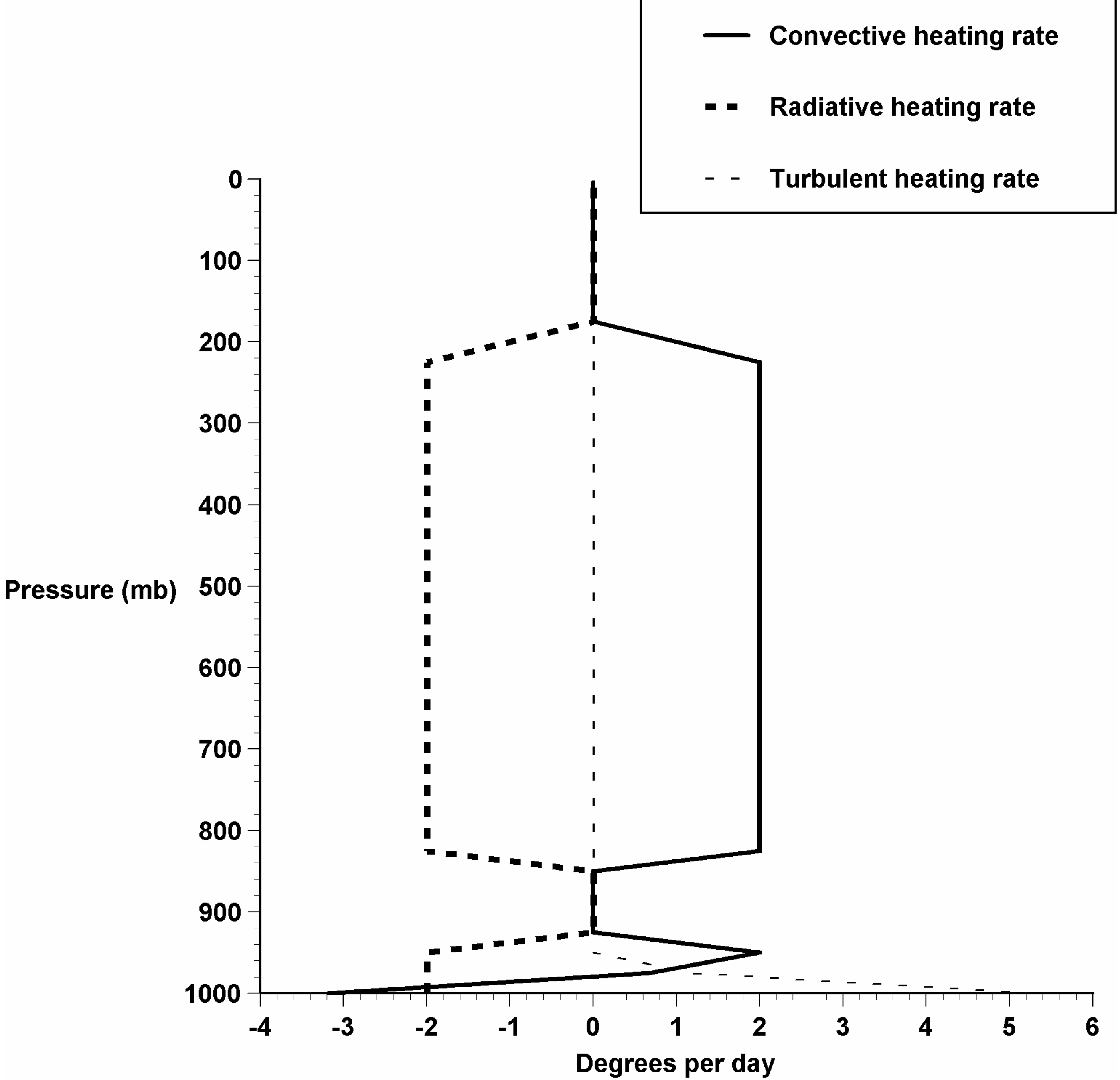

Pressure (mb)
0
100
200
300
400
500
600
700
800
900
1000
325
330
335
340
345
350
355
Moist static energy/c_p (K)

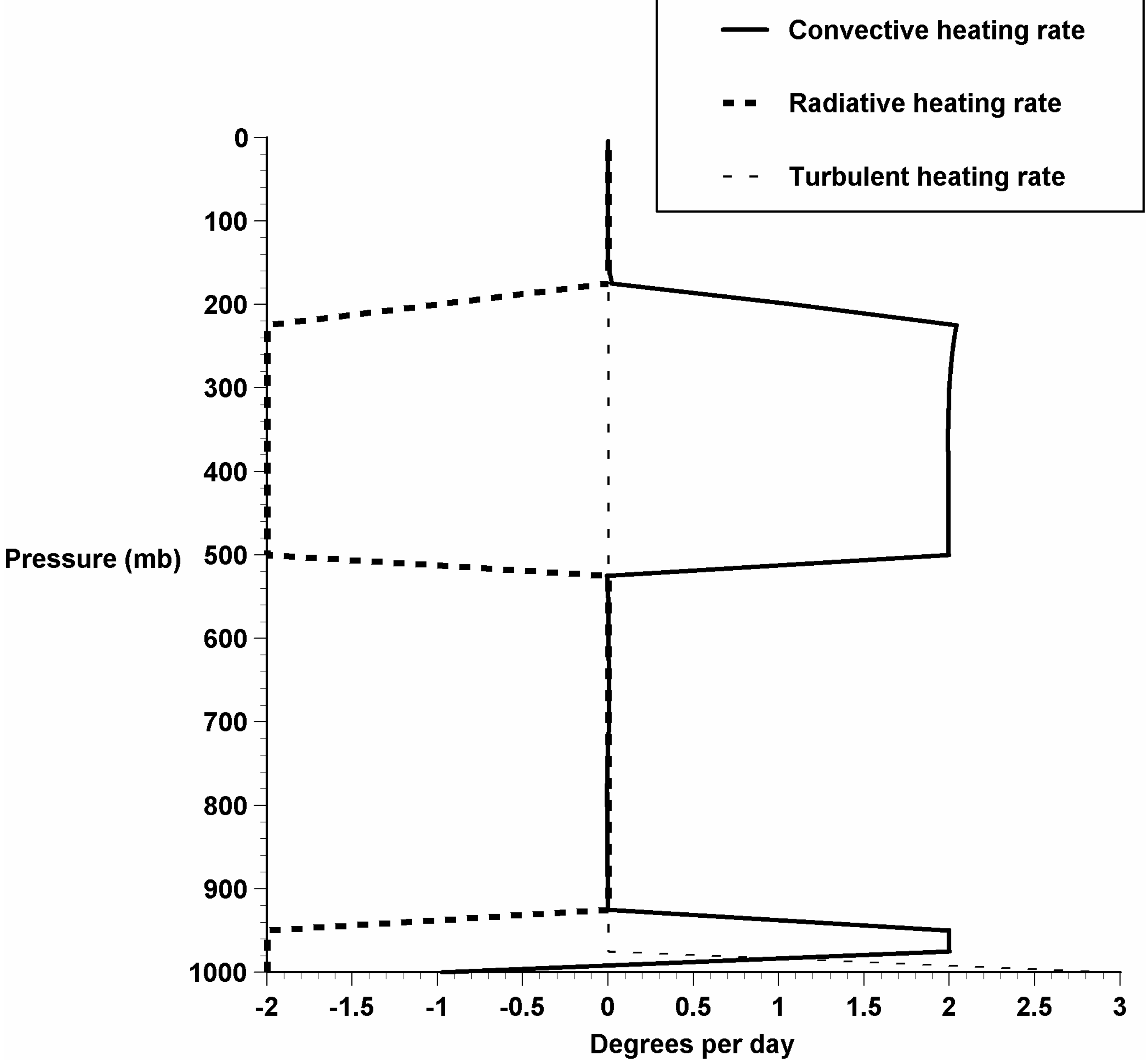


Upward turbulent enthalpy flux (W/m²)
Net upward radiative energy flux (W/m²))
Pressure (mb)
0
100
200
300
400
500
600
700
800
900
1000
-150
-100
-50
0
50
100
150
Wm -2


Convective heating rate
Radiative heating rate
Turbulent heating rate
Pressure (mb)
0
100
200
300
400
500
600
700
800
900
1000
-4
-3
-2
-1
0
1
2
3
4
5
6
Degrees per day


Convective heating rate
Radiative heating rate
Turbulent heating rate
Pressure (mb)
0
100
200
300
400
500
600
700
800
900
1000
-4
-3
-2
-1
0
1
2
3
4
5
6
Degrees per day


Convective heating rate
Radiative heating rate
Turbulent heating rate
Pressure (mb)
0
100
200
300
400
500
600
700
800
900
1000
-4
-3
-2
-1
0
1
2
3
4
5
6
Degrees per day



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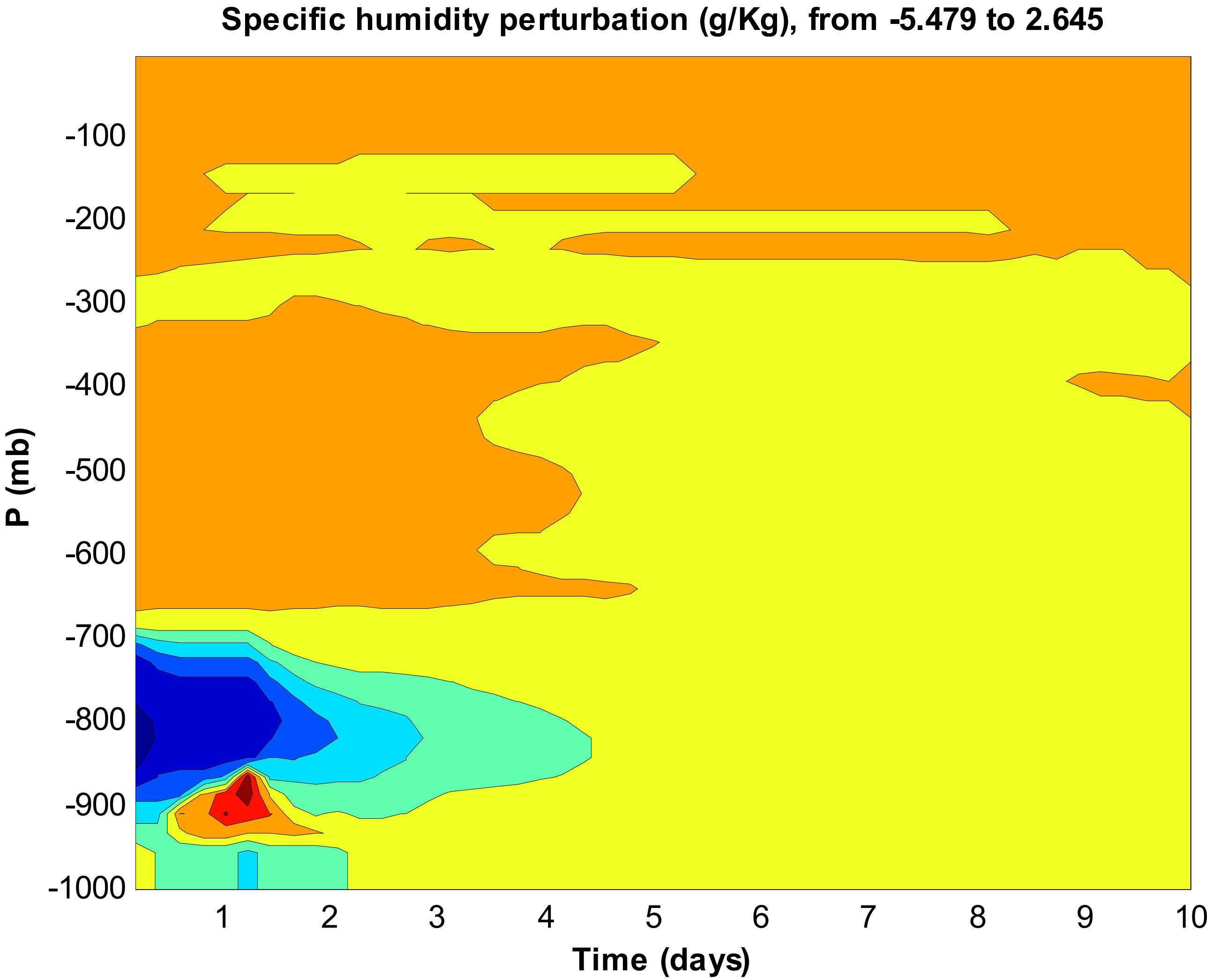
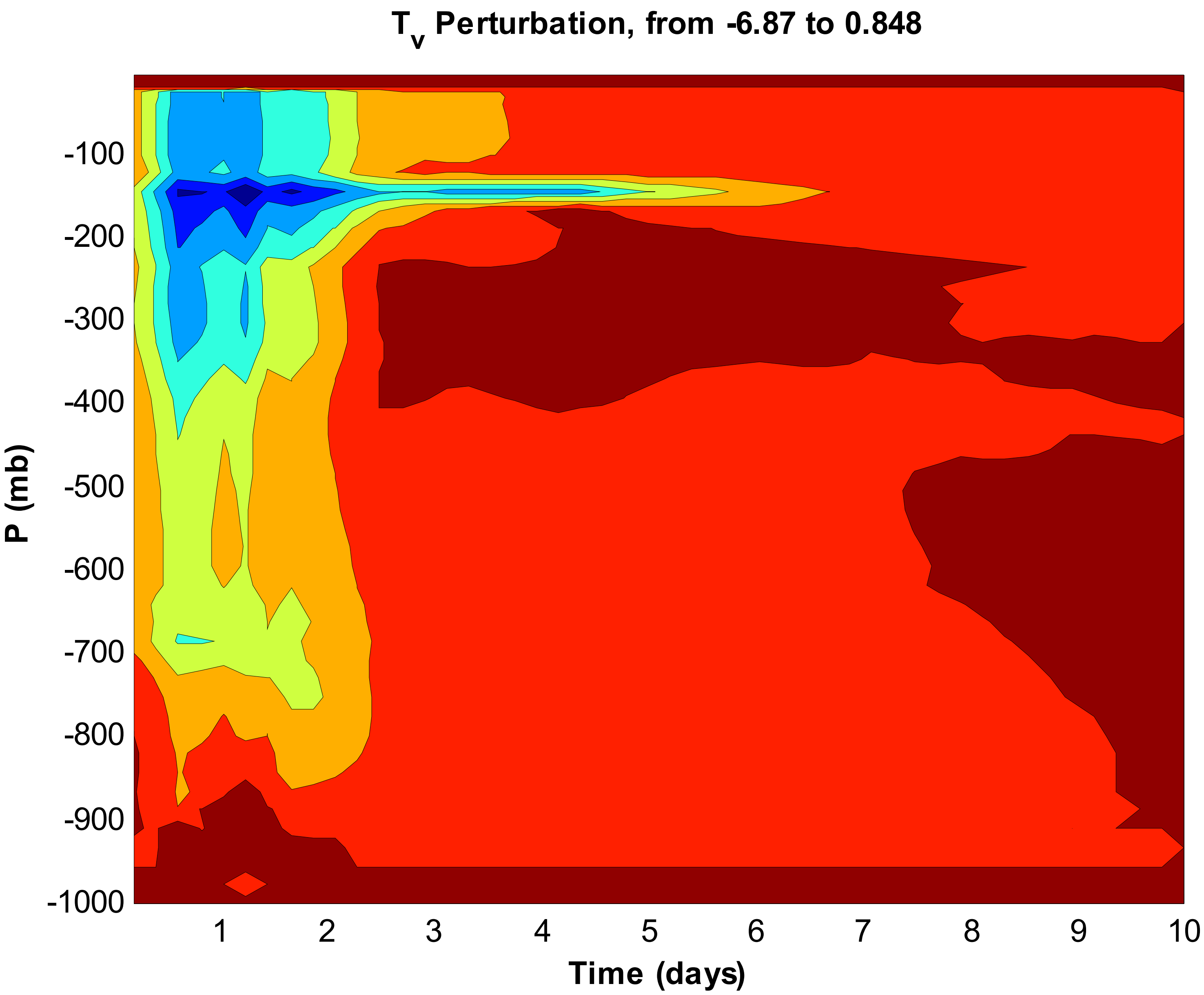
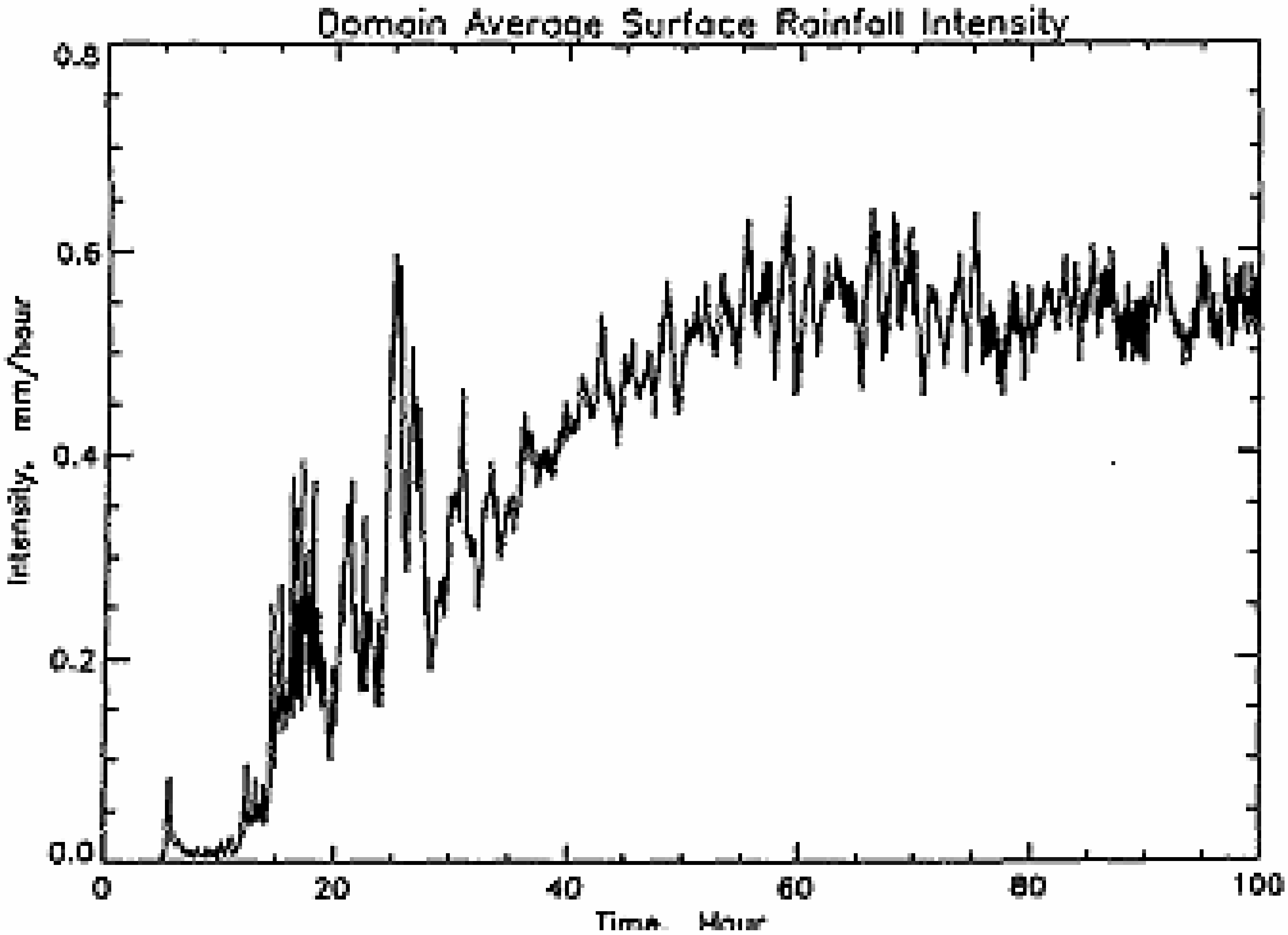
Recovery from mid-level specific humidity perturbation




T_v Perturbation, from -6.87 to 0.848
P (mb)
-100
-200
-300
-400
-500
-600
-700
-800
-900
-1000
1
2
3
4
5
6
7
8
9
10
Time (days)


Domain Average Surface Rainfall Intensity
0.8
0.6
0.4
0.2
0.0
Intensity, mm/hour
0
20
40
60
80
100
Time, Hour

Robe and Emanuel, J. Atmos. Sci., 1996

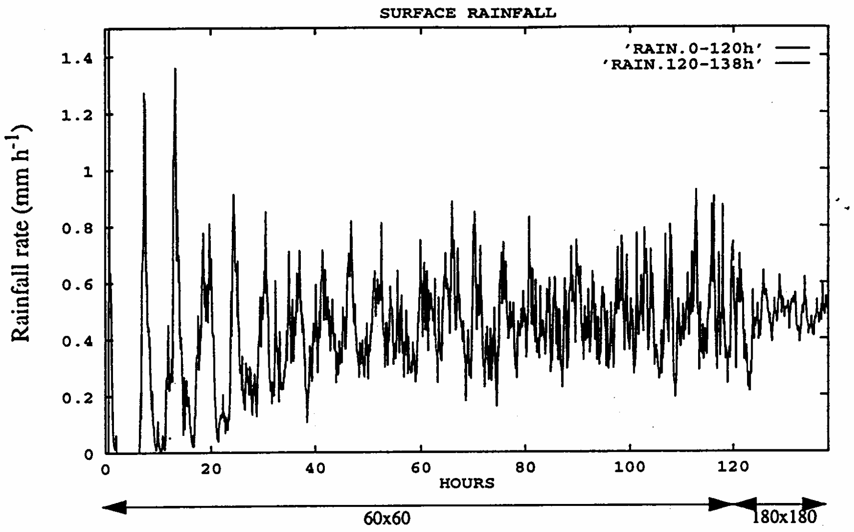
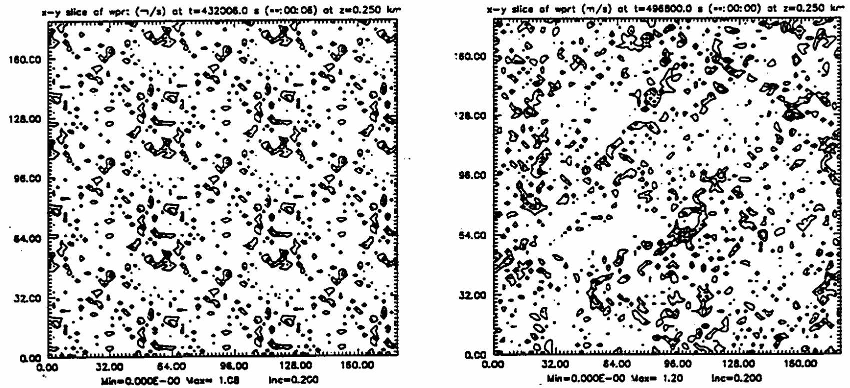


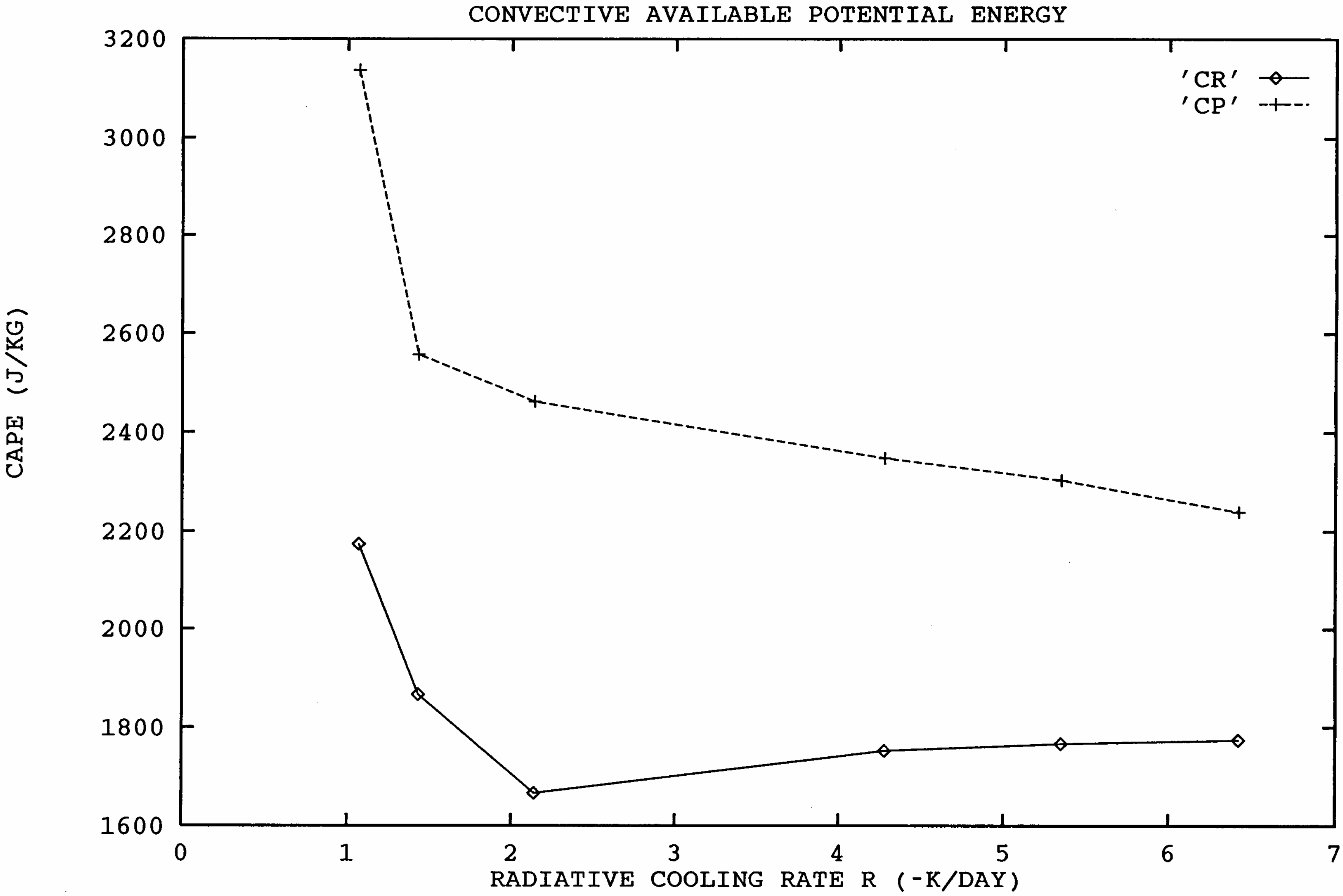
Figure 4.5: time-series of the horizontally averaged rainfall at the ground for R= -5.4 K/day. The domain extends over 60 x 60 km^2 for the first 120 hours, and over 180 x 180 km^2 for the last 18 hours.

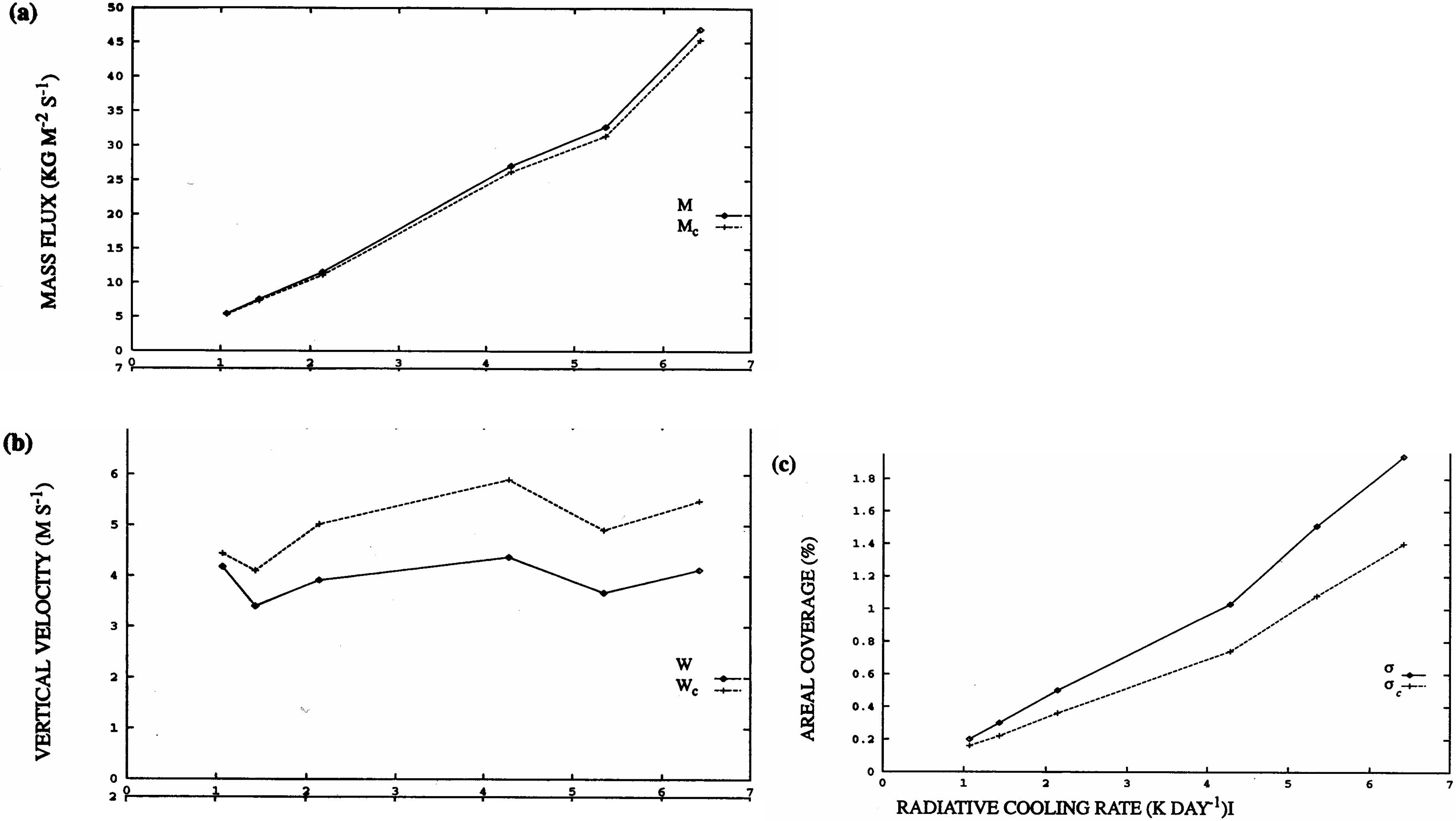


120 h

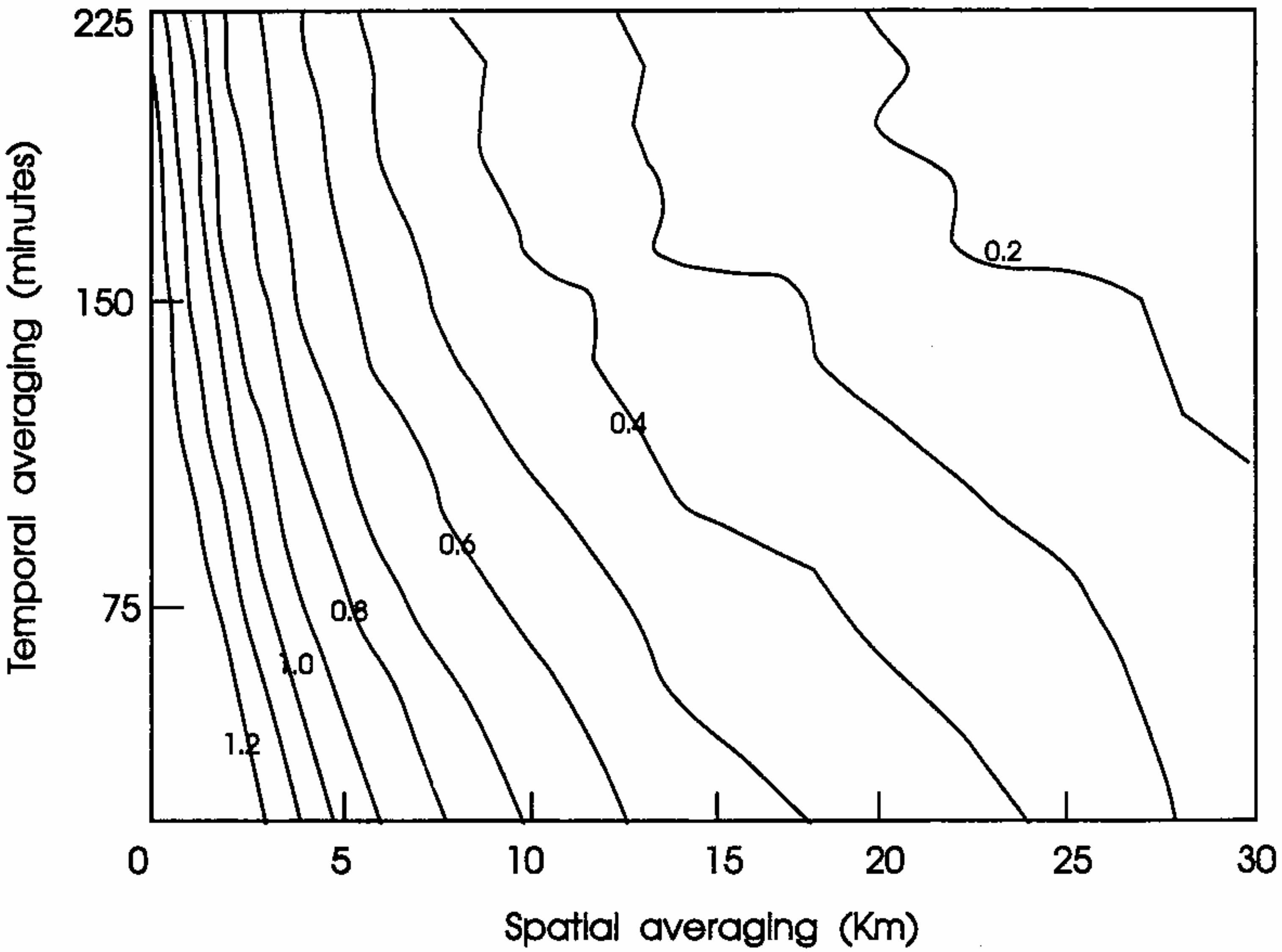
138 h

Image courtesy of American Meteorological Society.


CONVECTIVE AVAILABLE POTENTIAL ENERGY
'CR'
'CP'
CAPE (J/KG)
3200
3000
2800
2600
2400
2200
2000
1800
1600
0
1
2
3
4
5
6
7
RADIATIVE COOLING RATE R (-K/DAY)


(a)
MASS FLUX (KG M^{-2} S^{-1})
0
5
10
15
20
25
30
35
40
45
50
M
M$_c$
0
1
2
3
4
5
6
7
(b)
VERTICAL VELOCITY (M S^{-1})
0
1
2
3
4
5
6
W
W$_c$
0
1
2
3
4
5
6
7
(c)
AREAL COVERAGE (%)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
σ
σ$_c$
0
1
2
3
4
5
6
7
RADIATIVE COOLING RATE (K DAY^{-1})I

Islam et al. Predictability Experiments



Robe and Emanuel, *J. Atmos. Sci*., 2001

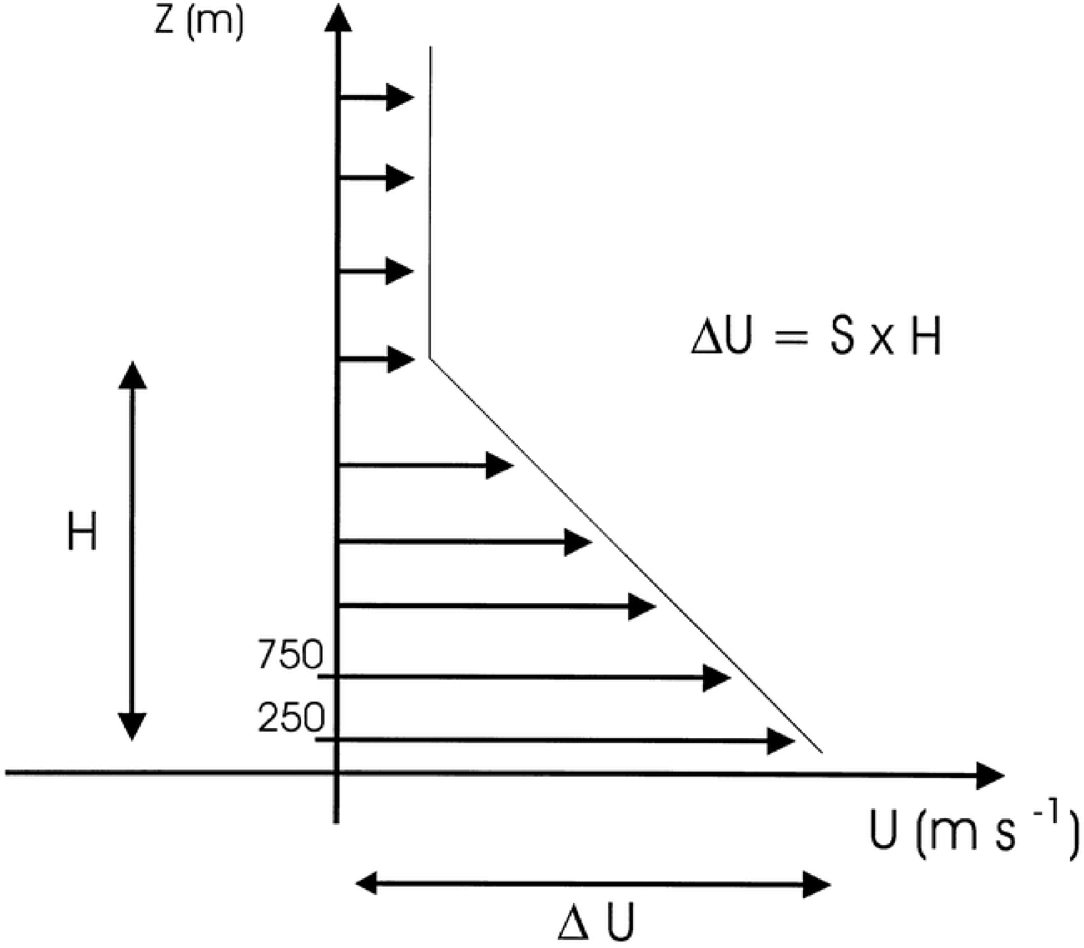


Image courtesy of American Meteorological Society.

Fovell and Ogura, *J. Atmos. Sci*., 1988, 1989

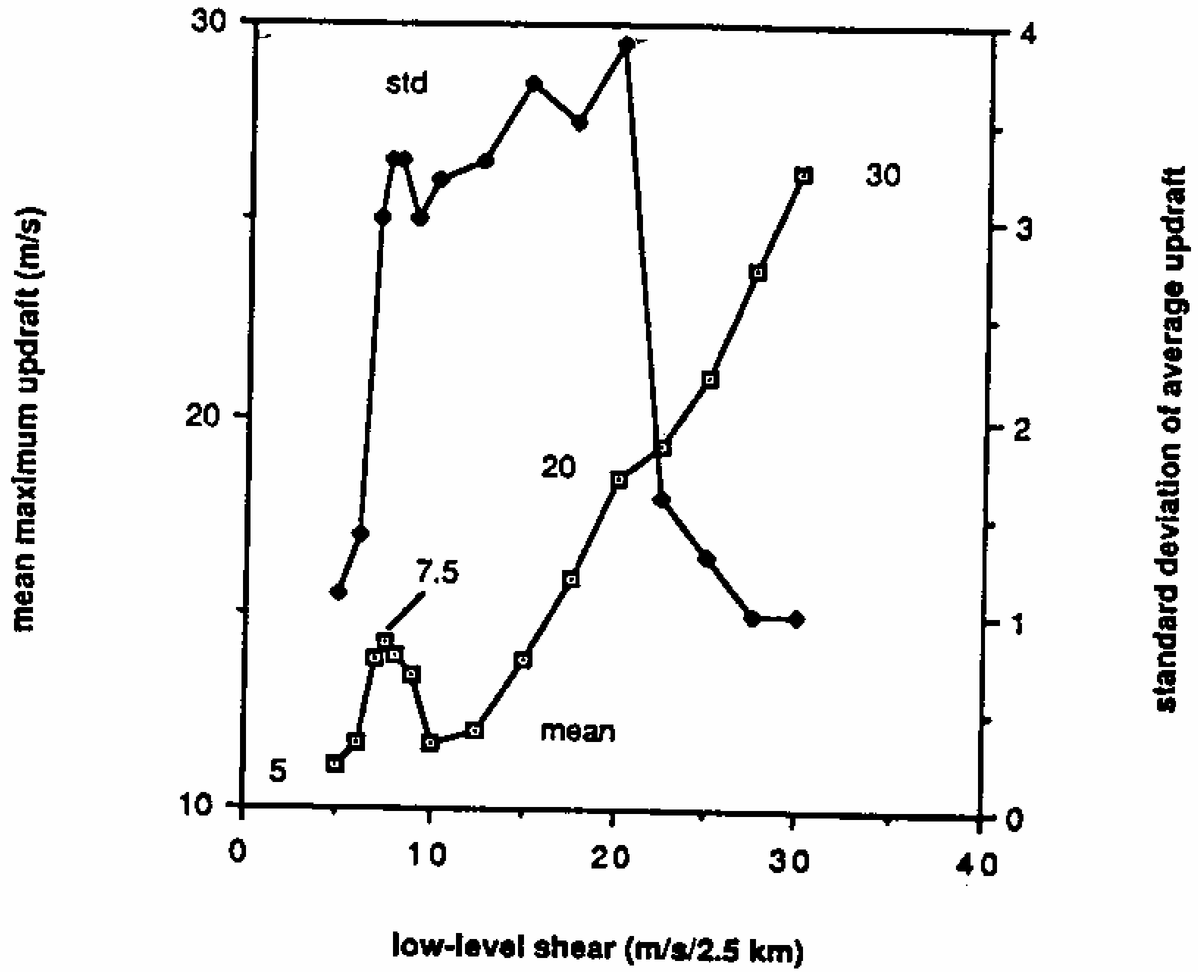


Image courtesy of American Meteorological Society.

Rotunno, Klemp and Weisman, *J. Atmos. Sci*., 1988

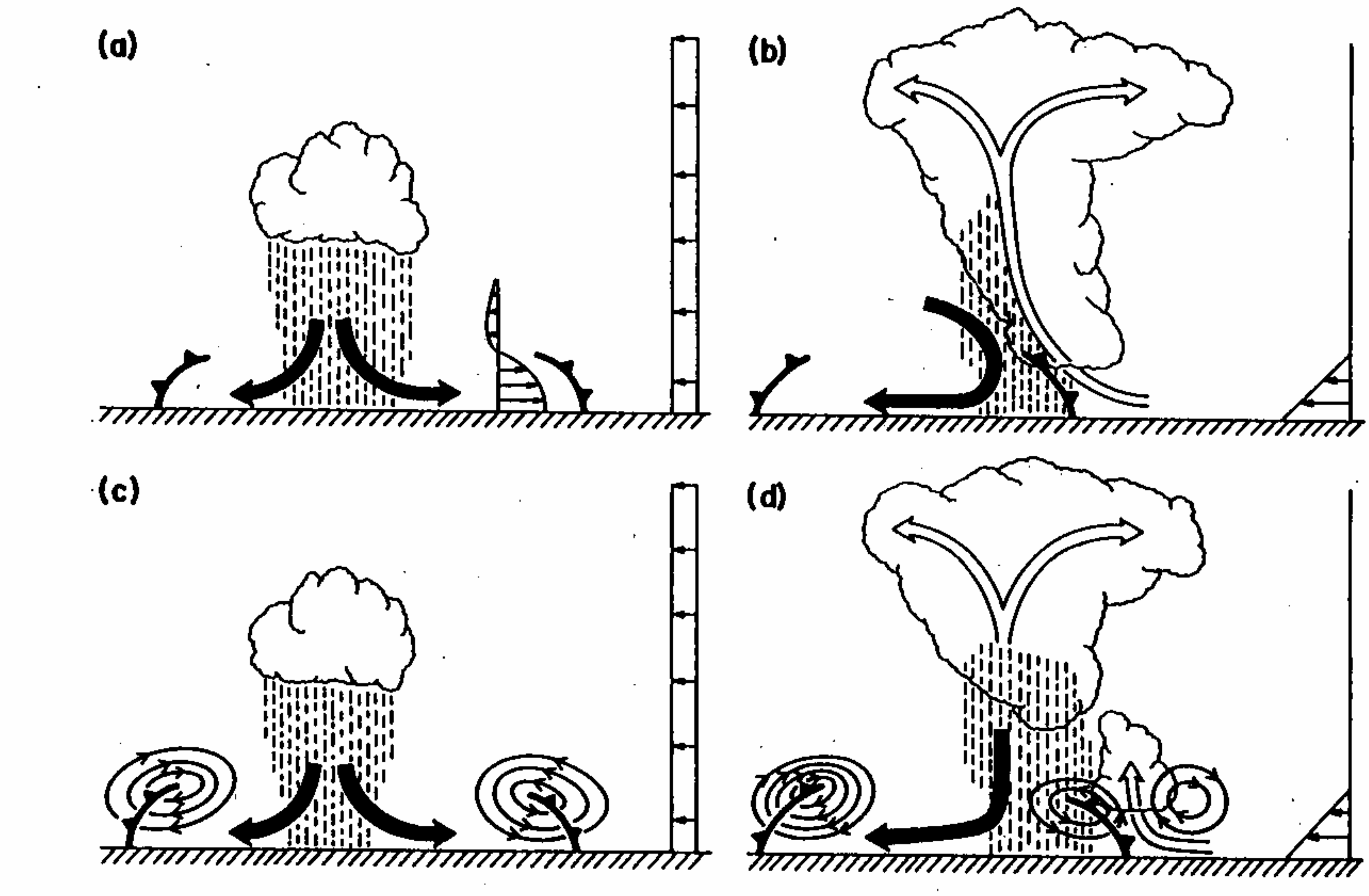
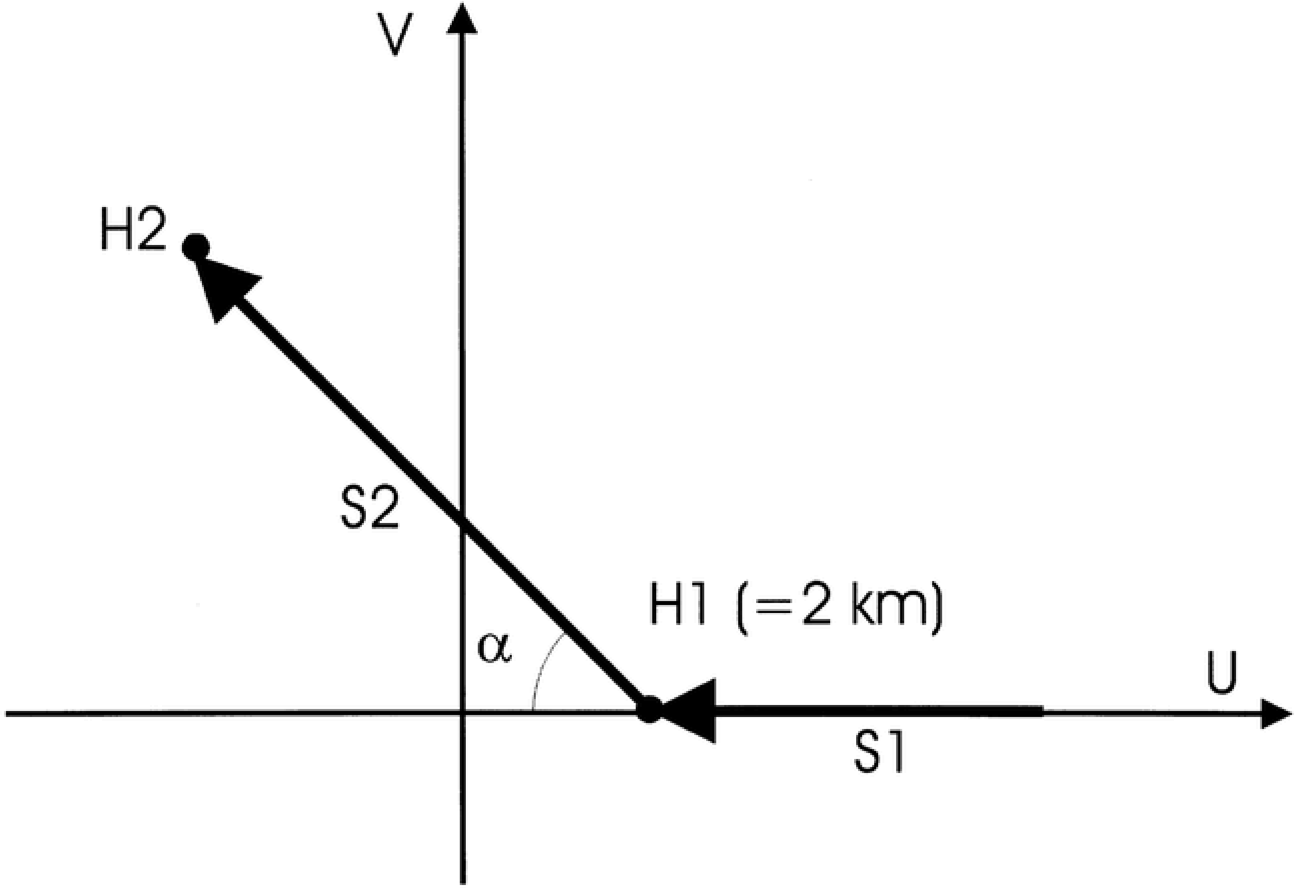


Image courtesy of American Meteorological Society.


V
H2
S2
H1 (=2 km)
α
U
S1

Speculative Regime Diagram

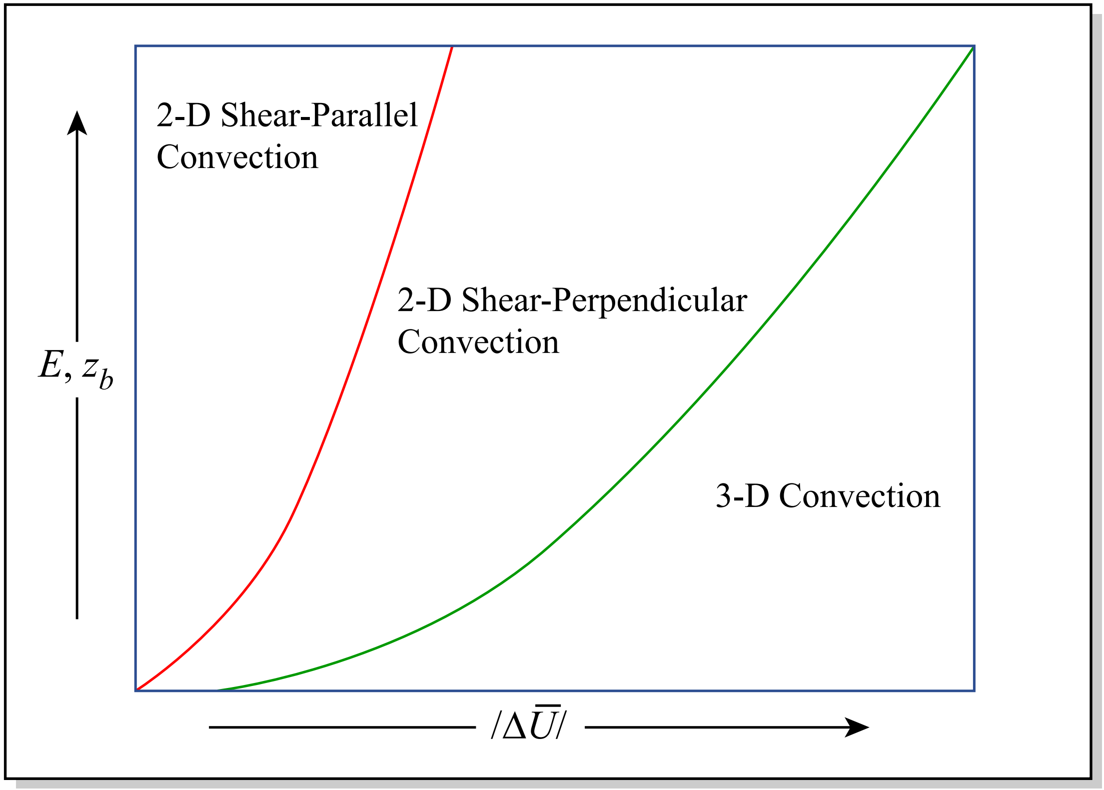


Figure by MIT OCW.

Non-equilibrium Convection

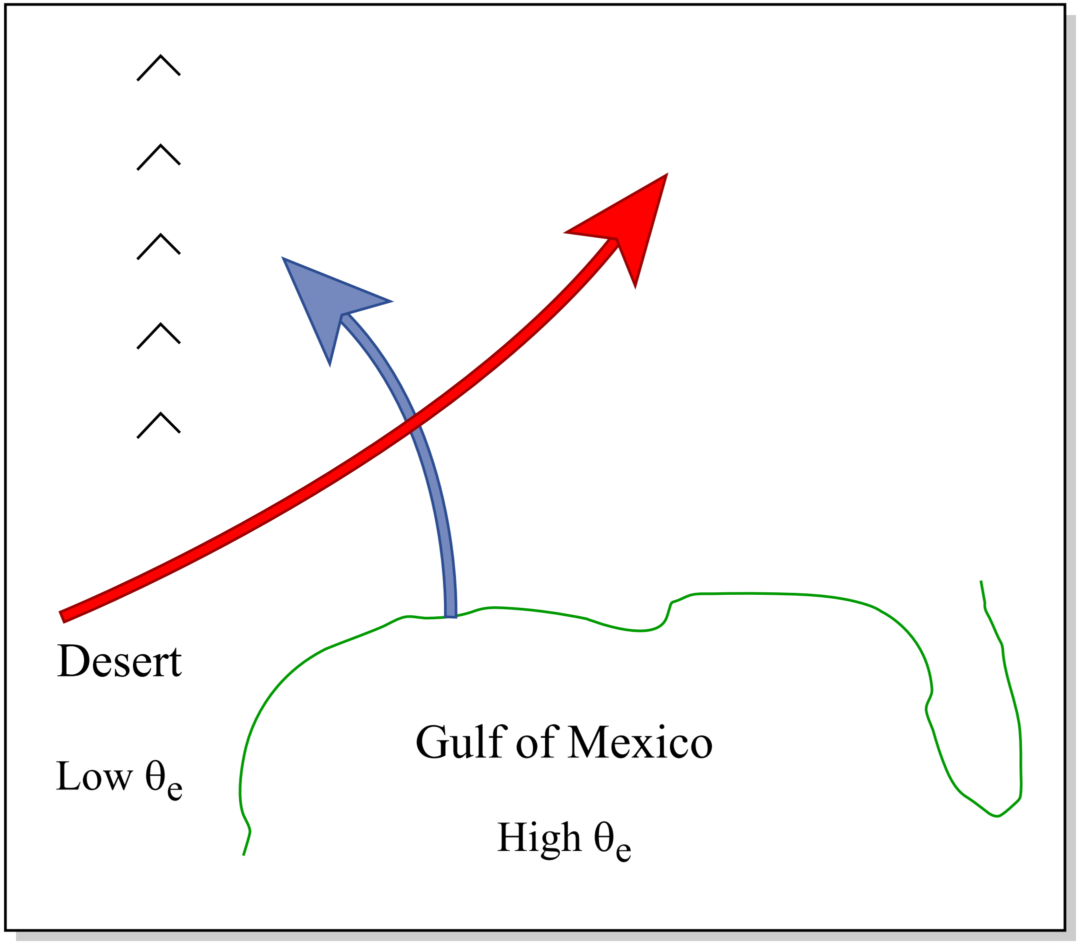
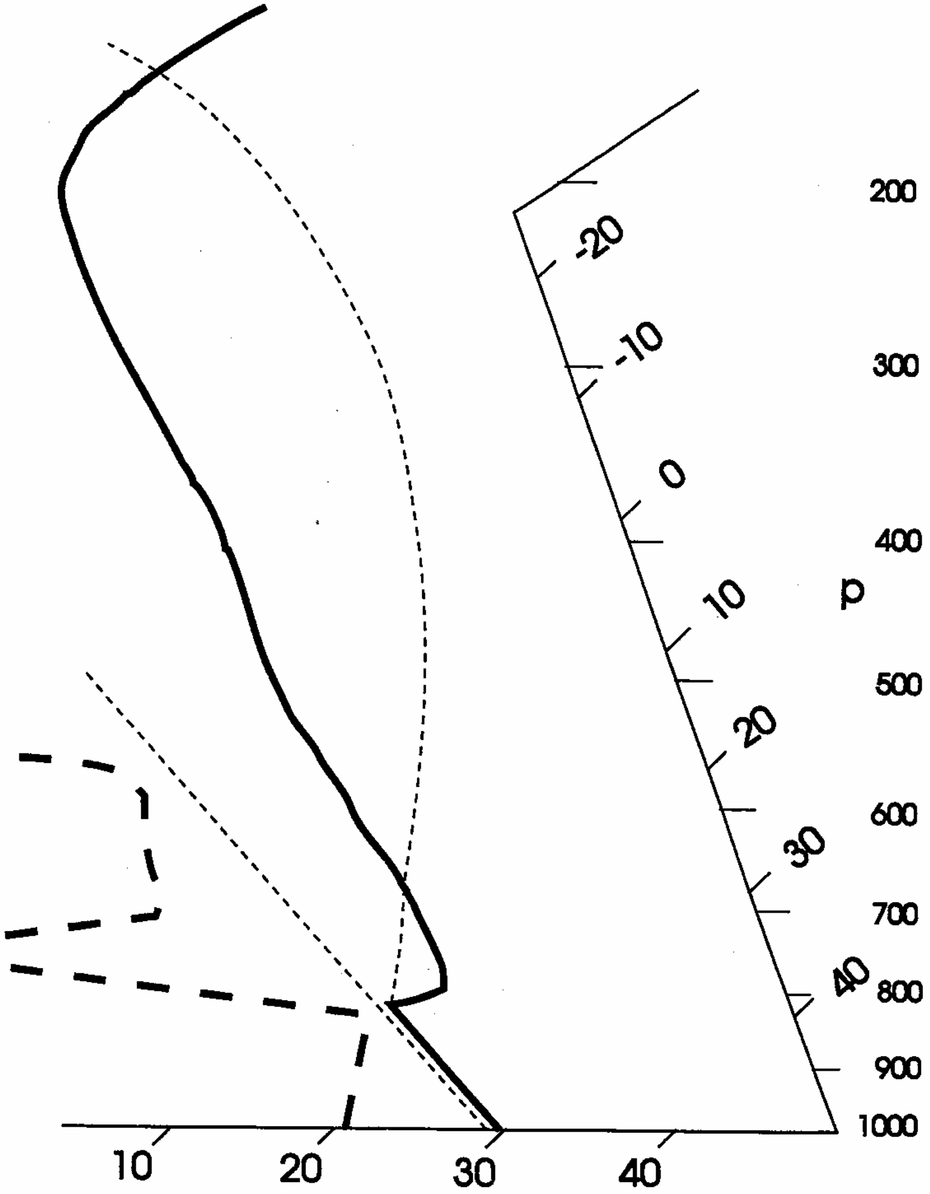
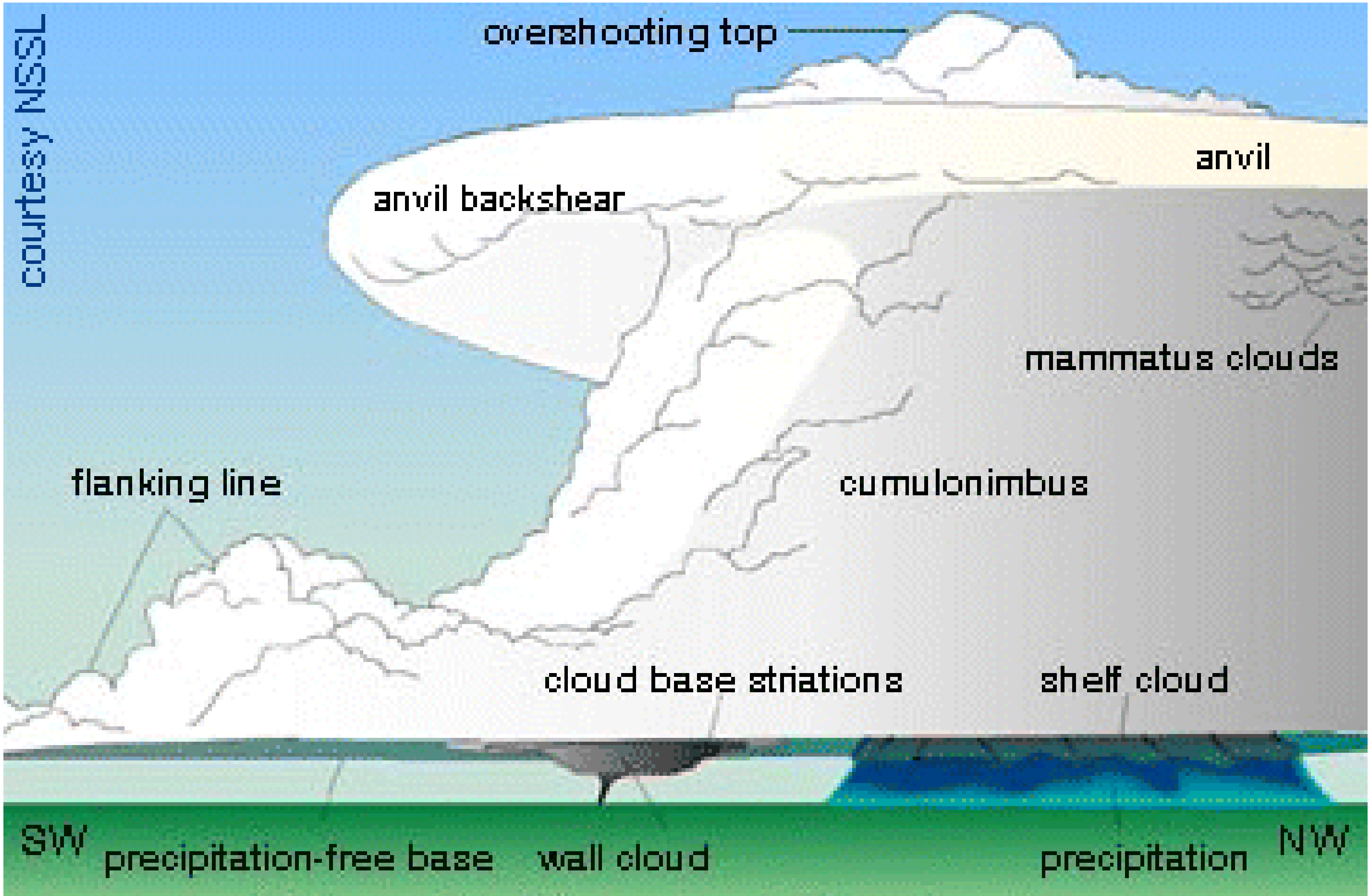


Figure by MIT OCW.


200
300
400
500
600
700
800
900
1000
p
-20
-10
0
10
20
30
40
10
20
30
40



courtesy NSSL

Idealized view of a "classic" supercell, looking west

Klemp and Wilhelmson, *J. Atmos. Sci.*, 1978

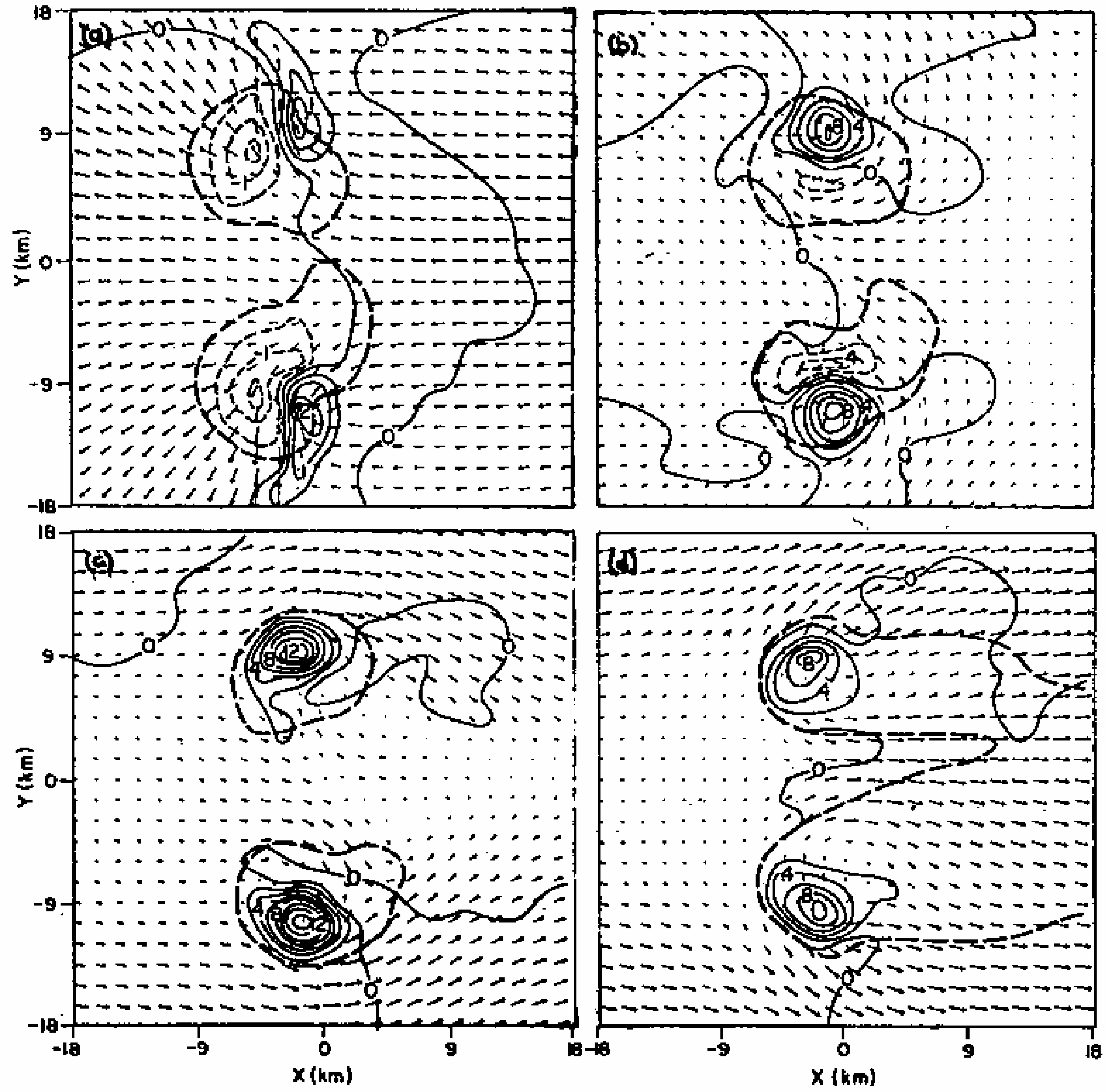


Image courtesy of American Meteorological Society.