12.215 Modern Navigation

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Review of last Class

- Review of linear Algebra. Class will be based on the book “Linear Algebra, Geodesy, and GPS”, G. Strang and K. Borre, Wellesley-Cambridge Press, Wellesley, MA, pp. 624, 1997
- Topics to be covered will be those later in the course
- General areas are:
  - Vectors and matrices
  - Solving linear equations
  - Vector Spaces
  - Eigenvectors and values
  - Rotation matrices

Today’s class

- Analysis of Sextant measurements
- Homework was broken into a number of small steps:
  - Determining the maximum observed angle to the sun and time this maximum occurred
  - Obtaining the mean index error
  - Computing maximum elevation to the sun
  - Computing the atmospheric bending correction
  - Computing the latitude
  - Computing the longitude

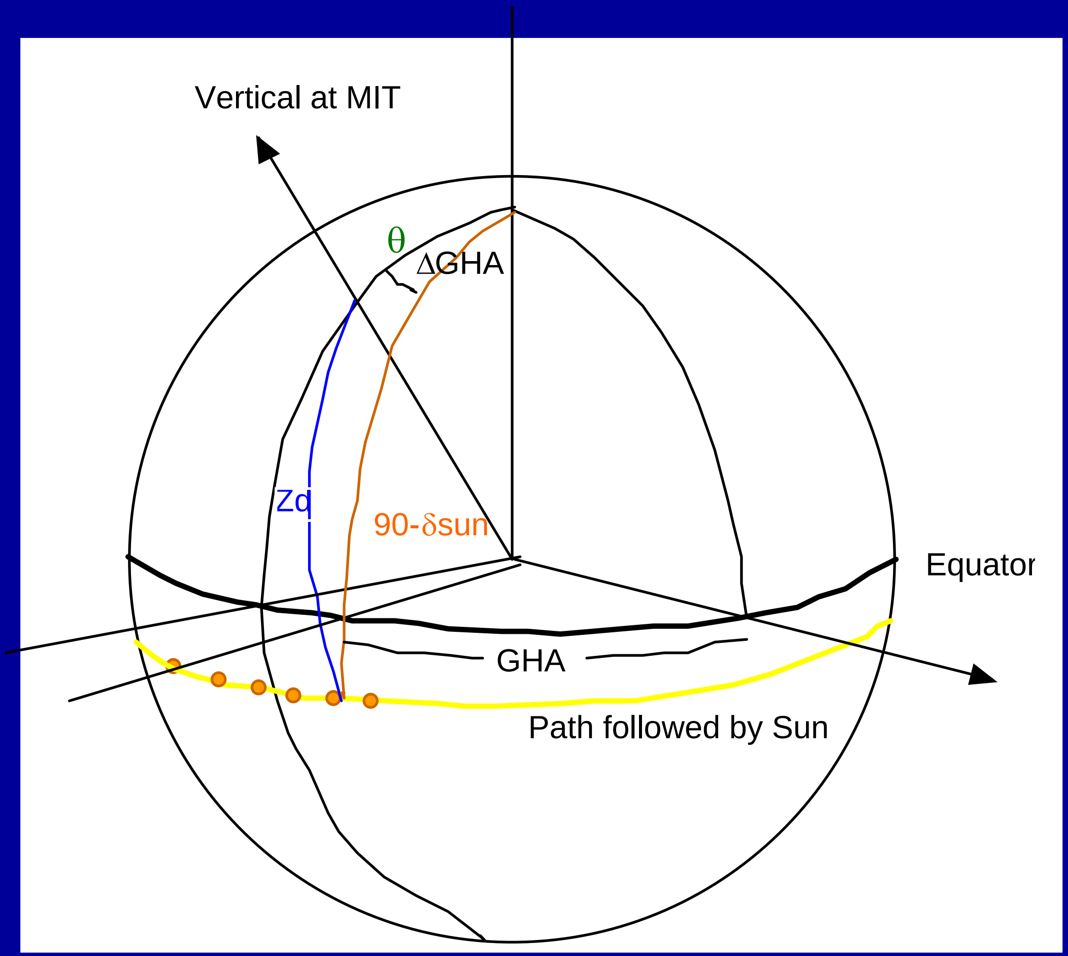
Simpler parts of calculation

- **Mean of index error:** Simply the sum of the values divided by the number of values
- Also we can compute a standard deviation about the mean (also called a root-mean-square (RMS) scatter). This gives is an indication of how well we can make measurements with the sextant. The standard deviation of our measurements was 0.9’
- We use this today and in later lectures we will show how to use this to allow us to estimate the uncertainty of our final latitude and longitude determination.

Atmospheric refraction

- We can use the simple formula given in class or we can look up the values in the Nautical Almanac.
- The formula result is slightly greater than 1’ since $\tan(\varepsilon) \sim 1$
- Using the almanac we can explore how much this value will vary due to atmospheric conditions.
- (For latitude determination, atmospheric refraction becomes a bigger problem the closer we get to the pole where the meridian crossing elevation angle will be much smaller. It will also be a bigger problem in mid-winter than in mid-summer).

Geometry of measurement



- Spherical trigonometry that we can solve (we interpret on the meridian and so easy)

Spherical Trigonometry

- Based on the figure, we can write the solution for the zenith distance to the sun:

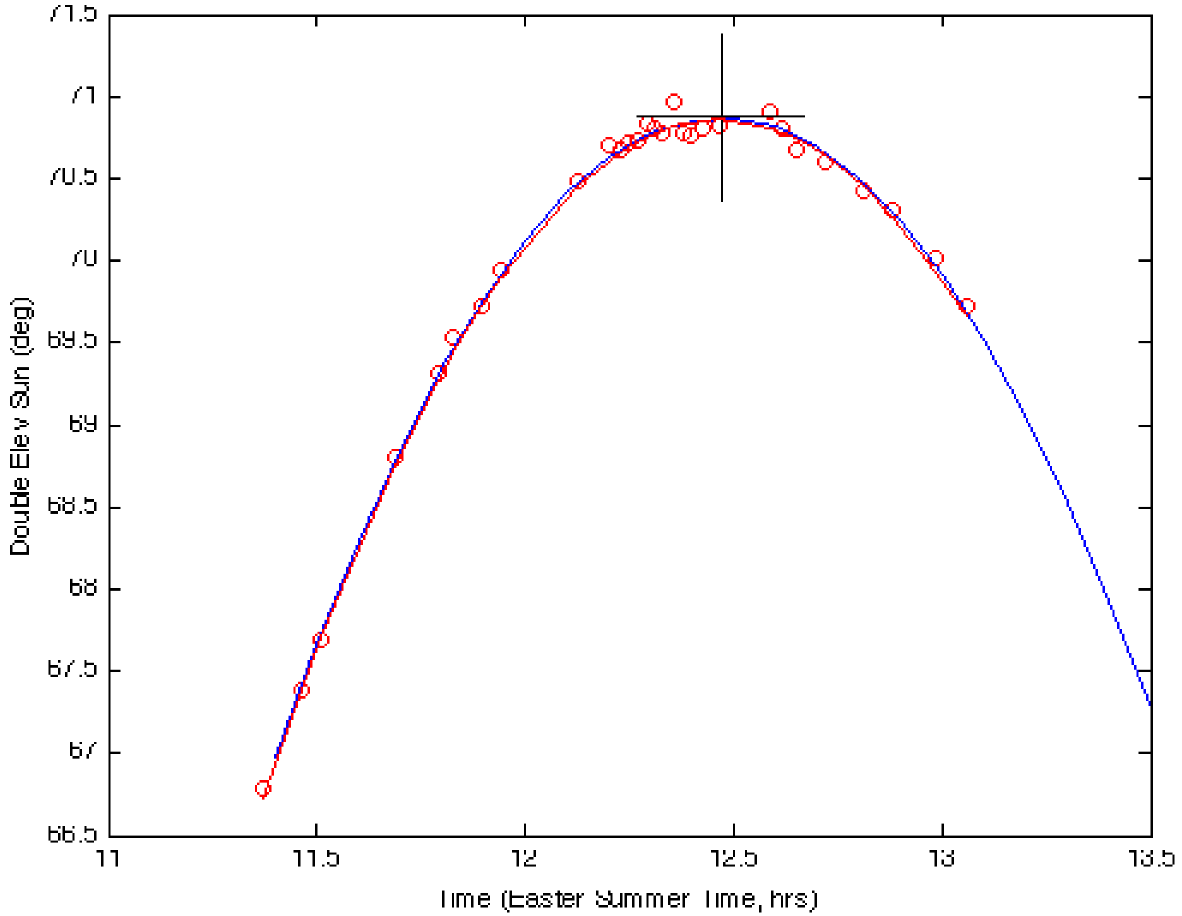
$$\cos Zd = \cos\theta\cos(90-\delta) + \sin\theta\sin(90-\delta)\cos(\Delta GHA)$$

- If we assume we know our latitude and longitude then we can compute the expected variations in the zenith distance to the Sun
- In addition, since we measured 2*(elevation to sun+refraction)+ index error , we can include this in what is called a “forward model”

Results of forward model

- GPS latitude 42.36; longitude -71.0890
- Declination of Sun at MIT meridian crossing -12.2 deg
- Interpolating the Almanac GHA, UT meridian crossing 16.470 hrs (-4 hrs to EST)
- The forward model can be computed and compared to measurements.

Forward Model Calculation

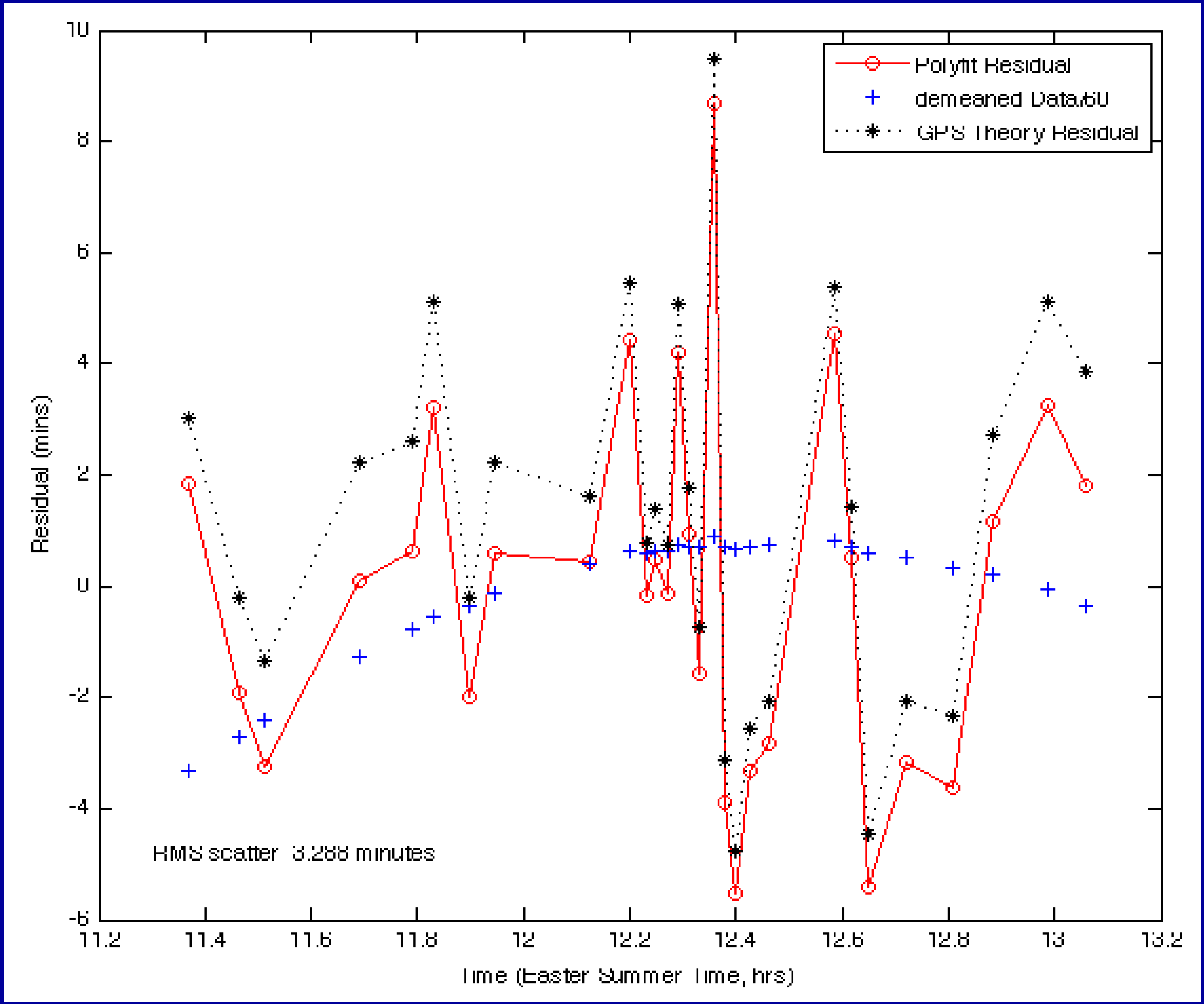


Blue: quadratic
Red: Forward Model

Comparison

- Agreement looks good but when totals are displaced the results can be be deceptive in that the details can not been seen.
- Normal to look at the difference between the observations and the model
- On the quadratic fit residuals we show “error bars” based on the index measurements. These are computed from sqrt(Sum(residuals^2)/(number-1)). Also called Root-mean-square (RMS) scatter
- In class we will vary the parameters of the model to see there effect on the fit to the data.

Residuals (Quadratic and Model)



Black Stars: Residual to model

Red circles: residuals to quadratic fit

RMS Fit: 3.3

Some neglected effects

- Refraction and index error not included in forward model but these can be easily added into Matlab code.
- Motion of Sun during measurements was accounted for during the run
- Later we will use the forward model to obtain rigorous estimate of latitude and longitude.

Summary:

- Today we explored the latitude and longitude problem in more detail looking at the actual data collected with the sextant.
- Introduced the notion of a forward model for comparing with data and varying the parameters of the model to better match the observations.
- Differences between observations and models can be quantified with an estimated standard deviation or RMS scatter.
- These issues are returned to when we address statistics and estimation.