Strain and single-crystal elasticity

Assigned Reading:

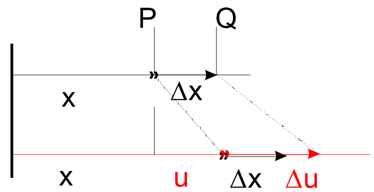
Nye JF (1957) Physical Properties of Crystals. Oxford University Press, Oxford, UK (Chapters 5 and 6).

Resource reading (If you would like to brush up on stress, strain, and elasticity):

Means, W.D., *Stress and Strain*, 338pp pp., Springer-Verlag, New York, 1976.
. (Many problems on which to practice.)

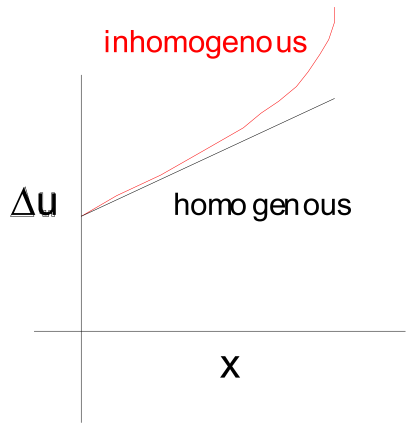
Infinitesimal Strain

One Dimensional Strain



One dimensional distortion

Consider a one-dimensional object that is stretched. It is easy to imagine that the tail of the line segment, Δx, will move by an amount *u*, and that the tip of the line segment will be moved to $u + \Delta x + \Delta u$, where *u* is the rigid body translation and Δu is the amount that the length of the line segment has increased.



Strain at a point P is the increase in length/original length

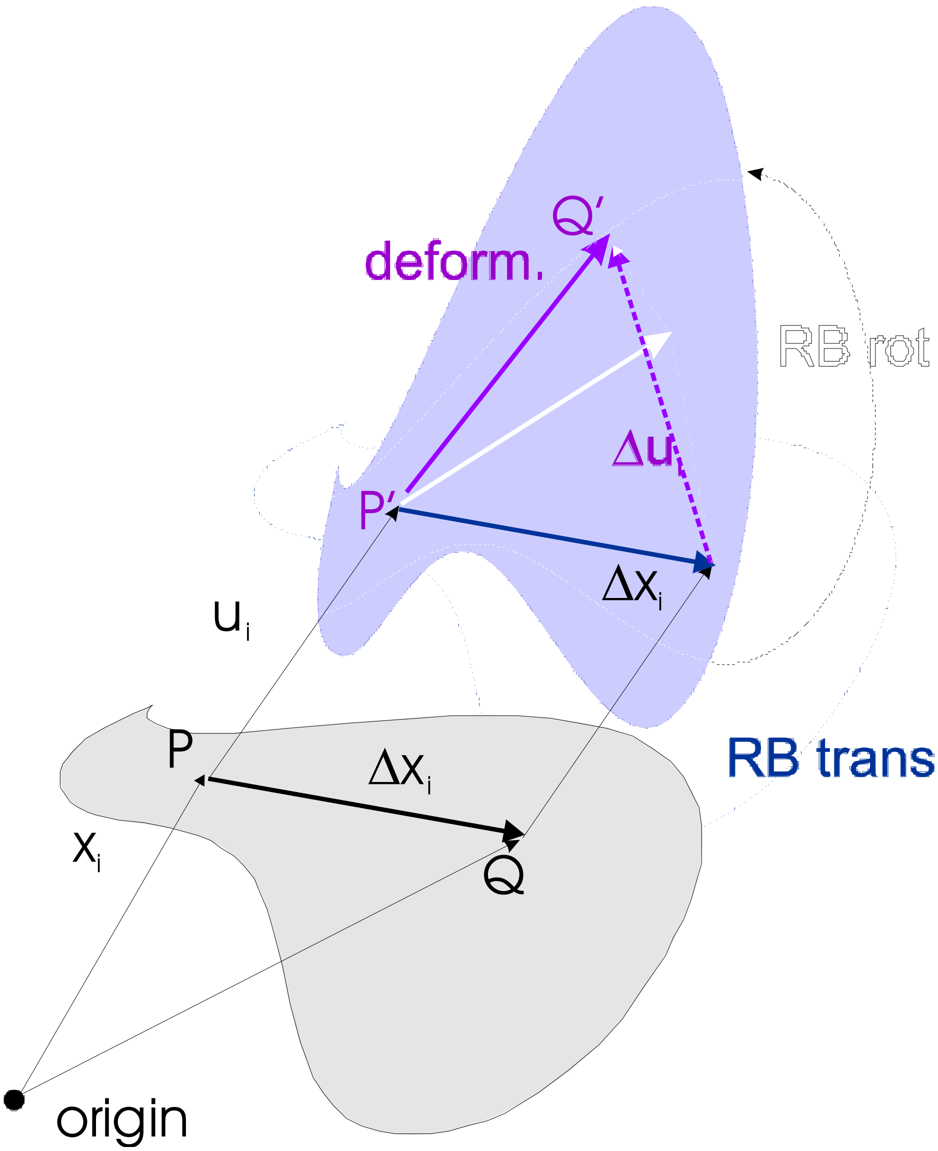
$\varepsilon = \dfrac{\Delta u}{\Delta x}$ and the total length of the stretched line segment $\overrightarrow{P'Q'}$ is

$\Delta x + \Delta u = \Delta x(1+\varepsilon)$

Goal: For a given arbitrary deformation of a body, define a tensor that describes the change of direction and length of any vector in the undeformed body.

Question: Why must the object be a tensor? What rank?

Infinitesimal Three Dimensional Strain



Point P is carried to P'; Point Q is carried to Q'. Since we are not interested in the rigid body translation, the deformation can be characterized by the coefficients Δu_i.

For any continuous differentiable variable,

$$\Delta u_i = \frac{\partial u_i}{\partial x_j} \Delta x_j$$

and we can define the displacement gradient tensor

$$\boxed{e_{ij} = \frac{\partial u_i}{\partial x_j}}$$

Comments and Questions: Because the tensor is written as a differential we are implicitely assuming small deformations (hence infinitesimal deformation gradient tensor).

e_{ij} is a second rank tensor. How do you know?

As an exercise, explicitly write out the expression for the Δu_i using the deformation gradient tensor and Δx_i.

Now divide the tensor into two parts:

$$e_{ij} = \frac{1}{2}\left(e_{ij} + e_{ji}\right) + \frac{1}{2}\left(e_{ij} - e_{ji}\right) = \boxed{\varepsilon_{ij} + \omega_{ij} \equiv e_{ij}}$$

Where ε_{ij}, ω_{ij} are the infinitesimal strain and rotation tensors, respectively. The former is symmetric and the latter antisymmetric.