Homework problems on Fluid Dynamics

(1.63J/2.21J)

Chiang C. Mei, 2002

MT-fanjet.tex.

2. **Jet from a broken pipe.**

Consider a broken circular pipe the interior of which is filled with air at high pressure. There is a slit from which air leaks and forms a fan jet outside. Because of the strong initial momentum at the exit, the moving fluid is thin in the z direction and the radial velocty is much greater than the vertical velocity. For sufficiently great r the radius of the pipe can be neglected. Assume axial symmetry. so that

$$\frac{\partial}{\partial\theta} = 0, q_\theta = v = 0, \quad \text{but} \quad q_r = u, q_z = w \neq 0 \tag{1}$$

the exact N-S equations are

$$\frac{\partial(ur)}{\partial r} + \frac{\partial(rw)}{\partial z} = 0 \tag{2}$$

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{r^2}\right) \tag{3}$$

$$u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right) \tag{4}$$

1. for $u \gg w, r \gg \delta$, deduce th approximate boundary layer equations in the jet.

2. Use the given jet momentum at the exit as one of the boundary conditions and show that a similarity solution exists. Deduce the form of the similar soitiuon and infer from it the variation of the boundary layer thickness and the maximum radial velocity at the center plane $z = 0$ i.e., $u(r, 0)$.

3. Solve the problem explicitly and discuss the physical implications.

4. Observe that the total mass flux accross a circular cylindrical contraol surface is not constant. Why?

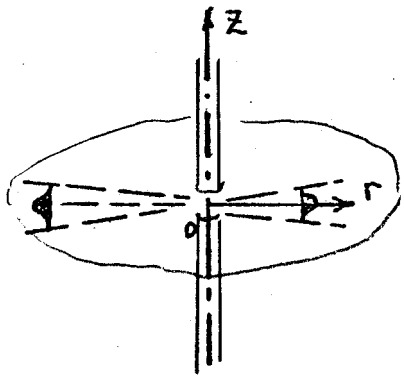

Figure 1: A fan jet