12.800: Fall 2004 Problem set 5

1) Produce the Boussinesq continuity, momentum, and energy equations by starting from the full versions of the equations we derived in class. The purpose of the problem is to motivate you to think critically about scaling arguments, so please justify each simplification with a scaling argument, and comment on the quality of each assumption.

2) Solve the Dirchlet problem, $\nabla^2\phi = 0$, for a square basin using Matlab and the following boundary conditions:

a)

Top boundary:	bctop[1:n]=1
Bottom boundary:	bcbottom[1:n]=2
Left boundary:	bcleft[1:n]=3
Right boundary:	bcright[1:n]=1

b)

Top boundary:	bctop[1:n]=cos((1:n)/(2*pi)+(t/pi))
Bottom boundary:	bcbottom[1:n]=sin((1:n)/(2*pi)+(t/pi))
Left boundary:	bcleft[1:n]=cos((1:n)/(2*pi)+(t/pi))
Right boundary:	bcright[1:n]=sin((1:n)/(2*pi)+(t/pi))

For t=1:9

You should make your code general enough that it will support any value of n, but for the work you hand in, let n=20 (hint: KC04 has an entire chapter on potential flow).

Turn in a printout of your code (complete with plenty of (relevant) comments), a plot of the potential (see pcolor in Matlab), and a plot of velocity (see quiver in Matlab) for each of the 10 cases given above. For part (b), put the nine plots of potential on a single page and the nine plots of velocity on a single page (see subplot and “orient tall” in Matlab)

3) A hemispherical vessel of radius *R* has a small rounded orifice of area *A* at the bottom. Show that the time required to lower the level from h_1 to h_2 is given by

$$t = \frac{2\pi}{A\sqrt{2g}}\left[\frac{2}{3}R(h_1^{3/2} - h_2^{3/2}) - \frac{1}{5}(h_1^{5/2} - h_2^{5/2})\right].$$

[KC04: Ex 4.10]