4. Remote Sensing

A. Introduction

Electromagnetic waves, interacting with a medium leave a signature dependent on the composition and thermal structure of the medium.

Let M=Medium and S=Signal respectively. Then, we have:

$$S=F(M) \tag{1}$$

Where F represents a function that is not necessarily linear. The function will generally relate to absorption, scattering, emission and polarization. In turn, absorption and scattering will depend on composition (molecular and particulate) and emission will depend on composition and temperature. Polarization will depend on scattering properties including the size, shape and orientation of particulates.

The inverse of Eq. 1 gives:

$$M=F^{-1}(S) \tag{2}$$

Where F^{-1} represents the inverse of the function F.

A fundamental obstacle in all inverse problems of remote sensing is the <u>uniqueness</u> of solution. The non-uniqueness arises because the medium under investigation may be composed of a number of unknown parameters whose various physical combinations may lead to the same radiation signature. There are also mathematical problems associated with the stability of solution.

We will need to consider both active and passive remote sensing:

<u>Active Remote Sensing</u> uses a radiation source generated by artificial means – e.g., lasers used for LIDAR or microwaves used in RADAR. Generally a beam of radiation is sent out and a back-scattered signal is measured, although it would also be possible to have the source and receiver located at different places. Range-gating enables the user to determine the location of scatterers.

<u>Passive Remote Sensing</u> uses natural radiation sources such as the sun or the emission of the earth and atmosphere itself. Thus, wavelengths of the solar spectrum from the UV to the IR are accessible and thermal emission by the earth atmosphere system is available from about 4 μm to the far infrared and microwave region.

B. Scattered Sunlight as a Means of Remote Sensing

Consider sunlight incident on the top of the atmosphere with a measurement system located at the earth's surface. We have:

$$dF_\lambda(z) = -F_\lambda(z)dz \sec\theta_0 \left[k(\lambda)\rho(z) + \sigma_s^R(\lambda)N(z) + \sigma_e^M(\lambda)N_a(z) \right] \tag{3}$$

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where θ_0 is solar zenith angle.

k is absorption coefficient associated with molecules.

ρ is the density of molecular absorbers.

σ_s^R is the Rayleigh scattering cross-section.

$N(z)$ = number of molecules/cm^3 at height z.

$N_a(z)$ = number of aerosols/cm^3 at height z.

$\sigma_e^M(\lambda)$ = Mie extinction cross-section

The solution to Eq. 3 is as follows:

$$\ln\left[\frac{F_\lambda(0)}{F_\lambda(\infty)}\right] = -\int_\infty^0 k(\lambda)\rho(z)\sec\theta_o\,dz - \tau^R(\lambda)\sec\theta_o - \tau^M(\lambda)\sec\theta_o \tag{4}$$

where we have introduced the optical depths associated with Rayleigh and Mie extinctions as τ^R and τ^M respectively.

Let us consider the example of seeking the total ozone concentration in the atmosphere so that $k(\lambda)$ is the absorption coefficient due to ozone at wavelength λ and $\rho(\lambda)$ is the density of ozone at altitude z. The total ozone concentration will then be:

$$\Omega = \int_0^\infty \rho(z)\,dz = \int_\infty^0 \rho(z)\,dz \tag{5}$$

Let us also simplify our problem by recognizing that ozone is concentrated in a layer centered near 22 km, so we can set $\theta_o = Z$ where Z = the solar zenith angle at 22 km. Otherwise, we'll use m = sec θ_o.

If we now select two wavelengths (λ_1 and λ_2) in the Hartley-Huggins ozone absorption band where there is little absorption by other atmospheric molecules and if we assume that the aerosol optical depths are about the same at these two wavelengths we can evaluate Eq. 4 for each wavelength. Subtracting the equation for λ_2 from the equation for λ_1 then enables a solution for Ω.

$$\Omega = \frac{\ln\left[F_{\lambda1}(\infty)/F_{\lambda2}(\infty)\right] - \ln\left[F_{\lambda1}(0)/F_{\lambda2}(0)\right] - m\left[\tau^R(\lambda_1) - \tau^R(\lambda_2)\right]}{\sec Z\left[k(\lambda_1) - k(\lambda_2)\right]} \tag{6}$$

In this equation, everything is known except $F_{\lambda1}(0)/F_{\lambda2}(0)$ which can be measured at the surface.

The determination of this quantity is routinely measured via a standard instrument known as a Dobson spectrometer at 80 ground stations around the world.

Turbidity Detection

Turbidity is a measurement of atmospheric aerosols, both natural and manmade. Let us write Eq. 3 for the situation where molecular absorption is ~ 0. The solution of Eq. 4 then becomes:

$$F_\lambda(0) / F_\lambda(\infty) = \exp\left\{-\left[\tau^R(\lambda) + \tau^M(\lambda)\right]m\right\} \tag{7}$$

$\tau^R(\lambda)$ can be theoretically calculated, enabling us to solve Eq. 7 for $\tau^M(\lambda)$. The quantity, $\tau^M(\lambda)$, depends on the number of particles and the particle size distribution. The solution for either requires an assumption for the other. So, we cannot truly obtain a unique solution here.

Water Vapor Determination

Measurement of incoming solar flux at two wavelengths in a near infrared water vapor band can be used to determine the total amount of precipitable water in the path from sun to ground. In this case, Eq. 4 outside of the water band becomes:

$$F_{\lambda 1}(0) / F_{\lambda 1}(\infty) = \exp\left\{-\left[\tau^M(\lambda_1) + \tau^R(\lambda_1)\right]m\right\} \tag{8}$$

The solution to Eq. 4 in the water vapor band is similarly given by Eq. 9:

$$F_{\lambda 2}(0) / F_{\lambda 2}(\infty) = \exp\left\{-\left[\tau^M(\lambda_2) + \tau^R(\lambda_2)\right]m - K\sqrt{um}\right\} \tag{9}$$

where $K = \sqrt{\pi S_0 \alpha}/\delta$, with S_0 the mean line intensity, α the mean line half-width and δ the mean line spacing. The path length (or precipitable water) is represented by u. If we assume that the extinction of molecules and aerosols is about the same at these 2 wavelengths, we can divide Eq. 9 by Eq. 8 and obtain Eq. 10.

$$\frac{F_{\lambda 2}(0) / F_{\lambda 2}(\infty)}{F_{\lambda 1}(0) / F_{\lambda 1}(\infty)} = \exp\left\{-K\sqrt{\mu m}\right\} = \frac{q_0}{q_\infty} \tag{10}$$

$$u = \frac{\left(\ln \dfrac{q_0}{q_\infty}\right)^2}{K^2 m} \qquad \text{where} \qquad \begin{aligned} q_0 &= F_{\lambda_2}(0) / F_{\lambda_1}(0) \\ q_\infty &= F_{\lambda_2}(\infty) / F_{\lambda_1}(\infty) \end{aligned} \tag{11}$$

Reflected Sunlight

Extinction can also be used to determine the amount of an absorbing gas in the atmosphere by making measurements at two wavelengths, appropriately spaced in an absorption band and outside of the absorption band. However, in this case we will consider the scenario of a satellite viewing the earth at the angle θ_s whereas the sunlight is incident on the atmosphere at angle θ_o. The solution to Eq. 3 now requires slight modification as indicated in Eq. 12:

$$\ln\left[\frac{F_\lambda(t)}{F_\lambda(\infty)}\right] = -\left[\int_\infty^0 k(\lambda)\rho(z)\sec\theta_o\, dz + \int_0^\infty k(\lambda)\rho(z)\sec\theta_s\, dz\right] - \left[\tau^R(\lambda)\sec\theta_o + \tau^R(\lambda)\sec\theta_s\right]$$
$$- \left[\tau^M(\lambda)\sec\theta_o + \tau^M(\lambda)\sec\theta_s\right] \tag{12}$$

Selecting 2 wavelengths in an absorption band sufficiently closely spaced in wavelength that Rayleigh and Mie extinction and surface (or cloud-top) reflection can be considered independent

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of wavelength, but with an appreciable difference in absorption by a molecular component of the atmosphere (such as ozone or water vapor), we can write two equations again take the difference and obtain Eq. 13:

$$\ln F_{\lambda_1}(t) - \ln F_{\lambda_1}(\infty) - \ln F_{\lambda_2}(t) + \ln F_{\lambda_2}(\infty) = -\left[\int_\infty^0 k(\lambda_1)\rho(z)\sec\theta_0\,dz + \int_0^\infty k(\lambda_1)\rho(z)\sec\theta_s\,dz\right]$$

$$+\left[\int_\infty^0 k(\lambda_2)\rho(z)\sec\theta_0\,dz + \int_0^\infty k(\lambda_2)\rho(z)\sec\theta_s\,dz\right] \qquad (13)$$

and if $k(\lambda)$ is independent of z and since $\int_\infty^0 \rho(z)\,dz = \int_0^\infty \rho(z)\,dz$, we have:

$$\ln\left[\frac{F_{\lambda_1}(t)}{F_{\lambda_2}(t)}\right] - \ln\left[\frac{F_{\lambda_1}(\infty)}{F_{\lambda_2}(\infty)}\right] = -\Omega k(\lambda_1)(\sec\theta_0 + \sec\theta_s) + \Omega k(\lambda_2)(\sec\theta_0 + \sec\theta_s) \qquad (14)$$

Again, the ratio $F_{\lambda 1}(\infty)/F_{\lambda 2}(\infty)$ is known from measurements of the solar constant, $k(\lambda_1)\ \&\ k(\lambda_2)$ are known from laboratory spectroscopic measurements, θ_0 and θ_s are determined from the time of day and geometry of measurement, $F_{\lambda 1}(t)/F_{\lambda 2}(t)$ are measured by the satellite and Ω is the integrated abundance of the atmospheric gas in question. Notice that the procedure will not work if $k(\lambda)$ is dependent on z.

Cloud Properties Inferred from Reflected Polarization

From Mie Theory, based on single scattering, we have:
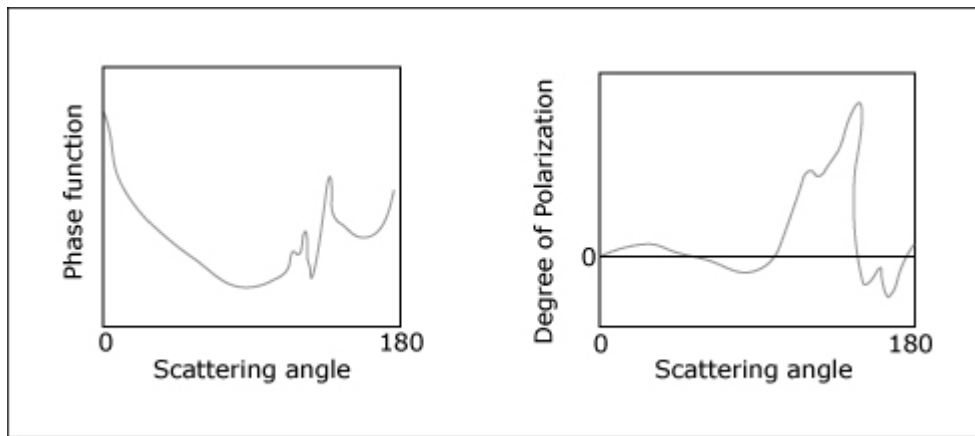
The results of multiple-scattering calculations, however, lead to the following figures for Intensity and Polarization (where the several curves are for different optical depths and the origin here represents a scattering angle of 80°).

Scattering Angle
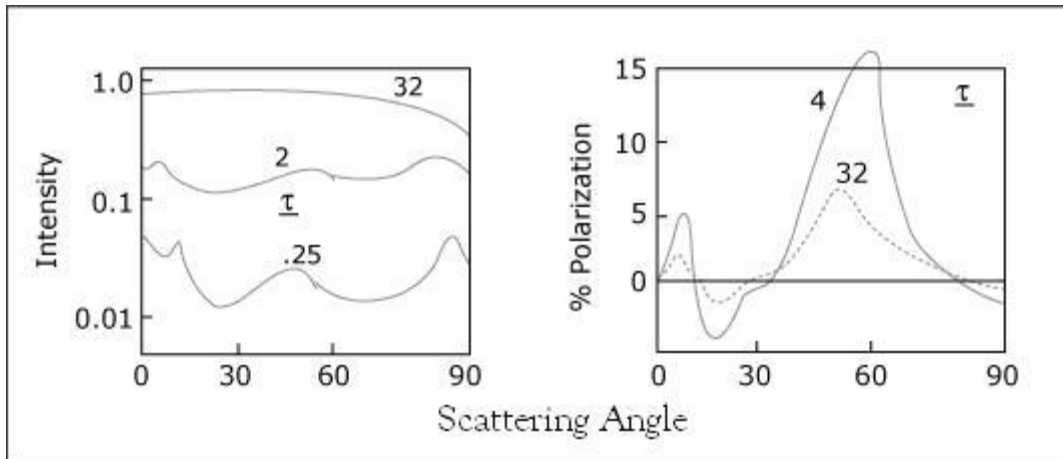
Remote Sensing of Cloud Optical Depth and Particle Size

1. Cloud covers more than 50% of the earth.

2. Many variables involved:

   a) Particle size distribution
   b) Complex index of refraction
   c) Particle phase
   d) Particle shapes
   e) Cloud optical depth

3. Some General Principles:

   a) Reflection from clouds at a wavelength with little absorption (e.g. – visible wavelength) is primarily a function of their optical depth.

   b) Reflection from clouds in the near infrared is largely dependent on particle size.

   c) The measurement of polarization represents an additional technique for separating the several variables associated with the scattered radiation field.

Designing Remote Sounding Systems

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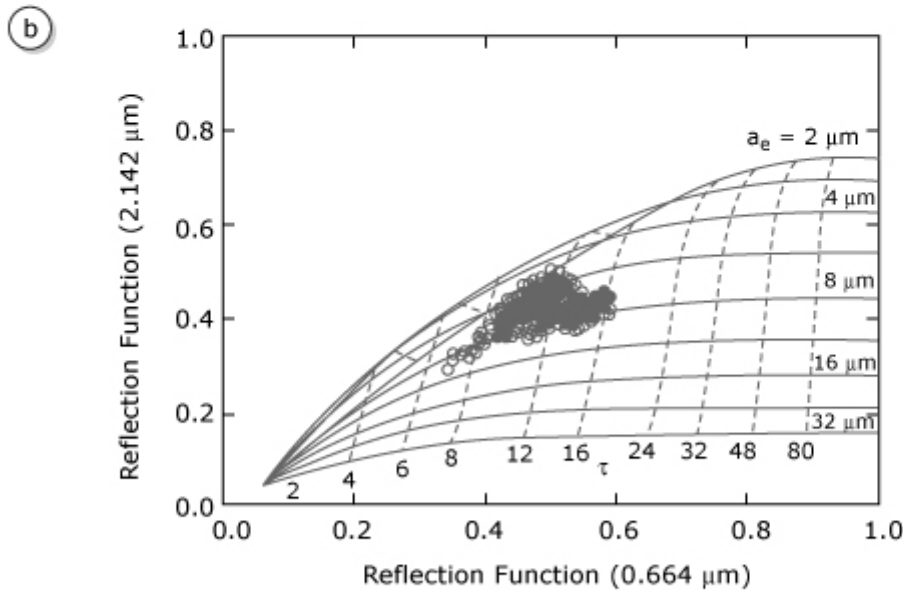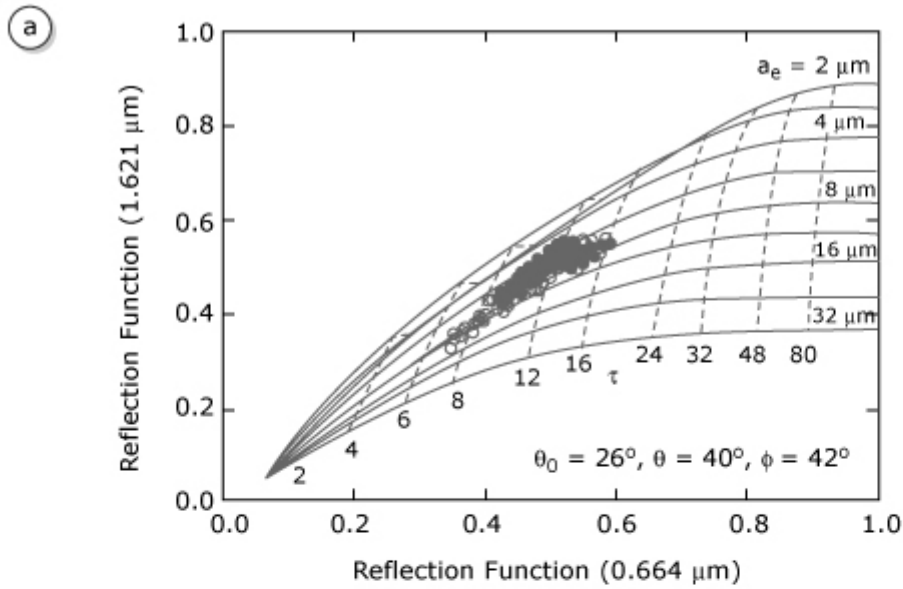
Theoretical relationship between the reflectance at 0.664 and (a) 1.621 μm and (b) 2.142 μm for various values of optical depth at 0.664 μm and effective droplet radius, for the specific geometry denoted in the diagram. Data from over 400 measurements obtained with the MAS that was aboard the NASA ER-2 aircraft are superimposed. These observations were obtained when the aircraft flew above marine stratocumulus clouds during a stratus cloud experiment on June 22, 1992.
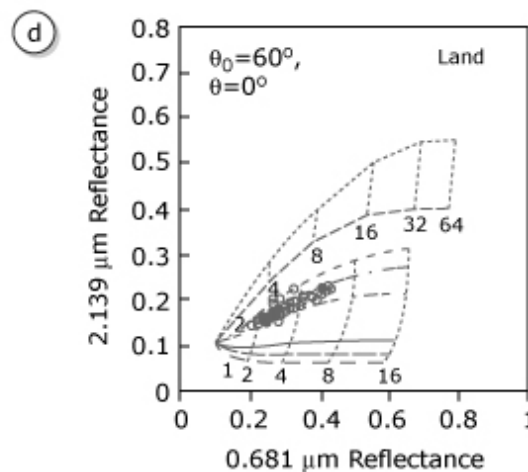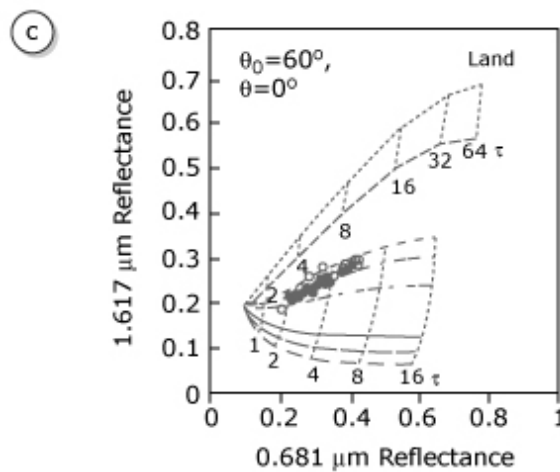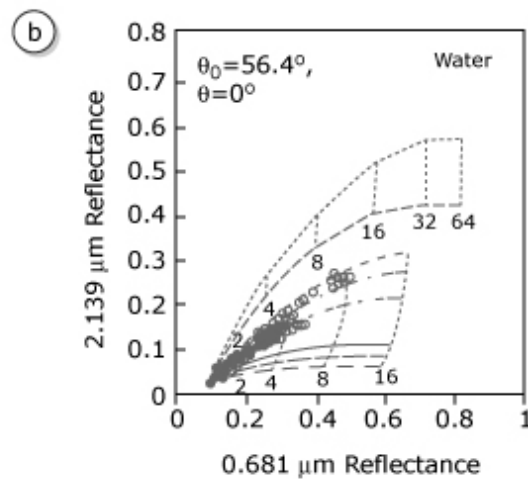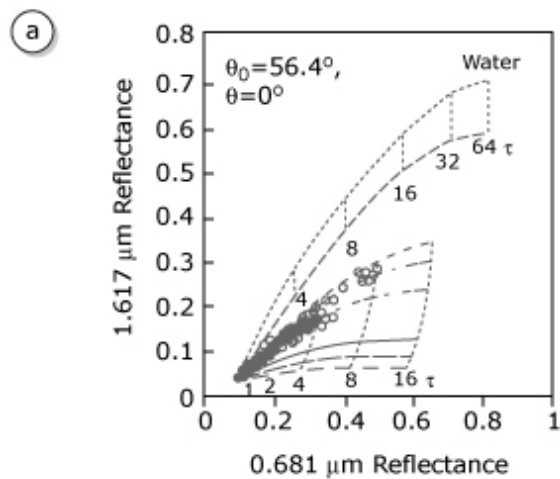
Correlation between the reflectances at 0.681/1.617 μm and at 0.681/2.139 μm. Radiative transfer calculations were performed for the six ice-crystal size distributions with mean effective sizes ranging from 24 to 124 μm, and for two water clouds with mean effective radii of 4 and 8 μm. The optical depth ranges from 0.5 to 64. Overlapped with the curves are the MAS data obtained from FIRE-II-IFO on 5 December 1991, over the northern Gulf of Mexico and eastern Oklahoma. Frames (a) and (b) are cases over water, and frames (c) and (d) are cases over land.

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C. Passive Remote Sensing Using Emitted Radiation

From the radiative Transfer Lecture Notes (Eq. 12) we have:

$$\mu \frac{d I_v\left(\tau_v, \mu\right)}{d\tau_v} = I_v\left(\tau_v, \mu\right) - B_v\left(\tau_v\right) \tag{15}$$

where $\mu = \cos\theta$, θ is the emergent angle, B_v is the Planck function and τ_v is the monochromatic optical depth:

$$\tau_v = \int_z^{z_\infty} k_v\left(z'\right)\rho_a\left(z'\right)dz' \tag{16}$$

with $\rho_a\left(z'\right)$ the density of absorbing gas, a, and k_v the absorption coefficient.

Since we're interested here in upwelling radiation (that can be measured from a satellite platform), the solution is given by Eq. 17.

$$I_v\left(\tau_v, \mu\right) = I_v\left(\tau_{v,s}\right)e^{-\tau_{v,s}/\mu} + \int_{\tau_v}^{\tau_{v,s}} B_v\left(\tau_v'\right)e^{-\left(\tau_v'-\tau_v\right)/\mu} \frac{d\tau_v'}{\mu} \tag{17}$$

where $\tau_{v,s}$ represents the monochromatic optical depth at the surface and $I_v\left(\tau_{v,s}\right)$ represents the emitted surface radiance which we generally take to be isotropic. Therefore, the first term represents the surface emission attenuated by the atmospheric absorption and the second term represents the emission of the atmosphere itself.
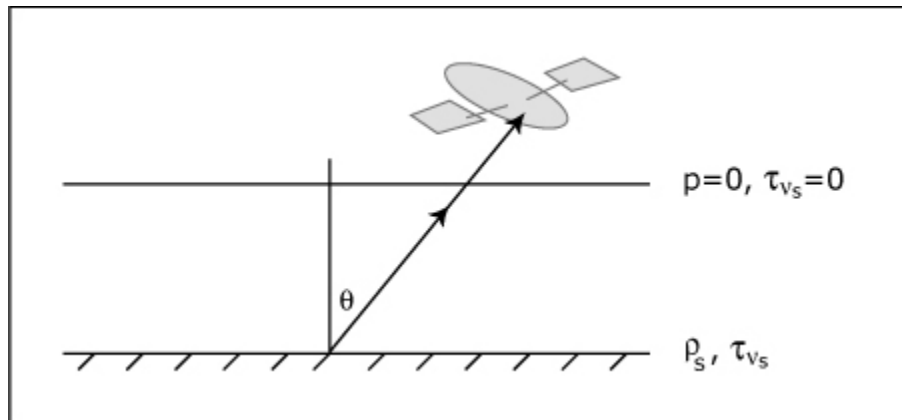
As we discussed earlier, the exponential terms are referred to as the transmission:

$$t_v\left(\tau_v\right) = e^{-\tau_v/\mu} \tag{18}$$

We can write the surface emission as:

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$$I_\nu\left(\tau_{\nu,s}, \mu\right) = \varepsilon_\nu \beta_\nu\left(T_s\right) \tag{19}$$

where T_s is the surface temperature and where ε_ν is the emissivity which can be taken = 1 in much of the thermal infrared region.

It is also customary to define a "weighting function" by:

$$\frac{dt_\nu\left(\tau_\nu\right)}{d\tau_\nu} = \frac{-e^{-\tau_\nu/\mu}}{\mu} \tag{20}$$

Thus the radiance at the top of the atmosphere is given by:

$$I_\nu\left(0, \mu\right) = \beta_\nu\left(T_s\right) t_\nu\left(\tau_{\nu,s}\right) + \int_{\tau_{\nu,s}}^{0} \beta_\nu\left(\tau\right) \frac{\partial t_\nu\left(\tau_\nu\right)}{\partial \tau_\nu} d\tau_\nu \tag{21}$$

In remote sensing, we usually write Eq. 21 in terms of height or pressure as independent variable. Let us define:

$$d\tau_\nu = -k_\nu \rho_a dz = -k_\nu q_a \rho dz \tag{22a}$$

$$= \frac{k_\nu q_a d\rho}{g} \quad \left(\text{Since } \frac{d\rho}{dz} = -\rho g\right) \tag{22b}$$

where q_a = absorber mass mixing ratio. We can then write Eq. 21 in terms of pressure as follows:

$$I_\nu\left(0, \mu\right) = \beta_\nu\left(T_s\right) t_\nu\left(\tau_{\nu,s}\right) + \int_{p_s}^{0} \beta_\nu\left(T\left(p\right)\right) \frac{\partial t_\nu\left(p\right)}{\partial p} dp \tag{23}$$

where p_s is now the surface pressure.

An instrument capable of measuring radiance as a function of frequency (or wavenumber) always has a finite bandwidth, $\Psi^{\left(\bar{\nu}, \nu\right)}$ where Ψ is the instrument response function and $\bar{\nu}$ its mean frequency (wavenumber).

The measured radiance is then given by Eq. 24:

$$I_{\bar{\nu}} = \int_{\nu_1}^{\nu_2} I_\nu \Psi\left(\bar{\nu}, \nu\right) d\nu \bigg/ \int_{\nu_1}^{\nu_2} \Psi\left(\bar{\nu}, \nu\right) d\nu \tag{24}$$

Since the Planck function varies slowly with frequency compared with the spectral response of a spectrometer, we can consider it to be constant over the interval $\Delta\nu = \nu_2 - \nu_1$

Therefore, if we now integrate Eq. 23 over the frequency interval, $\Delta\nu$ we obtain Eqs 25 & 26.

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$$I_{\bar{v}}(0, \mu) = B_{\bar{v}}(T_s)\,\bar{t}_{\Delta v}(p_s) + \int_{p_s}^{0} B_{\bar{v}}[T(p)]\frac{\partial \bar{t}_{\Delta v}(p)}{\partial p}\,dp \tag{25}$$

$$\text{or } I_{\bar{v}}(0, \mu) = B_{\bar{v}}(T_s)\,\bar{t}_{\Delta v}(p_s) + \int_{\bar{t}_{\Delta v}(p_s)}^{1} B_{\bar{v}}[T(p)]\,d\bar{t}_{\Delta v}(p) \tag{25a}$$

$$\text{where } \bar{t}_{\Delta v}(p) = \frac{\int_{v_1}^{v_2} \Psi(\bar{v}, v)\, \exp\left[-\frac{1}{g}\int_{0}^{p} k_v(p')\, q(p')\, dp'\right]}{\int_{v_1}^{v_2} \Psi(\bar{v}, v)\, dv} \tag{26}$$

Summary

1. The upwelling radiance has 2 components
   a) Surface term
   b) Atmospheric term

2. Temperature information is included in the Planck function whereas, absorbing gas profile information is included in the atmospheric transmission.

3. Observed radiances relate in a complex and inter-related way to the temperature and molecular constituent profiles in the atmosphere.

Choice of spectral regions for downward viewing

   i.   Available gaseous absorbers (thermal infrared)

$\lambda\,(\mu m)$	Gas	Applications
4.4–4.6	$N_2O(v_3)$	air temperature – tropo.
4.18–4.5	$CO_2(v_3)$	air temp. – tropo., strato.
13–18	$CO_2(v_2)$	air temp. – tropo., strato.
4300–5900 (5 mm)	O_2	air temp. – strato, meso.
5–8.3	$H_2O(v_2)$	H_2O abundance
12.5–13,500	H_2O(rot'n)	H_2O abundance
9–10	$O_3(v_3)$	ozone abundance
13.3–15.4	$O_3(v_2)$	ozone abundance
3.5–4.18	window (no gas)	surface & cloud-top temp.
10–13	window (no gas)	surface & cloud-top temp.
7,000–11,000	window (no gas)	surface & cloud-top temp.
>15,000	window (no gas)	surface & cloud-top temp.

ii.   Free of interference from clouds and aerosols (e.g., microwave region).

iii.   Maximum energy emission – strongly peaked near 15 μm (emission intensities are normalized.)

$\lambda(\mu m)$	200 K	300 K
4.3	1.25	200
15.0	5000	15,000
5 mm	1	1

iv.   Detector performance (normalized = emission intensity signal/detector noise signal)

$\lambda(\mu m)$	200 K	300 K
4.3	1	20
15	10	6
5	4	1

(i.e. – at 300K, 4.3 is best and at 200K, 15 is best.)

v.   Redundancy – because each spectral radiance measurement comes from a range of altitude, we can get redundancy.

D. Temperature Sounding using the 15 μm CO_2 Band

Let us specialize our problem to temperature sounding and the use of a CO_2 band for this purpose. And let us assume a spectral measurement over a 10 cm^{-1} bandpass. We can reasonably assume that carbon dioxide is uniformly mixed in the atmosphere in the altitude regime we're interested in (0-40 km). This means that

$\Delta m = C_0 \Delta p$ or (measuring down from the top of the atmosphere), $m = C_0 p$

Returning to the solution to the Radiative Transfer Equation on page 8,

$$I_v(0,\mu) = B_{\bar{v}}(T_s) t_{\bar{v}}(\mu, p_s) + \int_{p_s}^{0} B_{\bar{v}}(T(p)) dt_{\bar{v}}(\mu, p) dp$$

let us define a weighting function as $dt_v / d\ln p$ and perform our integration over $\ln p$ which is similar to z. And let us consider vertical viewing so we can for now ignore the μ dependence.

In order for the average transmission over 10 cm^{-1} to be significant (say transmission between 0.1 and 0.9), individual absorption lines will generally be completely absorbed at their centers and can be associated with the limiting case of strong absorption. As we showed earlier (See MODTRAN Lecture notes), for this

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situation, $\qquad t_{\Delta v} = \exp\left(-2\sum_i \left(S_v^i\, \alpha_L^i\right)^{\frac{1}{2}} m^{\frac{1}{2}}\right)$

where $\qquad S_v^i$ = intensity per unit mass of the ith line

$\qquad\qquad \alpha_L^i$ = Lorentz width of the ith line ($\alpha_L^i = \alpha_{Lo}^i\, \frac{p}{p_o}$)

$\qquad\qquad m$ = mass of absorbing gas in the path

Since α_L^i is proportional to pressure and m is also proportional to pressure, we have:
$t_{\Delta v} = \exp\left[-A_{\Delta v}p\right]$ from the top of the atmosphere to the pressure level, p.

Computing the Weighting Function, we obtain:

$$\frac{dt}{d\ln p} = W = A_{\Delta v}\, p e^{-Ap}$$

And the weighting Function maximum can be determined:

$$\frac{d^2 t}{d(\ln p)^2} = -A\, p e^{-Ap} + A^2 p^2 e^{-Ap} = 0 \qquad \text{Max is at A=1/p}$$

Thus we have $W = e^{-1} = 0.368$. This value is reached when p=1/A.

We can select spectral bands such that we achieve an array of weighting functions that span the atmosphere and are distributed in a reasonable manner.
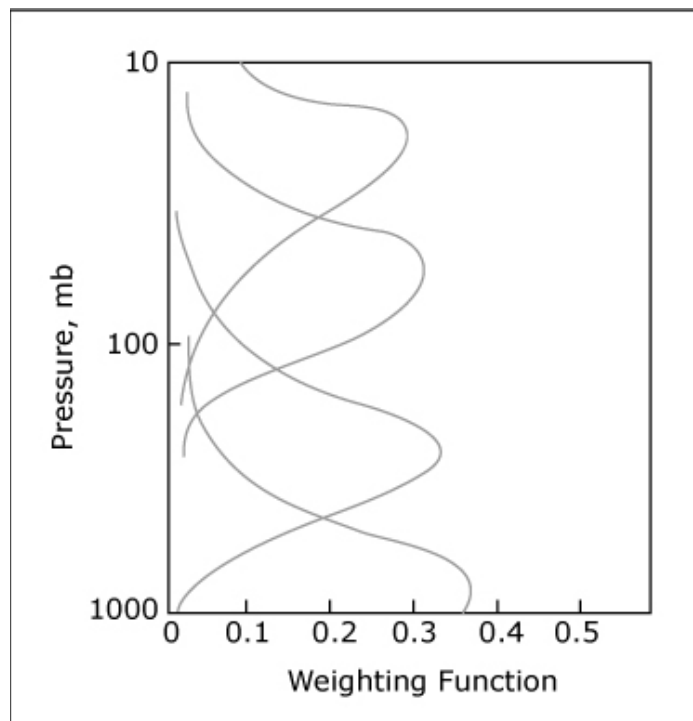
If we were to select spectral regions where we could look primarily at frequencies associated with line wings, we would have the situation that the transmission was proportional to p^2 instead of p. In this case, the weighting functions would be of the form $W = 2Ap^2 e^{-Ap^2}$ and the maximum would be at $2e^{-1} = 0.736$.

The weighting function and transmittance for the NOAA 2 Vertical Temperature Profile Radiometer (VTPR).

Remote Sensing Using Emitted Infrared Radiation

Outgoing radiance in terms of blackbody temperature in the vicinity of 15 μm CO_2 band observed by the IRIS on Nimbus 4. The arrows denote the spectral regions sampled by the VTPR instrument.
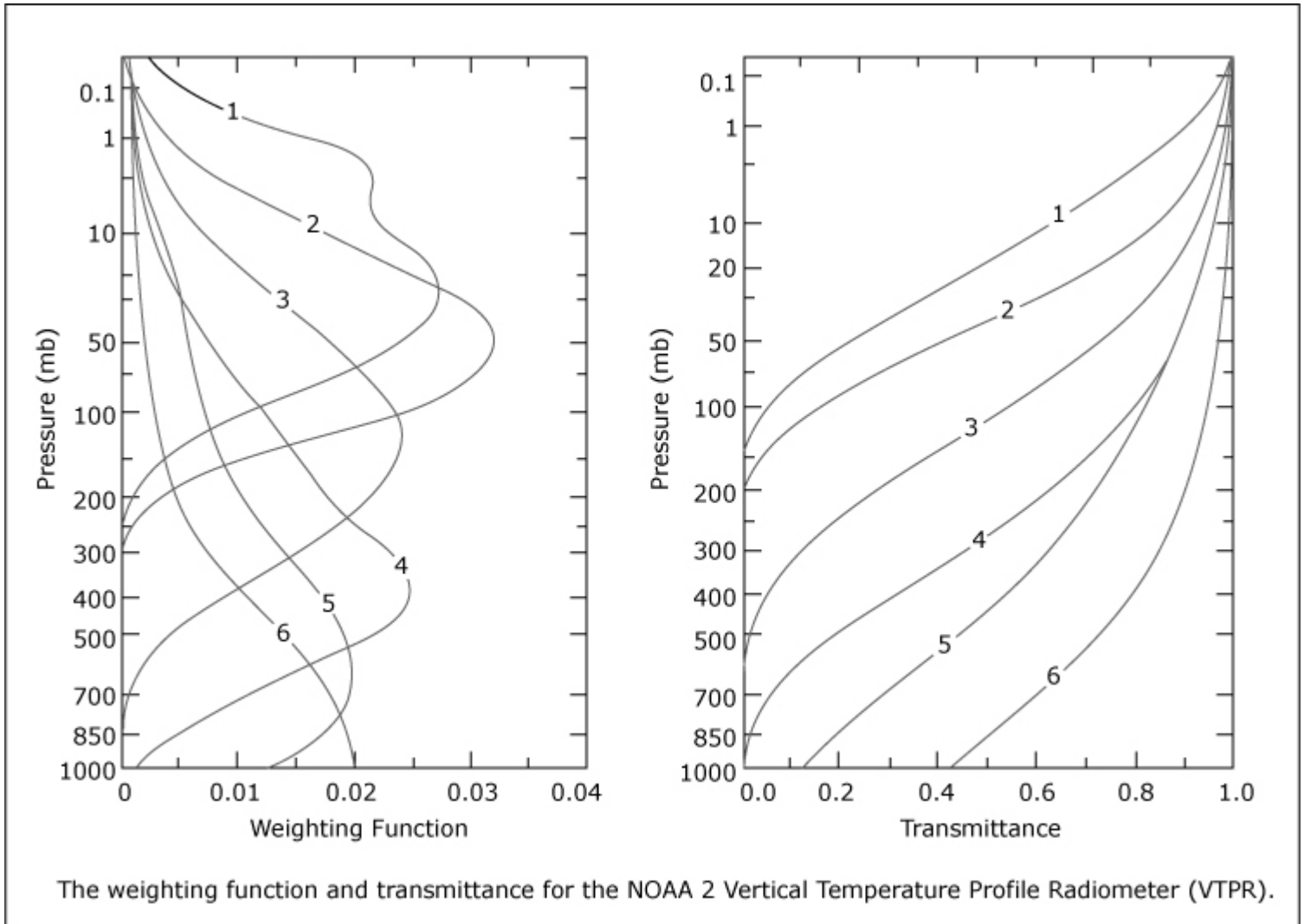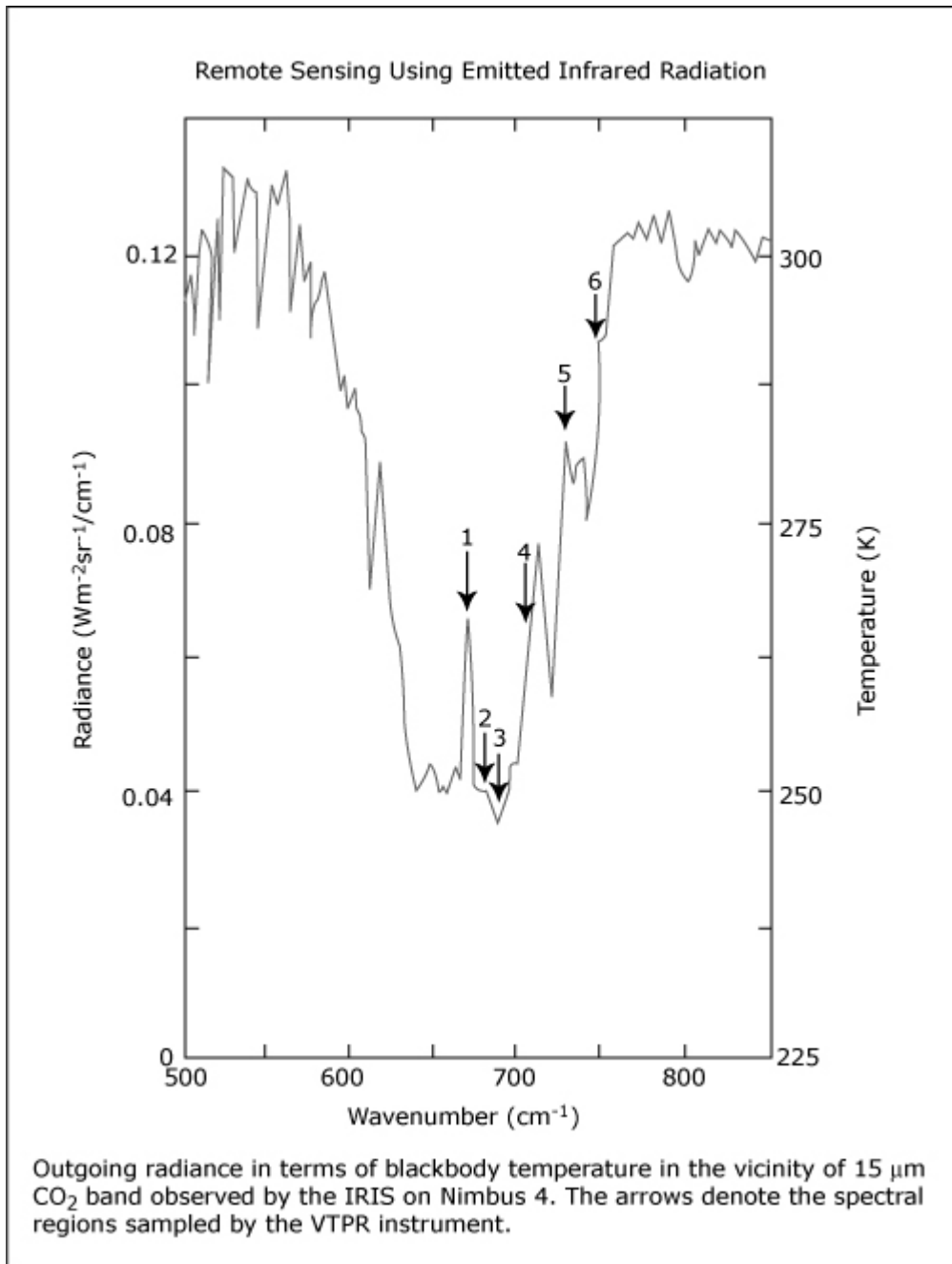
E. Retrieval of Atmospheric Temperature Profiles

a) Direct Inversion

We can <u>invert</u> the radiative transfer equation directly in the following manner. In the 11 μm "window", 85% of observed intensity is from the ground (depending on atmospheric humidity). Use an <u>initial guess</u> profile $\overline{T}_p$, to obtain T_s:

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$$B_{\bar{v}}(T_s)\bar{t}_v(\mu, p_s) = I_{\bar{v}}(\mu, 0) + \int_1^{\bar{t}_{\Delta v}(\mu p_s)} B_v(\overline{T}_p)d\bar{t}_v(\mu, p) \qquad (27)$$

(obs. At 11 μm)

Usually, $\overline{T}_p$ is climatological mean or better, a "forecast". Now put

$$B_{\bar{v}}(T_p) = B_{\bar{v}}(\overline{T}_p) + h_p \frac{dB_{\bar{v}}(\overline{T}_p)}{dT} \qquad \text{(Taylor approximation)}$$

$$\underbrace{\qquad\qquad}_{T_p - \overline{T}_p}$$

Thus, defining the observable, $r_{\bar{v}}(\mu)$ (for all other v).

$$r_{\bar{v}}(\mu) = \underset{\text{(observed)}}{I_{\bar{v}}(\mu, 0)} - \underset{\text{(guess)}}{\bar{I}_{\bar{v}}(\mu, 0)}$$

$$= I_{\bar{v}}(\mu, 0) - B_{\bar{v}}(T_s)\bar{t}_{\Delta v}(\mu, p_s) + \int_1^{\bar{t}_{\Delta v}(\mu p_s)} B_v(\overline{T}_p)d\bar{t}_{\Delta v}(\mu, p)$$

$$= -\int_1^{\bar{t}_v(\mu p_s)} \frac{dB_v(\overline{T}_p)}{dT} h_p \, d\bar{t}_{\Delta v}(\mu, p)$$

where we have used Equation 25a.

We can now approximate the integral with a summation:

$$r_i = \sum_{j=1}^N a_{ij} h_j = \sum_{j=1}^N \frac{\partial r_i}{\partial h_j} h_j \qquad (29)$$

where i is v and j is P, z, or $\bar{t}_v$.

Thus $\underset{\sim}{r} = A\underset{\sim}{h}$; where A is a "partial derivative" matrix and $\underset{\sim}{h} = A^{-1}\underset{\sim}{r}$ gives temperature profile by "direct inversion". Unfortunately, this method requires a <u>precise</u> knowledge of $\bar{t}_{\Delta v}$ which is not available from first principles (due to a lack of knowledge of atmospheric composition – especially, in terms of water vapor and clouds). Without this precise result, our efforts to retrieve the temperature profile by direct inversion will likely result in failure.

b) <u>Indirect Inversion – empirical regression method</u> – here, no transmission functions are required. Entirely empirical.

--L pairs of radiosonde temperature and satellite rediance measurements are available **at the same point.**

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--define $H = [h_{jk}]$ $\qquad$ $\underbrace{j = 1, N}$; $\qquad$ $\underbrace{k = 1, L}$

$\qquad\qquad\qquad\qquad\qquad\quad$ pressure $\qquad\qquad\quad$ number of
$\qquad\qquad\qquad\qquad\qquad\quad$ levels of $\qquad\qquad\qquad$ pairs
$\qquad\qquad\qquad\qquad\qquad\quad$ radiosondes

and $R = [r_{ik}]$ $\qquad\quad$ $\underbrace{i = 1, M}$; $\qquad\qquad$ $k = 1, L$

$\qquad\qquad\qquad\qquad\qquad\quad$ frequencies of
$\qquad\qquad\qquad\qquad\qquad\quad$ radiance
$\qquad\qquad\qquad\qquad\qquad\quad$ measurements

where as before

$h_{jk} = h\ (p_j)_k = T(p_j)_k - \overline{T}(p_j)$ $\qquad$ a guess-for example, the mean of all
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad$ radiosonde temperatures at p_j

$r_{ik} = r\ (\nu_i)_k = I(\nu_i)_k - \overline{I}(\nu_i)$ $\qquad$ a guess-for example, the mean of all
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad$ L satellite radiances at ν_i.

Here, we will correlate temperature profiles with radiances in a Least Square Sense.

Now, assume a linear relation between H and R.

(N x L) $\qquad\qquad$ (N x M) $\quad$ (M x L) $\qquad\qquad$ (N x L)

H $\qquad = \qquad$ C $\qquad$ R $\qquad\qquad$ or $\qquad$ O $\ = \ $ H − C R (error matrix)

$\qquad$ Define $\varepsilon = \sum_{j=1}^{N}\ \sum_{k=1}^{L}\left(h_{jk} - \sum_{i=1}^{M} C_{ji}\ r_{ik}\right)^2$

$\qquad\qquad\qquad\qquad\qquad$ known $\ $ (needed) $\ $ known

$\qquad\qquad = t\,r\ ([H - C\ R]\ \underbrace{[H - C\ R]'})$

sum of all
diagonal elements $\qquad\qquad\qquad\qquad\qquad$ "Error Covariance" matrix

where the prime represents the transpose matrix.

Ideally want $\varepsilon = 0$; therefore minimize to obtain best C in a <u>least Squares sense</u>.

i.e. want $\quad \dfrac{\partial\,\varepsilon}{\partial\,C_{ji}} = \dfrac{\partial}{\partial\,C_{ji}}\left\{ \sum_{k=1}^{L}\left(h_{jk} - \sum_{i^*=1}^{M} C_{ji^*}\ r_{i^*k}\right)^2\right\}$

$\qquad\qquad\qquad = \sum_{k=1}^{L} 2\left(h_{jk} - \sum_{i^*=1}^{M} C_{ji^*}\ r_{i^*k}\right)(r_{ik})$

$$= \sum_{k=1}^{L} \left[2 h_{jk} (r_{ki})' - 2 \left[\sum_{i^*=1}^{M} C_{ji^*} r_{i^*k} \right] (r_{ki})' \right]$$

$= j \; i^{th}$ element of $2HR' - 2CRR'$

$= 0 \quad \underline{\text{for all j and i}}$

(NxM)   (NxM) (MxM)

i.e. $2HR' - 2CRR' = 0$ and $C \quad = \quad HR' \; (R R')^{-1}$

where the ' indicates the "transpose matrix". $\boxed{\text{ij element } \sum_{k=1}^{L} r_{ik} (r_{kj})'}$ $\dfrac{\text{co-factor matrix of } RR'}{|RR'|}$

To prevent RR' (MxL, LxM) being singular, generally want L>>M, i.e. many more IR sounder-radiosonde coincidences than wavelengths – i.e. no redundant radiosonde-sounding coincidences.

We can now use this <u>empirically</u> – derived C (compare to the transmission function matrix, A^{-1} above) to invert all other satellite soundings:

i.e. $\underset{\sim}{h} = C \underset{\sim}{r}$

F. Remote Sensing Exercise

A 2-part Remote Sensing Exercise will be handed out here. The first part deals with temperature retrieval by empirical regression. The second part asks you to again use the MODTRAN code to examine weighting functions in the 15 μm band of CO_2 and use the information to design a system for temperature sounding.

G. Temperature Retrieval Based on Iteration

The retrieval procedures we've discussed so far are linear and empirical. The problem we face here is that the radiative transfer equation (a Fredholm equation with fixed limits) may not always have a solution. The radiances contain experimental uncertainty and so do the calculated transmission results. Furthermore, it is necessary to approximate the integral by a sum, again introducing errors in the computed radiances.

We should instead consider a non-linear approach. Re-writing the solution to the equation of radiative transfer, we have:

$$I_i = B_i (T_s) \bar{t}_i (p_s) + \int_{p_s}^{0} B_i [T(p)] \frac{\partial \bar{t}_i (p)}{\partial \ln p} d\ln p \qquad (28)$$

where we've used ln p as independent variable and where i denotes the spectral channels.

Using the mean value theorem, the radiance can be approximated by Eq. 29.

$$\hat{I}_i - B_i(T_s)\bar{t}_i(p_s) \cong B_i[T(p_i)]\left[\frac{\partial \bar{t}_i(p)}{\partial \ln p}\right]\Delta_i \ln p \tag{29}$$

where p_i denotes the pressure level at which the maximum weighting function is located and $\Delta_i \ln p$ is the pressure difference at the ith level and is defined as the effective width of the weighting function. If the guessed temperature profile is $T^*(p)$, the expected radiance is then given by Eq. 30.

$$I_i^* - B_i(T_s)\bar{t}_i^*(p_s) = B_i[T^*(p_i)]\left[\frac{\partial \bar{t}^*(p)}{\partial \ln p}\right]_{P_i}\Delta_i \ln p \tag{30}$$

Dividing Eq. 29 by Eq. 30 then leads to Eq. 31:

$$\frac{\hat{I}_i - B_i(T_s)\bar{t}_i(p_s)}{I_i^* - B_i(T_s)t_i^*(p_s)} \cong \frac{B_i(T(p_i))}{B_i(T^*(p_i))} \tag{31}$$

At frequencies where the surface contribution to the upwelling radiance is small, Eq. 31 may be approximated by Eq. 32.

$$\frac{\hat{I}_i}{I^*} \cong \frac{B_i[T(p_i)]}{B_i[T^*(p_i)]} \tag{32}$$

This approach was pioneered by M. Chahine and is referred to as the relaxation equation. A number of variants of this procedure have been developed and are used operationally today.

Here is the recipe for applying this kind of procedure. The quantity, n, is used here to represent the order of iteration.

1. Make an initial guess for the temperature profile: $T^{(n)}(p_i)$, n = 0

2. Substitute $T^{(n)}$ into Eq. 28 and use an accurate quadrature formula to compute the expected upwelling radiance $I_i^{(n)}$ for each sounding channel.

3. Compare the computed radiance values $I_i^{(n)}$ with the measured data $\hat{I}_i$. If the residuals $R_i^{(n)} = \dfrac{\left|\hat{I}_i - I_i^n\right|}{\hat{I}_i}$ are less than a preset small value (e.g., 10^{-4}) for each sounding channel, then $T^{(n)}$ is a solution. If not, continue the iteration.

4. Apply the relaxation equation (Eq. 32) M (where M = number of spectral channels, and we have the same number of pressure levels as spectral channels) times to generate a new guess for the temperature values $T^{(n+1)}(p_i)$ at the selected i pressure levels, i.e., force the temperature profile to match the observed radiances.

    Since the Planck function can be written as Eq. 33 with a = $2hc^2$ and b= hc/k

$$B_i(T) = \frac{a v_i^3}{\left(e^{b v_i / T} - 1\right)} \tag{33}$$

we can write an iterative equation for the temperature in terms of the previous iteration as indicated in Eq. 34.

$$T_{(p_i)}^{(n+1)} = b v_i / \ln \left\{ 1 - \left[1 - \exp\left(b v_i / T^{(n)}(p_i) \right) \right] I_i^{(n)} / \hat{I}_i \right\} \tag{34}$$

$$i = 1, 2 \ldots\ldots M$$

5. Carry out the interpolation between the temperature value at each given level, p_i, to obtain the desired profile.

6. Finally, go back to step 2 and repeat until the residuals are less than the preset criterion.

How well will the relaxation method converge to the correct solution? Or will it fail to converge? A variety of testing has led to variants of this procedure that are used in practice.

H. Water Vapor Retrieval Based on Iteration

Let us go back to the solution of the Eq. Of Radiative Transfer Equation as expressed in Eq. 25 (repeated here as Eq. 35):

$$I_{\bar{v}}(0, \mu) = B_{\bar{v}}(T_s) \bar{t}_{\Delta v}(p_s) + \int_{p_s}^{0} B_{\bar{v}}\left[T(p)\right] \frac{\partial \bar{t}_{\Delta v}(p)}{\partial p} dp \tag{35}$$

Performing an integration by parts on Eq. 35, we obtain Eq. 36.

$$I_{\bar{v}} = B_{\bar{v}}(T(0)) - \int_{p_s}^{0} \bar{t}_{\Delta v}(p) \frac{\partial B_{\bar{v}}(p)}{\partial p} dp \tag{36}$$

Where T(0) denotes the temperature at the top of the atmosphere and the spectral transmission is given by Eq. 37.

$$t_{\Delta v}(p) = \int_{\Delta v} \exp\left[-\int_{0}^{P} k_v(p) dm(p)\right] \frac{dv}{\Delta v} \tag{37}$$

If the temperature profile has been retrieved through measurements in one of the CO_2 bands, then the remaining unknown is the transmission. We can then select spectral channels in an appropriate water vapor absorption band (e.g. – the 6.3 micrometer vibration-rotation band) and attempt to retrieve the concentration of water along the path.

Empirical procedures have been used with some success and more recently, iterative procedures are being applied. However, the accuracy of retrieval is poorer than for temperature retrieval and is generally in the range of 10-20%. Higher spectral resolution measurements of the future are thought to enable measurements somewhat better than 10%.

I. Limb Sounding

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1. Useful for measuring trace constituent profiles of the middle atmosphere $10 \leq Z \leq 60$ km.

2. Can look at transmission (absorption) or emission. Transmission measurements are limited by availability of the sun and so only occasional occultations are observed.

3. Emission measurements (in thermal infrared) can be made any time of day or night.

   a) Emission originates in the few km immediately above tangent point due to rapid decrease of atmospheric density and pressure. This, coupled with low-altitude cut-off leads to an inherent high vertical resolution of measurement.

   b) All radiation comes solely from atmosphere – no surface boundary contribution.

   c) Horizontal (tangent) path leads to high opacity (x37). Thus, it is good for minor gas detection.

   d) Disadvantages: 1) High clouds; and 2) Horizontal region sounded is large (~ 100-200 km).
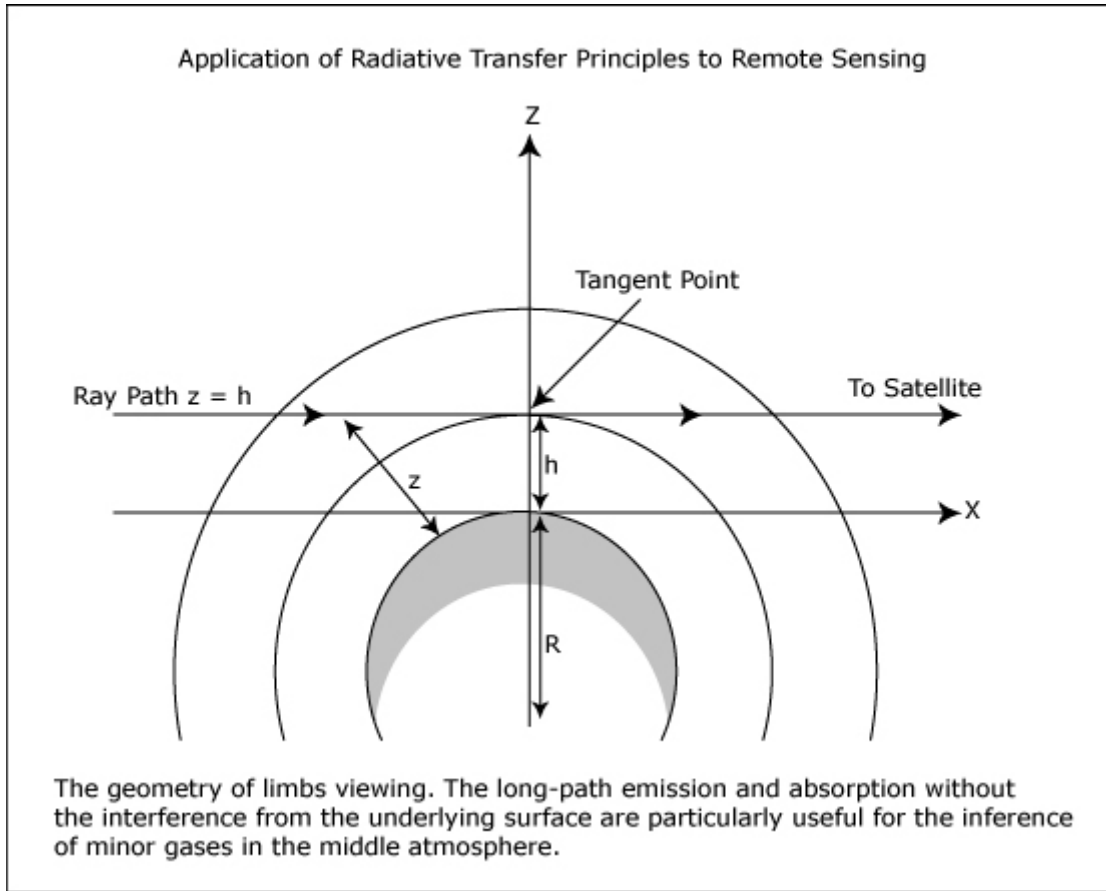
Eq. Of Radiative Transfer for Limb Infrared Emission:

$$I_{\bar{\nu}} = B_{\bar{\nu}}\left(T_s\right)\bar{t}_{\Delta\nu}\left(p_s\right) + \int_{p_s}^{0} B_{\bar{\nu}}\left[T\left(p\right)\right] \frac{\partial \bar{t}_{\Delta\nu}\left(p,0\right)}{\partial p} dp \tag{38}$$

Specializing radiative transfer equation to limb situation, we have:

$$I_{\bar{\nu}}(h) = \int_{-\infty}^{+\infty} B_{\bar{\nu}}\left[T\left(X\right)\right] \frac{\partial \bar{t}_{\Delta\nu}\left(h,\chi\right)}{\partial \chi} d\chi \tag{39}$$

where the boundary term has been set = 0 and the independent variable has been converted to x.

For determining gaseous composition as a function of height (z), we must change variable from x to z (using spherical geometry). We then have:

Application of Radiative Transfer Principles to Remote Sensing

The geometry of limbs viewing. The long-path emission and absorption without the interference from the underlying surface are particularly useful for the inference of minor gases in the middle atmosphere.

$$I_{\bar{v}}(h) = \int_0^\infty B_{\bar{v}}\left[T(z)\right] K_{\Delta v}(h, z)\, dz$$

(40)

where $K_{\Delta v}(h, z)$ is the weighting function (for an instrument with an infinitesimal vertical FOV).

J. Remote sensing using Emitted Microwave Radiation

Water Vapor and Oxygen are accessible in the microwave.

Again Radiative Transfer Equation is given by:

$$I_{\bar{v}}(0) = I_{\bar{v}}(p_s)\, \bar{t}_{\Delta v}(p_s, 0) + \int_{p_s}^0 B_{\bar{v}}\left[T(p)\right] \frac{\partial \bar{t}_{\Delta v}(p, 0)}{\partial p}\, dp$$

(41)

Emissivity in microwave region is generally less than unity (ranges from 0.4-1.0), so there is a significant reflection contribution from the surface. The radiance emitted from the surface is given by:

$$I_{\bar{v}}(P_s) = \varepsilon_{\bar{v}}\, B_{\bar{v}}(T_s) + \left(1 - \varepsilon_{\bar{v}}\right) \int_0^{p_s} B_{\bar{v}}\left[T(p)\right] \frac{\partial \bar{t}_{\Delta v}(p_s, p)}{\partial p}\, dp$$

(42)

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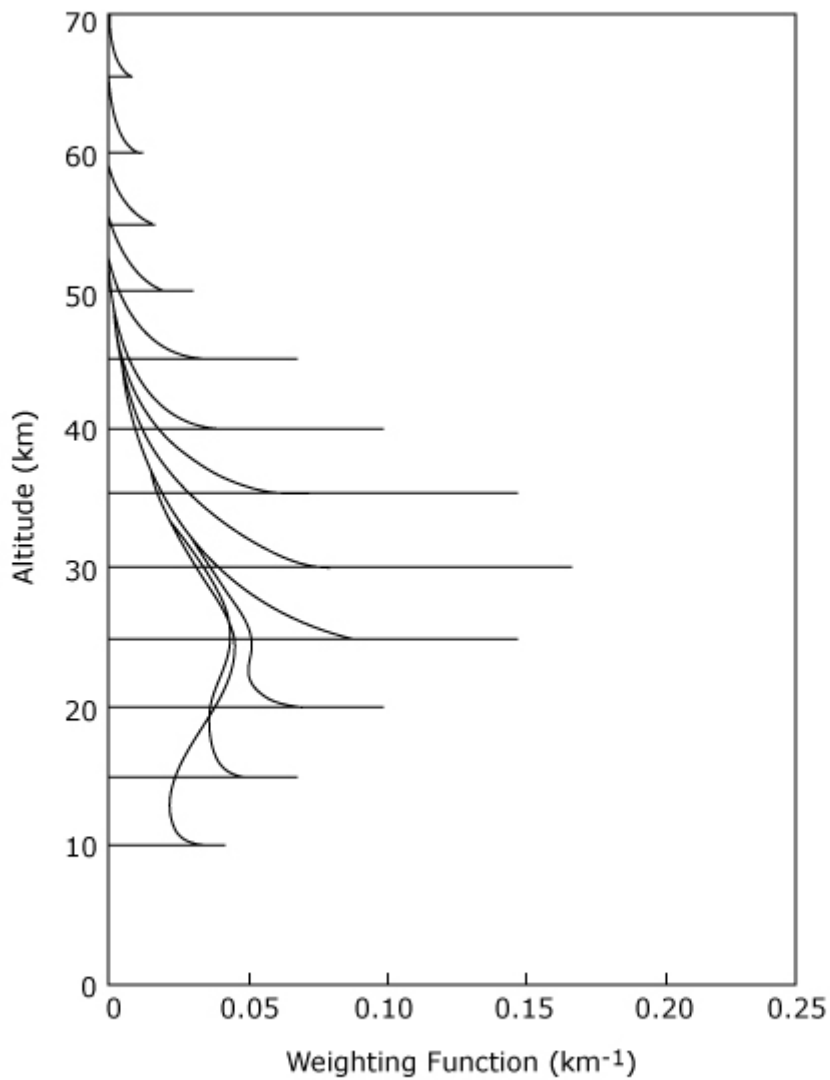
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The second term here represents emission from the atmosphere to the surface which is reflected back at the same frequency. The transmission here $\bar{t}_{\Delta v}(p_s, p)$ is now expressed with respect to the surface.
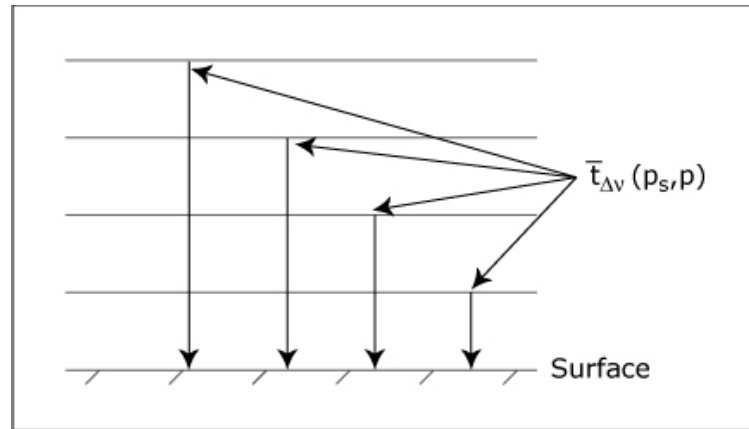
Upwelling radiance is now given by:

$$I_{\bar{v}} = \varepsilon_{\bar{v}} B_{\bar{v}}(T_s) \bar{t}_{\Delta v}(p_s, 0) + (1 - \varepsilon_{\bar{v}}) \bar{t}_{\Delta v}(p_s, 0) \int_0^{p_s} B_{\bar{v}}[T(p)] \frac{\partial \bar{t}_{\Delta v}(p_s, p)}{\partial p} dp + \int_{p_s}^0 B_{\bar{v}}[T(p)] \frac{\partial \bar{t}_{\Delta v}(p, 0)}{\partial p} dp \qquad (43)$$

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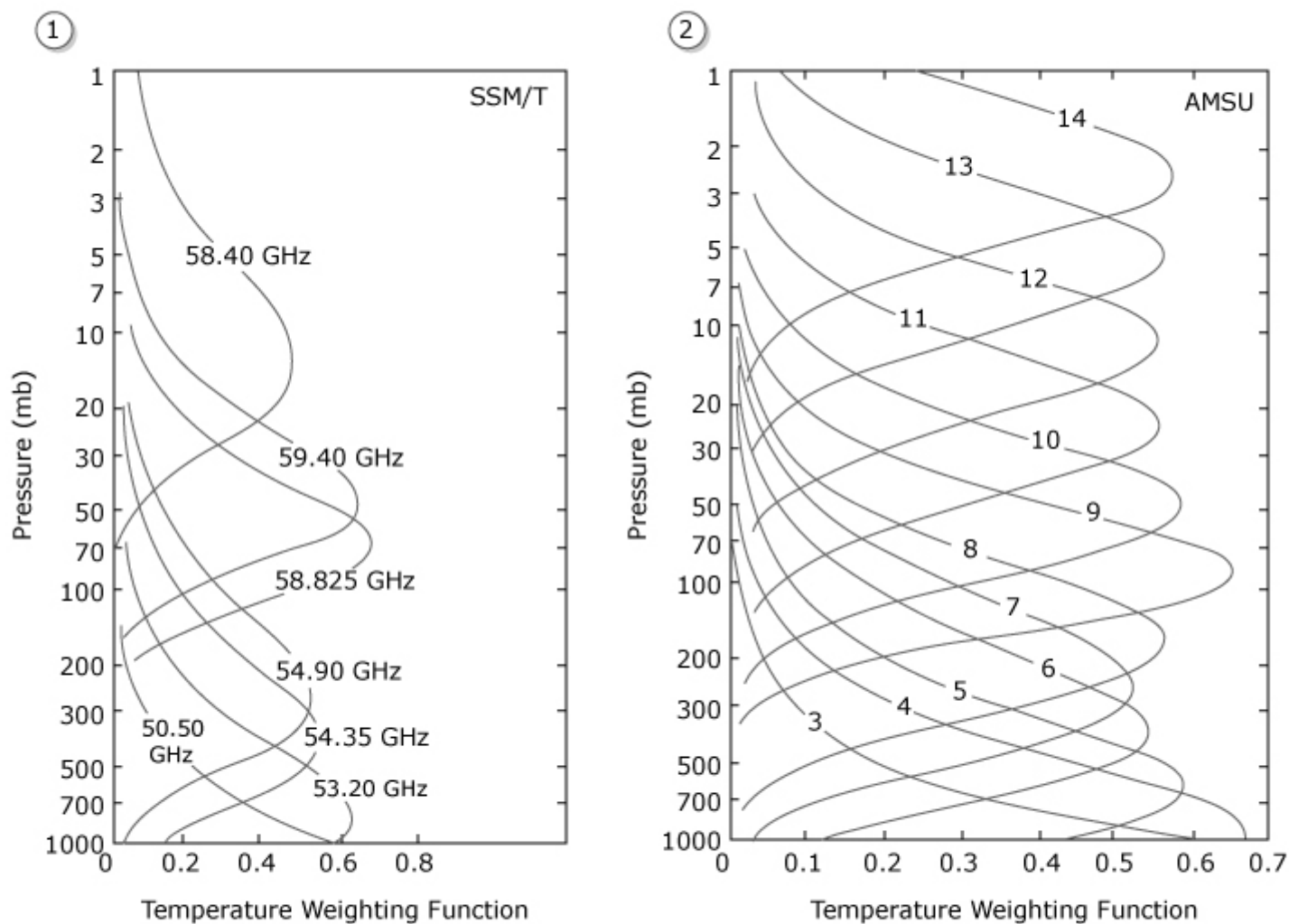Page 22 of 29

Remote Sensing Using Emitted Infrared Radiation

Limb viewing weighting function for the ideal case of an instrument with an infinitesimal vertical field of view for the spectral band 585-705 cm^{-1} covering most of the 15 μm band of CO_2.

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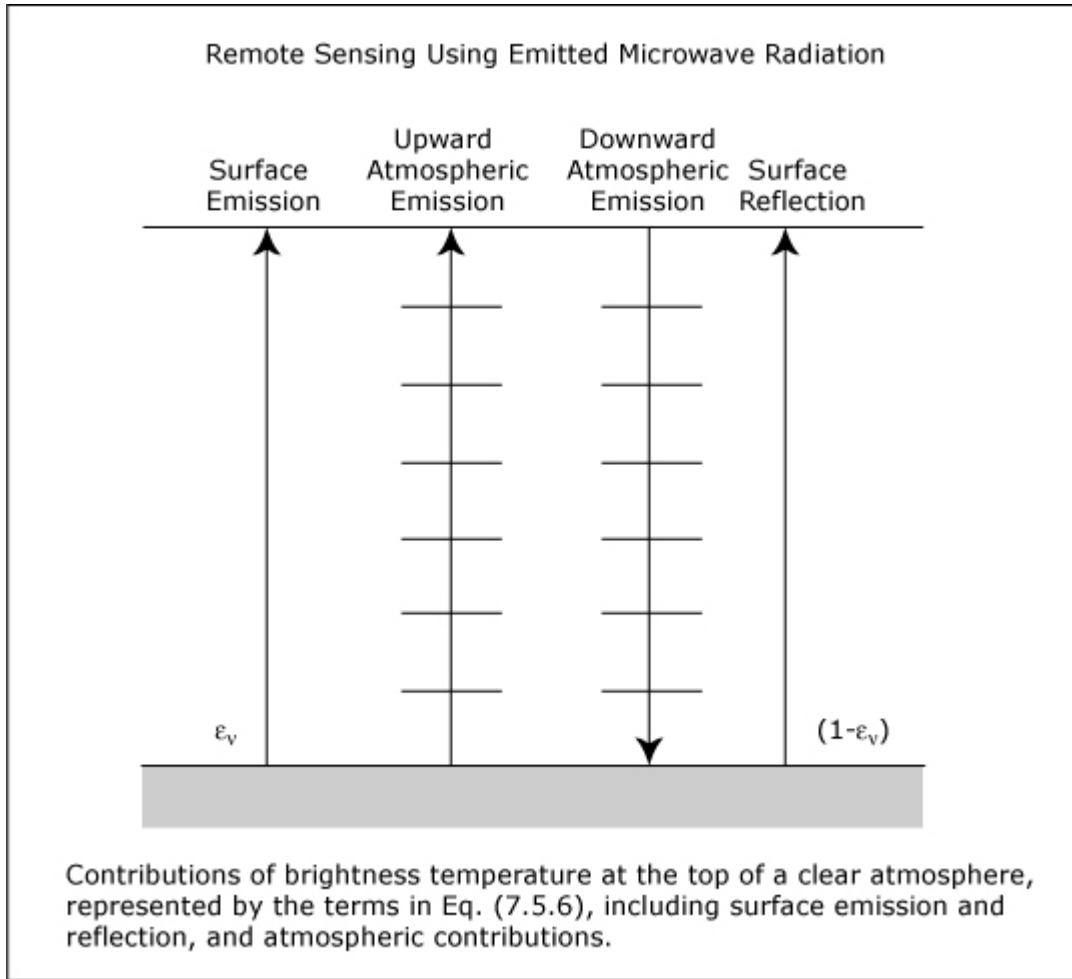Dr. Robert A. McClatchey and Prof. Ronald Prinn

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Demonstrative atmospheric transmittances (total, H_2O, and O_2) as a function of frequency and wavelength in the microwave region.

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Application of Radiative Transfer Principles to Remote Sensing

①

SSM/T

Pressure (mb)

58.40 GHz

59.40 GHz

58.825 GHz

54.90 GHz

50.50 GHz

54.35 GHz

53.20 GHz

Temperature Weighting Function

②

AMSU

Pressure (mb)

14

13

12

11

10

9

8

7

6

5

4

3

Temperature Weighting Function

(1) Weighting functions at nadir position over land for seven SSM/T channels; (2) Weighting functions at nadir viewing over land for 12 AMSU channels.

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Contributions of brightness temperature at the top of a clear atmosphere, represented by the terms in Eq. (7.5.6), including surface emission and reflection, and atmospheric contributions.

In the microwave region we have that $hc\nu/hT \ll 1$.

Therefore $B_{\bar{\nu}}(T) = 2hc\nu^3 / \left[e^{hc\nu/kt-1} \right] \simeq 2k\nu^2 T$ (44)

as we are in spectral region where the Rayleigh-Jeans Law applies. This <u>very good</u> approximation then leads to the following:

$$T_B(\bar{\nu}) = \varepsilon_{\bar{\nu}} T_s \bar{t}_{\Delta\nu}(p_s, 0) + \left(1 - \varepsilon_{\bar{\nu}}\right) \bar{t}_{\Delta\nu}(p_s, 0) \int_0^{p_s} T(p) \frac{\partial \bar{t}_{\Delta\nu}(p_s, p)}{\partial p} dp + \int_{p_s}^0 T(p) \frac{\partial \bar{t}_{\Delta\nu}(p, 0)}{\partial p} dp \qquad (45)$$

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L. Designing Remote Sounding Systems

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