VIII. HILLSLOPE EVOLUTION

A. Definitions: Transport Limited and Weathering Limited Landscapes

Transport-limited hillslopes: delivery of sediment to streams is limited by the rate at which soil and rock can be transported (supply >> capacity). Hillslope form dictated by transport processes and their spatial variability (conservation of mass; divergence of sediment flux).

Weathering-limited (detachment-limited) hillslopes: delivery of sediment to streams is limited by the rate of sediment production (supply << capacity) by the various mechanisms of chemical weathering, physical weathering, and erosional detachment (overland flow; mass movement). Hillslope form is dictated by weathering and erosional processes, divergence of sediment flux is not relevant.

B. Introduction to Hillslope Hydrology

Flow Pathways:

1. Horton Overland Flow (HOF). Rainfall intensity exceeds infiltration capacity, overland flow occurs regardless of soil saturation state. Typically arid regions and bare bedrock slopes (small fraction of Earth's surface today). Sharply peaked hydrographs (minutes to channel): storm flow. HOF may have dominated early Earth history before landplants.

2. Subsurface Storm Flow (SSF) ("Throughflow"). Shallow groundwater flow. Infiltration capacity (rate) exceeds rainfall intensity, downslope flow in saturated zone, usually a thin soil above bedrock or other discontinuity in hydraulic conductivity. Flow rates cm/s - cm/hr, contributes to storm flow and base flow (most flow gets downslope in hrs - 10's hrs) -- strong rise in hydrographs. Dominant mechanism in humid/temparate regions (most of Earth's surface).

3. Return Flow and Saturated Overland Flow (RF, SOF). Variable source area concept: saturated zones at base of hills (concave topography) grow during wet season/storms. When subsurface flow capacity ("transmissivity") is exceeded, SSF is forced to return to the surface, contributing to overland flow and storm flow component of hydrographs.

4. Groundwater flow. Slow vertical percolation of water in soil/rock. Rates from cm/hr to cm/yr. Important to chemical weathering of bedrock, contributes to base flow only (minimal storm response).

C. Hillslope Transport Processes

Slow/Continual Processes

1. Soil Creep (humid/temperate - SSF)

   Biogenic Mechanisms (Burrowing, Tree Throw, etc); Frost Heave; Shrink/Swell (clays); Rheologic Creep (slow plastic flow; solufluction -- freeze thaw or wet/dry)

2. Rainsplash/Sheetwash (arid - HOF)

   Rainsplash - Rainflow - Sheetwash Continuum. Rain drop impacts displace sediment "splash", net down-slope transport. "Rainflow" is transport caused by disturbance of thin, laminar sheet flow by rain drop impacts. Will consider only unchanneled sheetwash initially.

Rapid/Stochastic Processes

1. Masswasting

   Slumps, Earth flows, Landslides, Debris flows, Rock Fall, Rock Avalanches

D. Mathematical Description of Processes

Conservation of Mass

[Sketch] $h = z\cos\alpha$ $\Delta s = \dfrac{\Delta x}{\cos\alpha}$

Volume of sediment (V); Volume flux of soil (q_v) [x-component]; Depth of soil (h); Elevation soil surface (z); Δs; Δx; Δt Note for creep processes volume flux of soil includes the pore space (not volume flux of sediment), for sheetwash can define transport rate of soil or transport rate of sediment; the latter requires a porosity correction in mass balance equation.

Volume of sediment in box (unit width) $V = h\Delta s = z\cos\alpha \dfrac{\Delta x}{\cos\alpha} = z\Delta x$

Change in volume of sediment in box: $\Delta V = \Delta h \Delta s = \Delta z \Delta x = q_{v_{in}}\Delta t - q_{v_{out}}\Delta t$

$$\frac{\Delta h}{\Delta t} = -\frac{q_{v_{out}} - q_{v_{in}}}{\Delta s} \quad ; \quad \frac{\Delta z}{\Delta t} = -\frac{q_{v_{out}} - q_{v_{in}}}{\Delta x} \quad ; \quad \frac{\partial z}{\partial t} = -\frac{\partial q_v}{\partial x}$$

E. Soil Mantled Slopes: Steady State Forms

Generic Transport Relationship

Kirkby (1971): $q_v = kq^{m'}S^n = k'x^m S^n$

Kirkby gives empirical evidence for m,n values for different hillslope processes.

We will (later) examine evidence and derive values from theory, develop understanding of geologic, climatic and biotic control of various parameters. First we examine implications of different forms of the transport relationship for equilibrium hillslope form. Lab exercise will pursue numerical implementation to allow investigation of boundary conditions (link to rest of landscape) and hillslope responses to transients (e.g. change in climate or river incision rate)

Coupled with continuity equation - can derive relationships for steady-state slope forms developed under different sets of processes (e.g. different climates)

Soil Creep

Generally Humid/temperate conditions: $I_c >> R_i$; m=0, n=1

Transport law: $q_v = K_c S = K_c\left(-\dfrac{\partial z}{\partial x}\right)$

K_c = f(rainfall, windiness (tree throw), freeze-thaw cycles, soil texture, clay mineralogy, etc) $[L^2/T]$

Continuity: $\dfrac{\partial z}{\partial t} = -\dfrac{\partial q_v}{\partial x}$

Substitute: Diffusion Eqn $\dfrac{\partial z}{\partial t} = -\dfrac{\partial}{\partial x}\left(-K_c \dfrac{\partial z}{\partial x}\right) = K_c \dfrac{\partial^2 z}{\partial x^2}$

Steady State Condition: $\frac{\partial z}{\partial t} = -\dot{\varepsilon} = K_c \frac{\partial^2 z}{\partial x^2}$

Integrate (w/ respect to x): $-\dot{\varepsilon}x = K_c \frac{\partial z}{\partial x} + C_1$

B.C.: no flux across ridge: S=0 at x=0 $C_1 = 0$; $-\dot{\varepsilon}x = K_c \frac{\partial z}{\partial x}$

Note this is solution for steady-state slope: $\frac{\partial z}{\partial x} = \frac{-\dot{\varepsilon}}{K_c} x$

Separate variables, integrate $-\frac{1}{2}\dot{\varepsilon}x^2 = K_c z + C_2$

B.C.: z=z_o at ridge top, x=0 $C_2 = -K_c z_o$

Steady-state solution (parabola) $-\frac{1}{2}\dot{\varepsilon}x^2 = K_c z - K_c z_o$

$$z = z_o - \frac{\dot{\varepsilon}x^2}{2K_c}$$

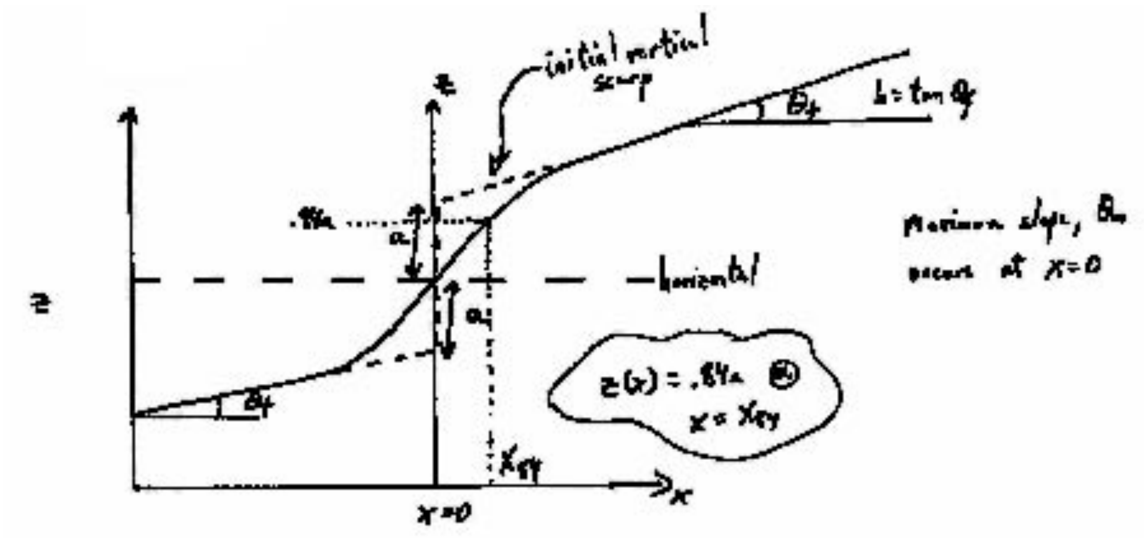
How are boundary conditions reflected in this formulation? Geologic and climatic factors?

Use in Fault-Scarp Age Determination

Analytical solution for transient behavior: error function solution. Initial profiles assumed to be at angle of repose.

Diffusion models "fit" to observed slope profiles -- "fit" only derives estimate of *K*t* product -- need independent estimate of t to "calibrate" K for field setting.

Sketch: Hanks et al, 1984 definition sketch.



Initial Condition: Vertical Scarp.

$$z(x,t) = a\,erf\left[\frac{x}{2\sqrt{Kt}}\right] + bx$$

Where *a* is scarp amplitude, *b* is far-field slope, and *erf* is the error function:

$$erf\left[\frac{x}{2\sqrt{Kt}}\right] = \frac{2}{\sqrt{\pi}} \int_0^{x/\left(2\sqrt{Kt}\right)} e^{-\eta^2}\, d\eta$$

Maximum scarp angle (θ_m) occurs at $x = 0$ and has the convenient formulation:

$$\left[\frac{dz}{dx}\right]_{x=0} = \tan\theta_m = \frac{a}{\sqrt{\pi K t}} + b$$

Another convenient relation is found by recognizing that *erf*(1) = 0.84. Thus if we define X_{84} as the position at which 84% of scarp offset *a* is reached, we can write:

$$X_{84} = 2\sqrt{Kt} \qquad \text{or} \qquad t = \frac{X_{84}^{\;2}}{4K}$$

$$K = \frac{X_{84}^{\;2}}{4t}$$

Pierce and Colman (1986) – Paper available – present a solution for a similar problem, with these differences: no background slope ($b = 0$); initial condition = scarp angle α (not a vertical scarp, usually the angle of respose, ca. 33º). They solve for diffusivity, *K*, in terms of scarp age (*t*) and morphologic measurement of the maximum scarp gradient, $\tan\theta_m$, for a scarp of total height (*h*) (not to be confused with the amplitude = ½ height used by Hanks et al):

$$K = \left[\frac{h}{4t^{1/2} erf^{-1}\left(\tan\theta_m / \tan\alpha\right)\tan\alpha} \right]^2$$

The Hillslope lab project will in part evaluate Pierce and Colman's findings using this relation in a field site in Idaho.

Scarp studies (southwestern US: semi-arid; granular soils; sparse vegetation) generally find: $10^{-3} < K_c < 10^{-2}$ m^2/yr.

Dietrich et al. in humid/temperate N. California and Oregon coastal mountains (moist winter conditions, dense vegetation) find: $K_c \sim 5 \times 10^{-3}$ m^2/yr.

Small et al. (Geomorphology, 1999) on frost-dominated, unvegetated summit flats eroding at rates of ~15 μm/yr estimate $K_c \sim 1.7 \times 10^{-2}$ m^2/yr.

<u>Non-linear Creep</u>

As hillslope gradient approaches the angle of repose (or critical threshold for landsliding) can expect transport rates to increase rapidly: increased probability of mass wasting events. Observations on fault scarps suggest non-linear transport processes or a scarp-height dependence on the coefficient of diffusion. Pierce and Colman (GSA Bull, 1986, 97, p. 869-885) find dependence of "diffusivity" on scarp height and scarp orientation (micro-climate). Several non-linear diffusion models have been proposed to capture this effect. Only that due to Roering et al is based on data.

Andrews and Hanks, late 1980's:

$$q_v = KS^n \quad ; \quad n \sim 2\text{-}3$$

Note: must recalibrate *K* for different values of *n* to get same transport rate at low slopes (different *units*!).

Anderson and Humphrey, 1989:

$$q_v = KS\left(1 + \frac{|S|}{S_c}\right)^a \quad ; \quad a \text{ is a "high power"}$$

Howard, 1994:

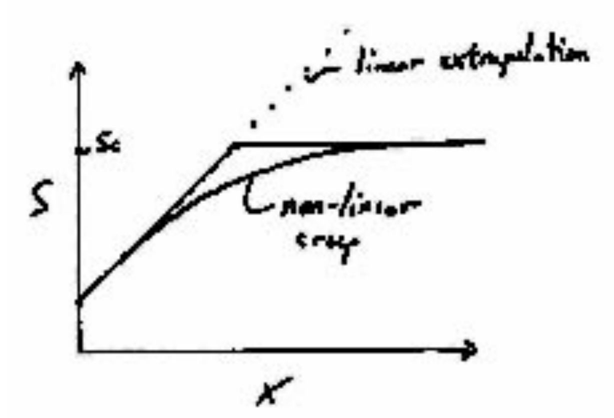
$$q_v = K_c S + K_m \left(\frac{1}{1 - \left(|S|/S_c \right)^a} - 1 \right) \quad ; \qquad a \sim 3$$

Roering et al., 1999:

$$q_v = KS \left(\frac{1}{1 - \left(|S|/S_c \right)^2} \right)$$

FIGURES: plots of various non-linear diffusion models

SKETCH: concept of linear term plus term that goes to infinity as slope reaches S_c, morphologic difference of linear with threshold vs non-linear.



Rainsplash-Sheetwash

Environmental Conditions: Thin or impermeable soils, sparse vegetation, $R_i > I_c$.

Near the ridge crest ($x < x_c$; v. shallow HOF) rainsplash dominates, away from ridge crest ($x > x_c$) sheetwash (unchanneled) dominates. Transition is due to: (1) sheetwash attains depth at which shear stress exceeds critical value to entrain sediment; (2) deepening sheetwash protects bed from raindrop impacts.

Transport Relations

Rainsplash (empirical: field/lab) $$q_s = k_r S = k_r \left(-\frac{\partial z}{\partial x} \right)$$

k_r = f(veg. cover, rainfall intensity, raindrop size, soil texture)

Sheetwash (empirical/theoretical) $q_s = k_w (x - x_c)^2 S^2 = k_w (x - x_c)^2 \left(-\frac{\partial z}{\partial x} \right)^2$

k_w = f(rainfall intensity, infiltration capacity, soil texture, veg cover, etc)

Why m ~ 2, n ~ 2 for sheetwash?

Definition sketch: thickening sheet of overland flow on hillslope

Conservation of Mass (water)

$$P_e = R_i - I_c$$

$$q_w(x) = P_e x = uh(x)$$

Conservation of Momentum

$$u = C_f^{-1/2} \sqrt{ghS} \quad ; \quad \tau_b = \rho g h S$$

Find h(x)

$$h(x) = \frac{P_e x}{u} = \frac{C_f^{1/2} P_e x}{\sqrt{ghS}} \quad \text{\{note } h \text{ on both sides\}}$$

$$h(x) = C_f^{1/3} g^{-1/3} P_e^{2/3} x^{2/3} S^{-1/3}$$

Find $\tau_b(x)$

$$\tau_b(x) = \rho C_f^{1/3} g^{2/3} P_e^{2/3} x^{2/3} S^{2/3}$$

Fluvial Transport Relation (for sheetwash: abrupt transition from no motion to suspension common, owing in part to transition from laminar to fully turbulent flow)

$$q_s = \alpha_s \tau_b^{\,a} \quad ; \quad a \sim 2.5 - 3 \text{ (take } a = 3\text{, for example)}$$

$$q_s = \alpha_s \tau_b(x)^3 = \alpha_s \rho^3 C_f g^2 P_e^2 x^2 S^2 = k_w x^2 S^2$$

$$k_w = \alpha_s \rho^3 C_f g^2 P_e^2$$

Note x_c represents position x at which $\tau_b = \tau_{cr}$

$$q_s = k_w (x - x_c)^2 S^2 = k_w (x - x_c)^2 \left(-\frac{\partial z}{\partial x} \right)^2$$

Hillslope Profiles

Rainsplash dominates for $x < x_c$, in Horton's (1945) "belt of no erosion". Diffusive transport, so steady state solution is same as for creep.

$0 < x < x_c$ $$z = z_o - \frac{\dot{\varepsilon} x^2}{2K_r}$$

Note IF rainsplash measured in terms of volume transport of sediment (not soil) then $K_r = k_r/(1-\lambda_p)$; else $K_r = k_r$.

for $x > x_c$ $$\frac{\partial z}{\partial t} = -\frac{k_w}{(1-\lambda_p)} \frac{\partial}{\partial x} \left((x - x_c)^2 \left(-\frac{\partial z}{\partial x} \right)^2 \right)$$

Steady state condition: $$\frac{\partial z}{\partial t} = -\dot{\varepsilon} = -K_w \frac{\partial}{\partial x} \left((x - x_c)^2 \left(-\frac{\partial z}{\partial x} \right)^2 \right)$$

Integrate (w/ respect x) $$\dot{\varepsilon} x + C_1 = K_w (x - x_c)^2 \left(-\frac{\partial z}{\partial x} \right)^2$$

For C_1, evaluate at $x = x_c$ $$-\dot{\varepsilon} x_c = C_1$$

$$\dot{\varepsilon}(x - x_c) = K_w (x - x_c)^2 \left(-\frac{\partial z}{\partial x} \right)^2$$

$$\frac{\partial z}{\partial x} = -\sqrt{\frac{\dot{\varepsilon}}{K_w (x - x_c)}}$$

Separate variables and integrate $$z = -2\sqrt{\frac{\dot{\varepsilon}(x - x_c)}{K_w}} + C_2$$

Use z at $x = x_c$ (rainsplash) for C_2 $$z = z_o - \frac{\dot{\varepsilon} x_c^{\ 2}}{2K_r} = C_2$$

Steady State Solution:

(x < x_c): $$z = z_o - \frac{\dot{\varepsilon} x^2}{2K_r}$$

(x > x_c): $$z = z_o - \frac{\dot{\varepsilon} x_c^{\,2}}{2K_r} - 2\sqrt{\frac{\dot{\varepsilon}(x - x_c)}{K_w}}$$

Convexo-Concave Profile Form.

Generic Controls on Profile Form (Smith and Bretherton, 1972)

Generic hillslope transport relation:

$$q_v = k q_w^{\,m'} S^n = k' x^m S^n$$

Conservation of mass (water) for HOF (implies m' = m) and sediment:

$$q_w = P_e x \qquad ; \qquad q_v(x, S) = \dot{\varepsilon} x$$

differentiate q_v with respect to x:

$$\dot{\varepsilon} = \frac{\partial q_v}{\partial x} + \frac{\partial q_v}{\partial S}\frac{\partial S}{\partial x}$$

multiply both sides by either q_w or $P_e x$:

$$\dot{\varepsilon} P_e x = q_w \frac{\partial q_v}{\partial x} + P_e x \frac{\partial q_v}{\partial S}\frac{\partial S}{\partial x} = P_e q_v$$

$$q_v - \frac{q_w}{P_e}\frac{\partial q_v}{\partial x} = x \frac{\partial q_v}{\partial S}\frac{\partial S}{\partial x}$$

LHS $x > 0$, $\partial q_v / \partial S > 0$, thus $\partial S / \partial x < 0$ (→ concave-up profile) if and only if RHS is < 0:

$$q_v < \frac{q_w}{P_e}\frac{\partial q_v}{\partial x}$$

True for any function $q_v(q_w, S)$.
For specific case of $q_v = k q_w^{\,m'} S^n = k' x^m S^n$ this condition is satisfied by the following conditions:

1< m < n+1 → concave
all m,n > 1 → concave
n>1; 0<m<1 → concavo-convex

F. Soil Creep Mechanics: SSF Regime

Conditions: soil mantled slopes, almost all climates: humid tropical, temperate, periglacial, arid (alluvial fan surfaces)

Processes

Biogenic: Tree Throw, Burrowing mammals/insects/worms, Foot tread (Non-Rheologic Creep)

Soil Mechanics:

Non-Rheologic: Wet/dry and Freeze/thaw; Frost heave (grain-by-grain lift and fall); Shrink-Swell clays.

Rheologic or "True" creep fluxes (mass flow). Liquefaction (solufluction -- extreme case of freeze-thaw). Plastic or visco-plastic flow (mostly in clays, slow deformation of sand embankments also modeled this way). Note: Mass movement processes (Earth flow, landsliding) averaged over space and time can be modeled as non-linear diffusion (K = f(s)).

All mechanisms flux are dependent on slope, soil texture, and soil moisture content (though in different ways), some dependent on temperature fluctuations, and/or soil depth.

Theoretical treatments: Davidson, 1889 (frost heave); Kirkby, 1967 (frost heave, shrink-swell); Mitchell ("Fundamentals of Soil Behavior": Statistical mechanics, analogy to thermal activation -- rheologic creep); Anderson, 2002 (frost-driven creep).

Velocity Profiles and the importance of differentiating non-rheologic from rheologic creep:

Sketch: Idealized non-rheologic soil creep (slope dependent only -- only surface flux)

Sketch: Typical non-rheologic soil creep (velocity profile develops due to depth dependence on frequency and magnitude of distrubance [biogenic or freeze/thaw])

Sketch: Possible rheologic soil creep velocity profiles.

Assume rheologic creep can be approximately modeled as slow viscous flow:

$$\frac{\partial v}{\partial z} = \frac{1}{\mu_{eff}}\tau_{zx} = -\frac{1}{\mu_{eff}}\alpha_1 h \frac{\partial z}{\partial x}$$

Integrate twice: $$q = -\alpha_2 \frac{h^3}{\mu_{eff}}\frac{\partial y}{\partial x}$$

Note: effective topo. diffusivity (K) $$K = \frac{\alpha_2 h^3}{\mu_{eff}} \quad x$$

Thus for rheologic creep effective diffusivity depends strongly on soil thickness, texture, moisture content, clay mineralogy.

Some non-rheologic creep processes also predict a soil-depth-dependent effective diffusivity. Anderson, 2002, recently published an analysis for freeze-thaw environments that have this effect.

Displacement per frost event i (Δx_i) is a maximum at the surface and linearly decrease with depth (ζ), reaching a value of zero at the depth of frost penetration (ζ_i):

$$\Delta x_i = \Delta x_{\max}(\zeta_i - \zeta)$$

Displacement at the surface depends on the surface gradient and both the soil moisture and the “frost susceptibility” of the soil:

$$\Delta x_{\max} = \beta \frac{\partial z}{\partial x}\zeta_i$$

Total soil flux over the long term depends on the soil flux per frost event i (q_i), the probability distribution of frost depths ($p(\zeta_i)$) and the frequency of frost events (f), which in turn depends on the temperature history, $T(t)$.

Transport = $G(\Delta x_{\max}, p(\zeta_i), f)$

Sediment flux per frost event is:

$$q_i = \int velocity\_profile = \frac{\beta}{2}\frac{\partial z}{\partial x}\zeta_i^2$$

Assume exponential distribution of frost depths, with scale depth ζ_*. Rationale: Temperature fluctuations drop off exponentially with depth (basic result from diffusive heat conduction). SKETCH

$$p(\zeta_i) = \frac{1}{\zeta_*}e^{-\zeta_i/\zeta_*}$$

Thus we can write total sediment flux as:

$$Q = f\int_0^\infty q_i p(\zeta_i)d\zeta_i = f\beta\zeta_*^2 \frac{\partial z}{\partial x}$$

This is correct if the soil is deeper than all frost events. If, however, we add the restriction that there is no frost-heave displacement of bedrock, only the soil, then total sediment flux is restricted to:

$$Q = f\beta\left[\zeta_*^2 - e^{-h/\zeta_*}\left(h\zeta_* + \zeta_*^2\right)\right]\frac{\partial z}{\partial x}$$

So we see that Q → 0 as h → 0, and Q → $f\beta\zeta_*^2 \frac{\partial z}{\partial x}$ for h → infinity

Thus soil flux is governed by a diffusive transport relation:

$$Q = k\frac{\partial z}{\partial x}$$

where $k = f\beta\left[\zeta_*^2 - e^{-h/\zeta_*}\left(h\zeta_* + \zeta_*^2\right)\right]$ is the effective diffusivity.

For this mechanism then, we have an explicit expectation of a depth-dependent diffusivity.

FIGURES: plots from Anderson (2002).

<u>Creep Rates (field/lab measurements)</u>

<u>Methods</u>:

Young Pit (pins or thin metal plates in side wall of trench, refilled);
Rudberg Column (column of wooden dowels inserted in auger hole);
Flexible pipe outfitted with tilt sensors (Fleming and Johnson, 1975).

Biogenic fluxes (burrowing and tree throw): estimate average volume (V) of material, average down-slope shift of center of mass (x), number of occurences (n) per unit time (dt) per unit area (A) in study site (Dietrich and Dunne).

Volume x distance x n times / unit time / (Area study)

$$q_v = \frac{nVx}{Adt}$$

Longterm accumulation rates in topographic hollows (sediment budget: Dietrich and Dunne, 1978). Estimates by Reneau, Benda in PNW, generally consistent with other methods. Hollows are periodically flushed out by landslides/debris flows; radiocarbon date from base of soil gives estimate of filling time.

Results:

Generally not that good: direct slope dependence difficult to demonstrate. Problems: distrubance, different density, moisture content, etc; inaccurate possition measurements; differential transport of different size objects (dowels >> soil grain size). [Sketches]

Examples: Kirkby (1969)

Creep Rates from Be^{10} Budgets

Be^{10} rapidly adsorbed onto soil particles (soil traps all of atmospheric flux delivered by rainfall, except overland flow loses). If no Be^{10} in bedrock (meteoric Be^{10} ~ 5 orders magnitude > *in situ*), simple Be^{10} mass budget can be used to estimate creep rates, and thus test slope dependence, depth dependence, soil moisture, etc. McKean et al (1993, Geology) describe simple balance assuming plug flow (constant velocity with depth -- flux should be depth dependent! -- but only 20-25% difference to Be^{10} budget). Mass balance at position x from the divide for steady-state soil and Be^{10} budget can be written:

$$\int_0^x P_{Be}(x)dx = \int_0^h \rho(x,z)V_s(x,z)\varepsilon_{Be}(x,z)dz$$

where V_s is the velocity of soil movement, ρ is soil bulk density, and ε_{Be} is the concentration of Be^{10}. Note that mass flux of soil is $q_s = \rho V_s h$, and mass flux of Be^{10} is $\rho V_s h\, C_{Be}$, where C_{Be} is the vertically averaged concentration.

Assumptions, integrating.

$$q_s = \frac{P_{Be}x}{\rho C_{Be}(x)}$$

G. Weathering and Soil Production

Transport-limited conditions require non-zero steady-state soil thickness.

Soil Production is often modeled as decaying exponentially with soil thickness: thicker soils insulate the soil-rock interface and inhibit weathering and mechanical soil production (including biogenic mixing):

$$W(h) = W_s e^{-W_a h}$$

W_a is the rate of soil production decline with depth.

At steady state, soil thickness is unchanging ($dh/dt = 0$), therefore the rate of soil production (rate of lowering of the soil-rock interface) must equal the rate of soil surface lowering:

$$W(h_{ss}) = \frac{\partial z}{\partial t} = -\dot{\varepsilon}$$

Thus faster erosion rates lead to thinner soils. Since at steady state the surface lowering rate is everywhere the same, this implies that steady state soil thickness is constant along the hillslope! Where $W_s < |\dot{\varepsilon}|$, the soil is stripped and bare bedrock weathering-limited slopes result. If erosion rate is high, bedrock-involved landsliding is the dominant transport process. New data from Arjun Heimsath indicates maximum values of W_s are on the order of 0.5 mm/yr.

Thus the rate of soil production is a fundamental control on landscape form, erosion process and hillslope hydrology. Soil mantled → SSF; Bare bedrock → HOF.

Keep in mind: hillslope erosion rate is set by the incision rate of bordering channels. In tectonically active landscapes, channel incision tends to adjust to match the rock uplift rate and many such landscapes are landslide-dominated (either $W_s < |\dot{\varepsilon}|$ or $S > \tan\phi$ [the angle of repose]).

Field Tests of the Soil Production Function

Heimsath, Dietrich, Nishiizumi, and Finkel, 1997, Nature

Assume: linear diffusive transport, soil thickness approximately constant in time (i.e., slowly varying or steady). Latter assumption implies that erosion rate matches the soil production rate.

Surface lowering rate by diffusive transport ~ $K\nabla^2 z$ (diffusion in 2D)

Hypothesis: weathering rate ~ $e^{-W_a h}$;

Thus anticipate an exponential relationship between surface curvature and soil thickness (essentially 2 measures of erosion rates if hypothesis is correct) – this is confirmed with field data: topographic surveys plus soil pits for $h(x,y)$.

Implication: spatially non-uniform erosion rates: faster where soil is thinner.

Issue: If erosion is spatially non-uniform, how is this a steady-state landscape as assumed (steady soil thickness). Argument: soil thickness reaches quasi-steady values long before landscape reaches steady-state; soil depth varies only slowly – insignificant amount over timescale of cosmogenic isotope accumulation (~100ka) [see below].

Independent Field Test: direct estimation of soil-rock interface lowering rate using *in-situ* cosmogenic isotopes (^{10}Be, ^{26}Al).

If soil thickness is steady, then expect a constant production rate at the soil-rock interface, which implies a constant rate of lowering of the bedrock.

Expect: Concentration of ^{10}Be, ^{26}Al to be a function of soil thickness and lowering rate. [*Note*: the production of cosmogenic isotopes decreases exponentially with depth].

So Heimsath et al. estimated soil production rate directly from the cosmogenic isotopes (assuming steady, but spatially variable, soil thickness) and plotted this against soil thickness.

FIGURE Heimsath et al data.

This analysis confirmed the expected exponential decline in production rate with soil thickness:

$$W(h) = W_s e^{-W_a h}$$

With no fudging, if the author's use the independently estimated linear soil diffusivity for the region (K = 50 cm^{2}yr^{-1}), to calculate erosion rate from $\varepsilon = -K\nabla^2 z$ they get matching rates – a nice consistent result.