12.005: Problem Set 6
5/3/06

Each problem is worth 20%.

1) For the small strains involved in most elastic problems, the mathematics describing the stresses and strains are linear. This means that the solution to a more complicated problem can be found by breaking the problem into smaller, simpler pieces and adding the solutions together. As an example, you can determine the relationship between the bulk modulus and other elastic moduli such as Young's modulus E, the Lamé parameters λ and μ (where $\mu = G$, the shear modulus), and Poisson's ratio ν in the following way: First consider the stresses and strains involved in uniaxial stress along the x_1 axis. That is, consider the experiment used to define Young's modulus E: for $\sigma_{11} = \sigma_0$ and all other $\sigma_{ij}=0$, write each component of the strain tensor, first in terms of λ and μ, then in terms of E and ν.

Next, consider uniaxial stress σ_0 along the x_2 direction. Write each component of the strain tensor, first in terms of λ and μ, then in terms of E and ν.

Next, consider uniaxial stress σ_0 along the x_3 direction. Write each component of the strain tensor, first in terms of λ and μ, then in terms of E and ν.

Finally, add these 3 solutions together to determine the total strains involved when a stress σ_0 is applied along each axis at the same time. From these relations between stress and strain, determine the bulk modulus K, first in terms of λ and μ, then in terms of E and ν.

2) The relationship between stress and strain for a simple isotropic elastic material is:

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu\varepsilon_{ij}$$

(λ and μ are constants, the Lamé parameters, and are called "moduli").

It is sometimes useful to represent ε_{ij}, rather than σ_{ij} , as the independent variable. One way of determining the equation for ε_{ij} in terms of σ_{ij} and σ_{kk} is to first solve the equation above for ε_{kk},then substitute this value for ε_{kk} and solve for ε_{ij}. First, determine the expression for ε_{kk}. Then write an equation equivalent to the one above, but with ε_{ij} on the left hand side and stresses on the right hand side. (The constants multiplying the stresses are called "compliances.")

3) Problem 3-17, Turcotte & Schubert. (See pp 119-120*) – original edition*

4) Problem 3-18, Turcotte & Schubert.

5) Problem 3-19, part b) only, Turcotte & Schubert. *) – original edition* - [Note that part a) of this problem discusses only one of multiple maxima in the bending moment. There are other maxima. You should identify the one with the largest bending stress.]