12.003 Atmosphere, Ocean and Climate Dynamics
Fall 2008

Problem set 6: Atmospheric circulation
Due date: November 12th, 2008

1. Using only the equation of hydrostatic balance and the rotating equation of motion, show that a fluid cannot be motionless unless its density is horizontally uniform. Do not assume geostrophic balance, but assume that a motionless fluid is subject to no frictional forces.

2. Consider a zonally symmetric circulation (i.e., one with no longitudinal variations) in the atmosphere. In the inviscid upper troposphere, one expects such a flow to conserve angular momentum, so that $DA/Dt = 0$, where

$$A = \Omega a^2 \cos^2 \phi + ua \cos \phi$$

   is the absolute angular momentum per unit mass (see Eq. 8-1 in the textbook), Ω is the Earth's rotation rate, u the eastward wind component, a the Earth's radius, and ϕ latitude.

   (a) Show that, for an inviscid zonally symmetric flow, the relation $DA/Dt = 0$ is consistent with the zonal component of the equation of motion (using our standard notation, with $\mathcal{F}_x$ the x-component of the friction force per unit mass)

$$\frac{Du}{Dt} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \mathcal{F}_x,$$

   in (x, y, z) coordinates, where $y = a\phi$ (see Fig. 6.19 in the textbook).

   (b) Use angular momentum conservation to describe how the existence of the Hadley circulation explains the existence of both the subtropical jet stream in the upper troposphere and the near surface trade winds.

   (c) If the Hadley circulation is symmetric about the equator, and its edge is at $20°$ latitudes, determine the strength of the subtropical jet stream.

3. An open dish of water is rotating about the vertical axis at one revolution per minute. Given that the water is 1°C warmer at the edges than at the center, at all depths, estimate, under stated assumptions and using the following data, typical azimuthal flow speeds at the free surface relative to the dish. Comment on, and give a physical explanation for, the sign of the flow. How much does the free surface deviate from its solid body rotation form?

   Briefly discuss ways in which this rotating dish experiment is a useful analogue of the general circulation of the Earth's atmosphere.

   Assume the equation of state for fresh water, $\rho = \rho_{ref}(1 - \alpha[T - T_{ref}])$, with $\rho_{ref} = 1000$ kg m^{-3}, $\alpha = 2 \times 10^{-4}$ K^{-1}, and $T_{ref} = 15$ °, the mean temperature of the water in the dish. The dish has a radius of 10 cm and is filled to a depth of 5 cm.

4. Consider the typical, zonally-averaged eastward flow component, u, shown at
   http://paoc.mit.edu/labweb/atmos-obs/zonalwind.htm
   and the typical overturning atmospheric circulation shown at
   http://paoc.mit.edu/labweb/atmos-obs/overturning.htm.
   Focus on two prominent features of the observed atmospheric circulation: (1)
   the zonal westerly jets at midlatitudes and (2) the Hadley cell circulation in the
   tropics. Discuss how each feature will change if:

   (a) The Earth's rotation rate is twice what it is today.

   (b) The Earth spins clockwise (instead of anticlockwise).

   (c) The Earth does not rotate.

   Give qualitative answers. Do not attempt detailed calculations, because the
   problem can become very messy.

5. Consider the typical, zonally-averaged flow, u, shown here:
   http://paoc.mit.edu/labweb/atmos-obs/zonalwind.htm.
   Concentrate on the vicinity of the subtropical jet near 30°N in winter (DJF). If
   the x-component of the frictional force per unit mass is:

   $$\mathcal{F}_x = \nu \nabla^2 u$$

   where the kinematic viscosity coefficient is $\nu = 1.34 \times 10^{-5}$ m^{2}s^{-1} and

   $$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

   Compare the magnitude of this eastward force with the northward or southward
   Coriolis force and thus convince yourself (and me!) that the frictional force is
   negligible. [Hint: 10° of latitude $\approx$ 1100 km; the jet is at an altitude of about
   10 km. You should find that an oder of magnitude calculation will suffice to
   make the point unambiguously.]

6. In class we showed that friction in the bottom Ekman layers tends to steer large-
   scale atmospheric flows towards low pressure. Derive a detailed solution for the
   flow in the Ekman boundary layers. Assume that the large-scale atmospheric
   flow has a small Rossby number and express the frictional force per unit mass
   as
   $$\mathcal{F}_x = \nu \frac{\partial^2 u}{\partial z^2}, \qquad \mathcal{F}_y = \nu \frac{\partial^2 v}{\partial z^2}.$$
   Solve for the structure of the agesotrophic velocity field as a function of depth.

   (a) The frictional force used in this problem is similar to that used in problem
       1, but the horizontal derivatives in ∇^2 have been dropped. Do you think
       this is a reasonable approximation? Discuss.

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(b) Show that the equation for the ageostrophic flow are:

$$-fv_a = \nu\frac{\partial^2 u_a}{\partial z^2},$$
$$fu_a = \nu\frac{\partial^2 v_a}{\partial z^2}.$$

(c) The full velocity at the bottom of the atmosphere ($z = 0$) must vanish due to friction. Therefore the sum of the geostrophic and ageostrophic velocity must be zero at $z = 0$, $u_a = -u_g$ and $v_a = -v_g$ (for simplicity consider a zonal mean flow with $u_g = U_g = const$ and $v_g = 0$). The ageostrophic velocity must also vanish as $z \to \infty$ because friction becomes less and less important as you get into the interior of the atmosphere. Using these bottom and top boundary conditions show that solutions of the equations above have the form,

$$u_a = V_0 e^{z/\delta}\cos(z/\delta),$$
$$v_a = V_0 e^{z/\delta}\sin(z/\delta).$$

Express the parameters δ and V_0 in terms of ν, f and U_g.

(d) Sketch the direction of the ageostrophic velocity at different depths in the atmosphere.

(e) Take the vertical integral of u_a and v_a between $z = 0$ and $z = \infty$. In which direction does the integrated ageostrophic transport go?

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