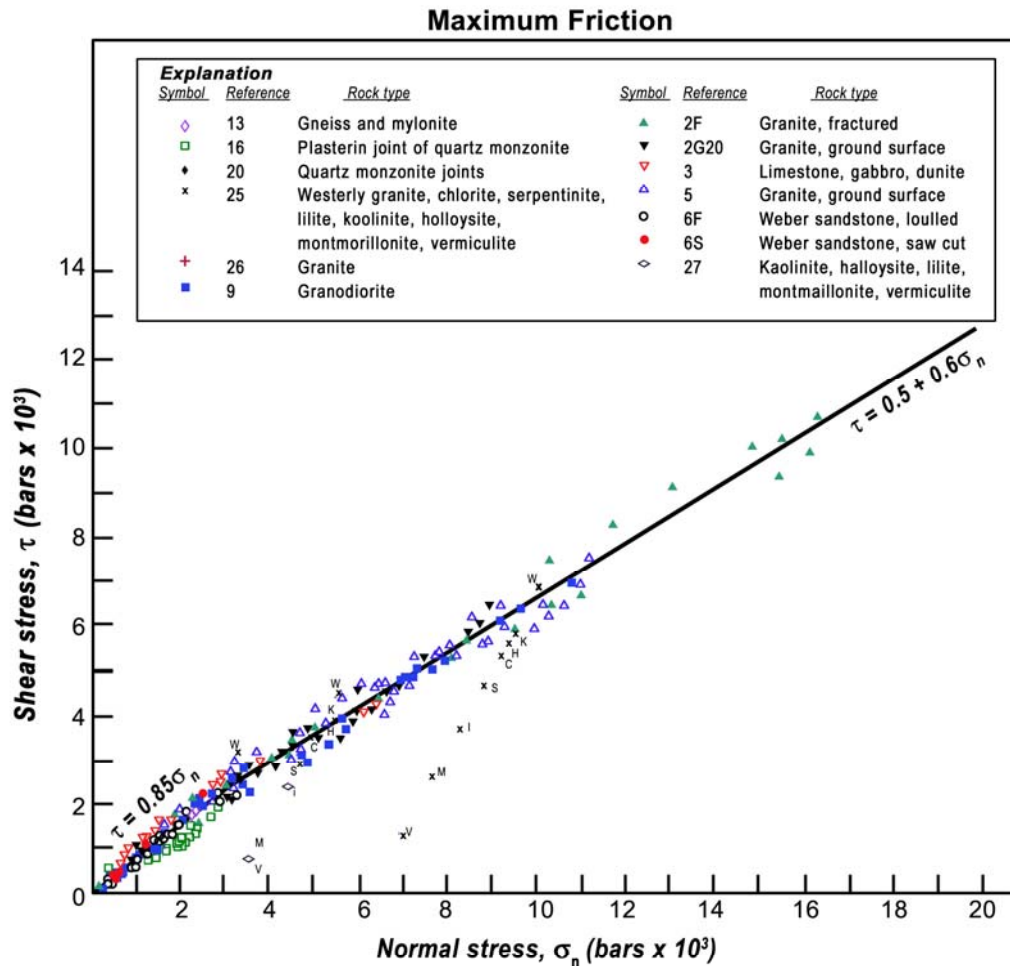


## 12.005 Lecture Notes 6

The coefficients of friction,  $\mu$ , for a wide variety of rocks, are comparable, in the range 0.6 – 0.9. The only rocks with low coefficients of friction are clays, which are not stable at the high pressures and temperatures characteristic of fault zones at depths of  $\sim 10$  km where earthquakes nucleate.



Shear stress plotted as a function of normal stress at the maximum friction for a variety of rock types at normal stresses to 20kb.

Figure 6.1

Note: the Navier-Coulomb failure law,  $|\tau| = \mu|\sigma| + \sigma_0$ , is also sometimes written

$$|\tau| = \mu|\sigma| + S_0.$$

For rocks,  $S_0 \sim 0.1 - 4$  kbars (10 - 400 MPa). How does this "breaking strength compare with the "frictional stress?"

Plot stress predicted for the initiation of faulting as a function of depth, assuming that  $\sigma$  is the lithostatic stress. (Recall that the "lithostatic stress gradient  $\sim 1$  kbar/3 km.)

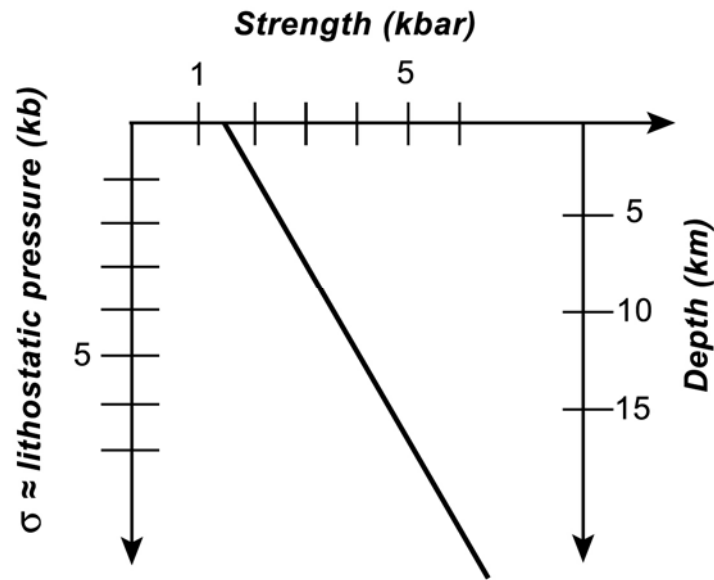


Figure 6.2

So, if Byerlee's law is correct, and if the stress in the lithosphere is not too far from lithostatic, the shear stress on faults should be  $\sim 5$  kbar at 15 km depth and about 250 kbar at 700 km depth.

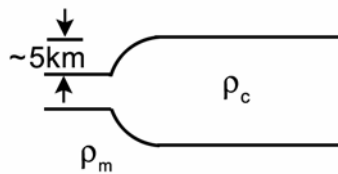


Figure 6.3

How can we test this prediction?

- Model stresses associated with holding up mountains (e.g., Himalayas > 1.5 kbar)
- Stress drops associated with earthquakes (usually ~ 3- 300 bars, very rarely > 1 kbar)
- Work available from convection (dynamic "engine," <~1 kbar)
- Heat flow in fault zones (frictional heating rate ~  $\tau v \Rightarrow \tau \sim 100$  bars)

Big discrepancies here - a frontier region of geodynamics.

Important factor - the role of **pore fluid pressure** in faults.

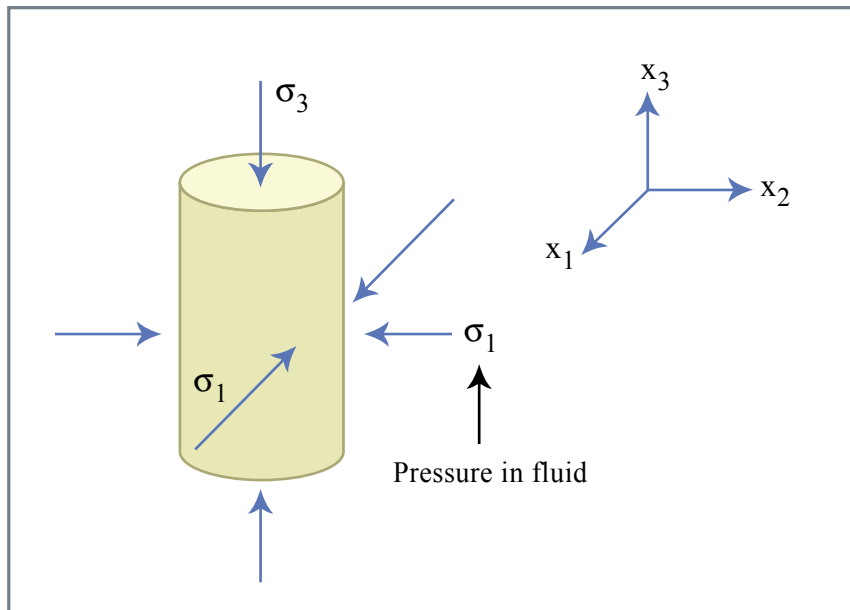


Figure 6.4

Figure by MIT OCW.

What is pressure in sample?

$$u_{ij} = \begin{bmatrix} -\sigma_1 & 0 & 0 \\ 0 & -\sigma_1 & 0 \\ 0 & 0 & -\sigma_3 \end{bmatrix}$$

$$P = \frac{-\text{Tr}(\underline{\underline{\sigma}})}{3} = \frac{\sum_{i=1}^3 \sigma_{ii}}{3}$$

Einstein summation:

If index repeated in a single term  $\Rightarrow$  implicit sum.  $\sum_{i=1}^3 \sigma_{ii} \Rightarrow \sigma_{ii}$

$$\underline{\underline{T}} = \underline{\underline{\sigma}} \cdot \underline{\underline{n}} \quad T_{i\hat{n}} = \sigma_{ij} n_{j\hat{n}} \quad P = \frac{-(2\sigma_1 + \sigma_3)}{3}$$

Consider a common experiment in soil mechanics or rock mechanics: Apply uniaxial stress  $\sigma_3$  to a cylindrical sample confined by  $\sigma_1$ . Continue increasing  $\sigma_3$  until failure.

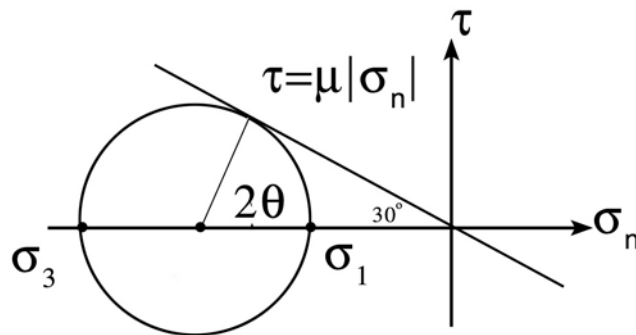


Figure 6.5

$\hat{n}$  of failure plane typically at  $\theta$ ;  $\approx 30^\circ$  from  $\sigma_1$ .

Empirical results: For a pore fluid pressure  $p$ , the Navier-Coulomb failure law becomes:

$$|\tau| = \mu|\sigma + p| + S_0$$

(Note that because compressive pressure is positive, while compressive normal stress  $\sigma$  is negative, the pressure decreases the amplitude of the "effective normal stress"  $|\sigma + p|$ .)

Graphically:

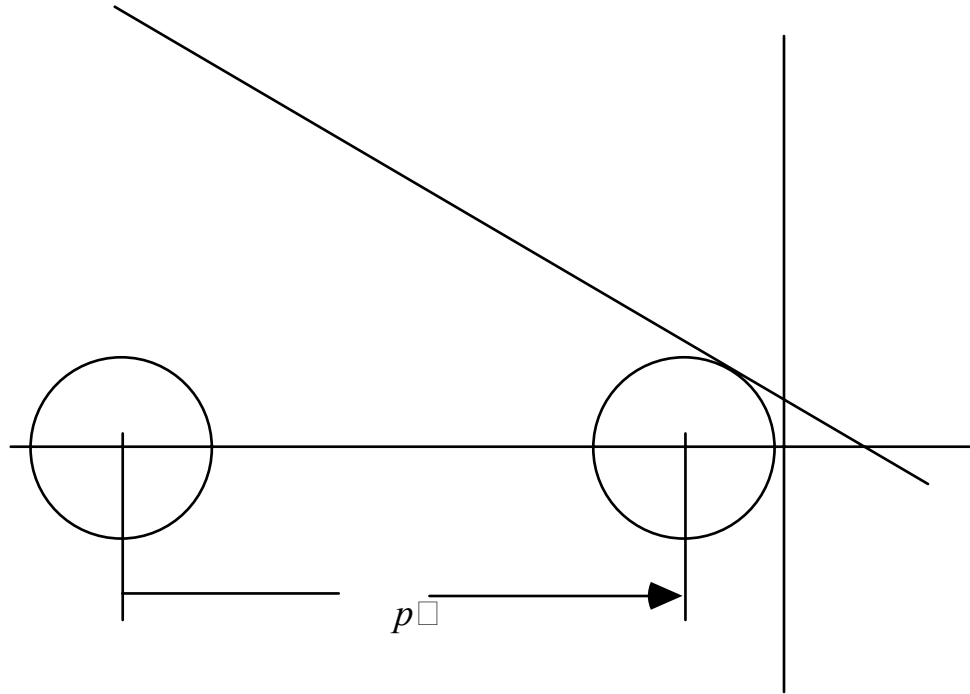


Figure 6.6

A qualitative explanation of the effect of pore fluid pressure is that the fluid helps to "support" some of the normal stress that is otherwise carried by solid grains. Consider a simple model of a continuum made up of dry sand. Let  $f_A$  represent the fraction of a surface area that is made up of solid grain contacts. Then for a macroscopic normal stress  $\sigma$ , the *average* (microscopic) normal stress at the solid grain contacts is  $\sigma/f_A$ , since the pore space in between can support no normal tractions. (Of course, in places the actual value will be much larger than the average value.) If Admonton's law applies to the contacts, then the microscopic shear traction needed to cause failure is  $\mu\sigma/f_A$ , and the macroscopic shear traction is  $\tau = \mu\sigma$ .

If pore fluid pressure is now introduced, the pore fluid will support some of the macroscopic normal stress, leading to a decrease to the normal stress at the grain contacts.

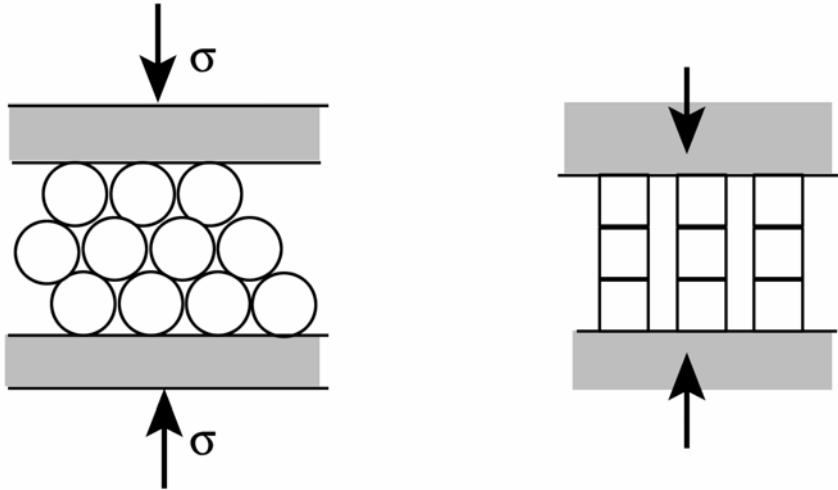


Figure 6.7

Adding pore fluid pressure effectively "stretches" the  $\sigma_n$  axis, giving, effectively, a lower coefficient of friction (except that angles no longer work out!)

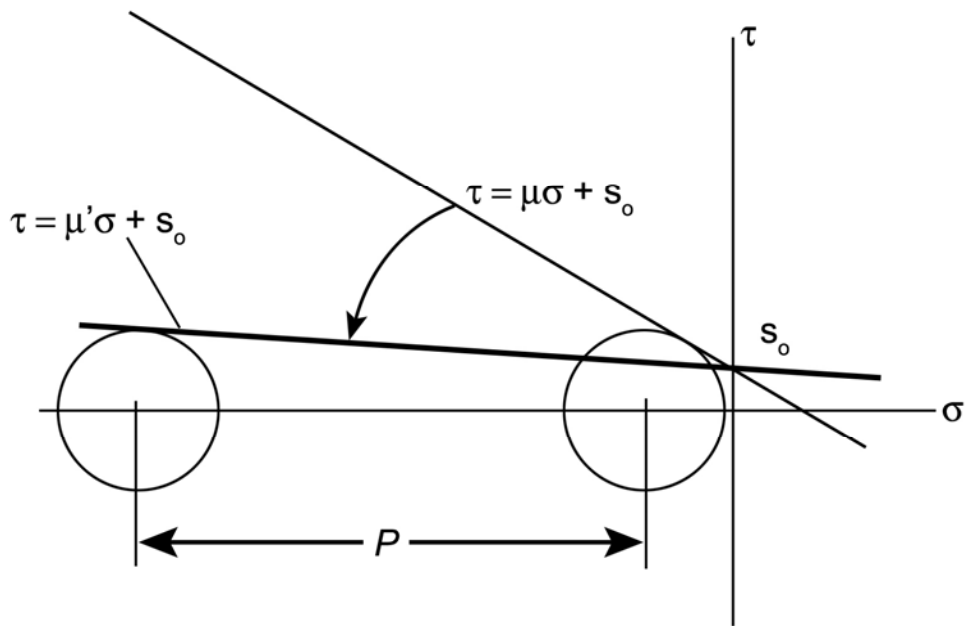


Figure 6.8

An alternative explanation for deep earthquakes is that the failure envelope bends over at large  $\sigma_n$ .

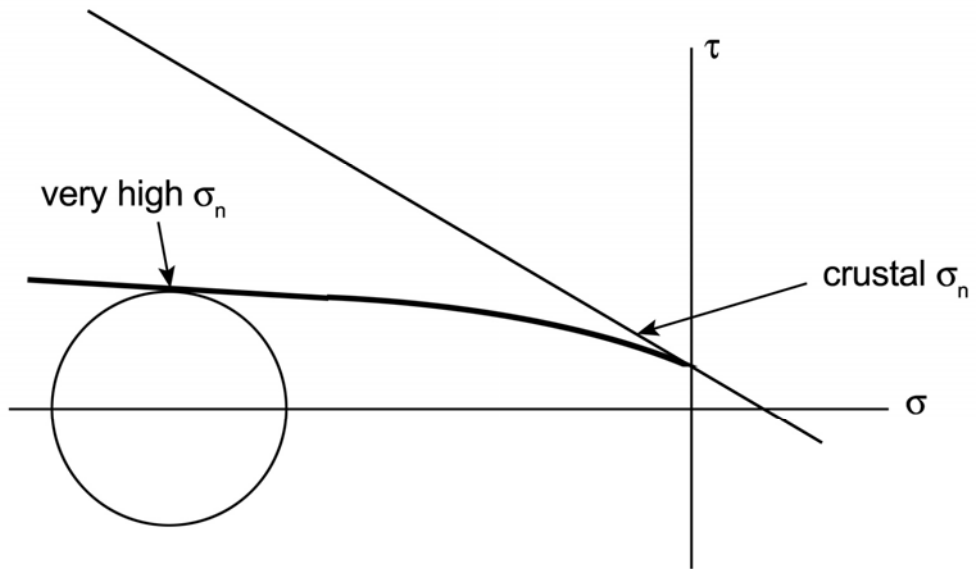


Figure 6.9

Finally, let's consider a medium that is anisotropic – perhaps one which has a preexisting fracture at an angle  $\theta$  to the least principal stress. (For a preexisting fracture, the strength  $S_0 = 0$ .) Then if the double angle  $2\theta$  is within the region shown, the rock will fail along the preexisting fracture.

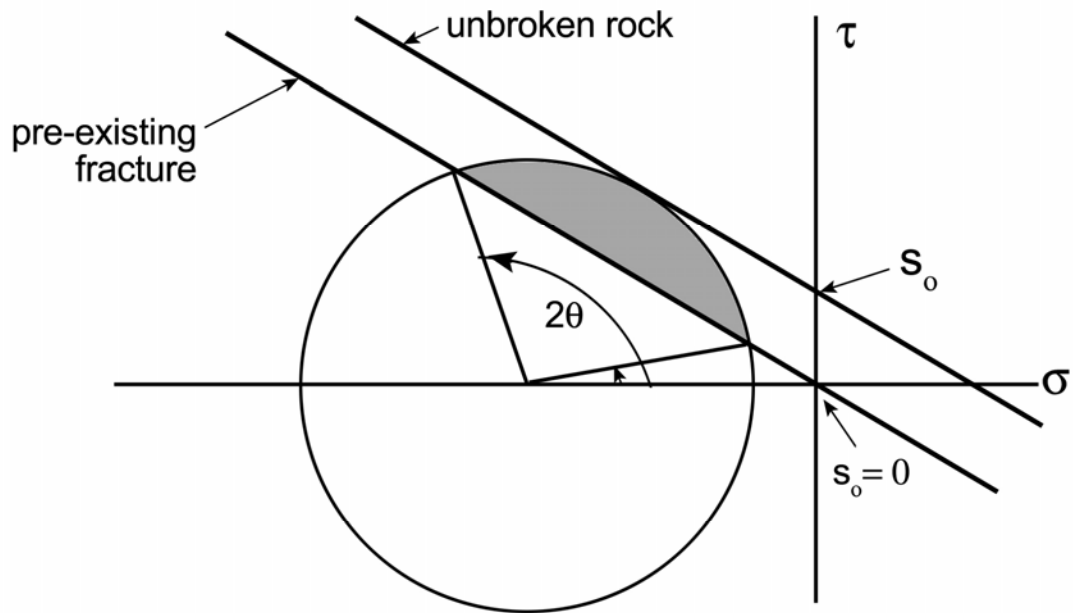


Figure 6.10