Vehicle Scheduling

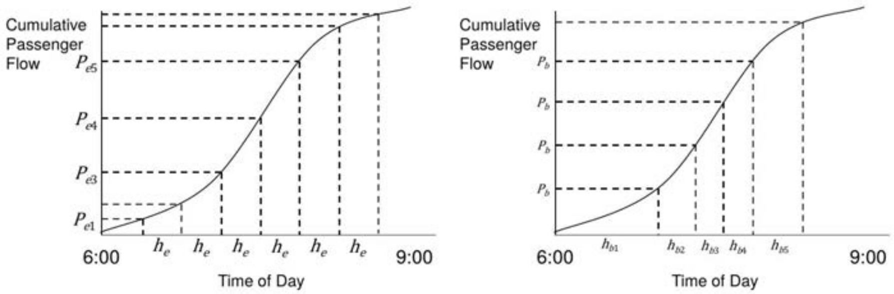
Outline

- Timetable Development
- Fleet Size Calculation
- Vehicle Scheduling

Timetable Development

Translating frequency into timetable by specifying headways as:

- equal – appropriate if demand is uniformly distributed across period
- balanced load – appropriate if there is substantial variation in demand over period
  - e.g. peak of the peak
- clockface or not – do headways repeat every hour?



Timetable Development

If we have *N* departures in the peak period:

equal headway solution $$H = \frac{\text{Peak Period Duration}}{N}$$

balanced load solution $$\text{Load} = \frac{\text{Total Passenger Flow}}{N}$$

More vehicles might be required for a balanced load solution.

Fleet Size Required

Salzborn's Fleet Size Theorem

Given:

$l(k,t,s)$ = # of departures from terminal *k* by time *t* following schedule *s*

$a(k,t,s)$ = # of arrivals at terminal *k* by time *t* following schedule *s*

and:

$d(k,t,s)$ = $l(k,t,s) - a(k,t,s)$

deficit function at terminal *k* at time *t* following schedule *s*

Assumptions

- no trip shifting
- no deadheading (to balance deficit among terminals)

Fleet Size Required

Salzborn's Fleet Size Theorem

Then:

$N(s)$, the minimum size fleet to serve schedule s, is given by:

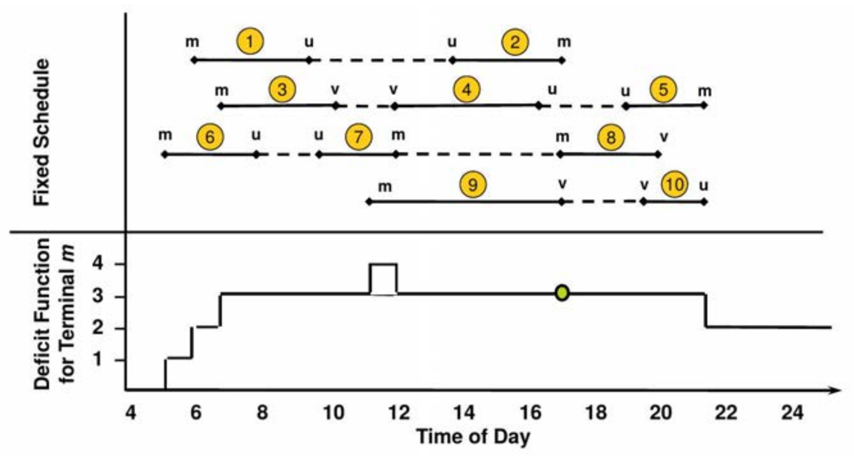
$$N(s) = \sum_{k \in K} \max_{t} \big(d(k,t,s)\big)$$

Where K is the number of terminals.

Fleet Size Required

The deficit function, or minimum required fleet size, may be reduced by:

- shifting departure and/or arrival times
- adding deadhead trips between terminals



Vehicle Scheduling Problem

Objective

- Define vehicle blocks (sequences of revenue and non-revenue activities for each vehicle) covering all trips so as to:
  - minimize fleet size
  - minimize non-revenue time and mileage

Observation

- These are proxies for cost, but a large portion of cost will depend on crew duties which are unknown at this stage of solution.

Vehicle Scheduling Problem

Input

- A set of vehicle revenue trips to be operated, each characterized by
  - starting point and time
  - ending point and time
- Possible *layover arcs* between the end of a trip and the start of a (later) trip at the same location
- Possible *deadhead arcs* connecting
  - depot(s) to trip starting points
  - trip ending points to depot(s)
  - trip ending points to trip starting at a different point

Vehicle Scheduling Problem

Observations

- there are many feasible but unattractive deadhead and layover arcs, so it is best to generate only plausible non-revenue arcs
- layover time affects service reliability, so set minimum layover (recovery) time

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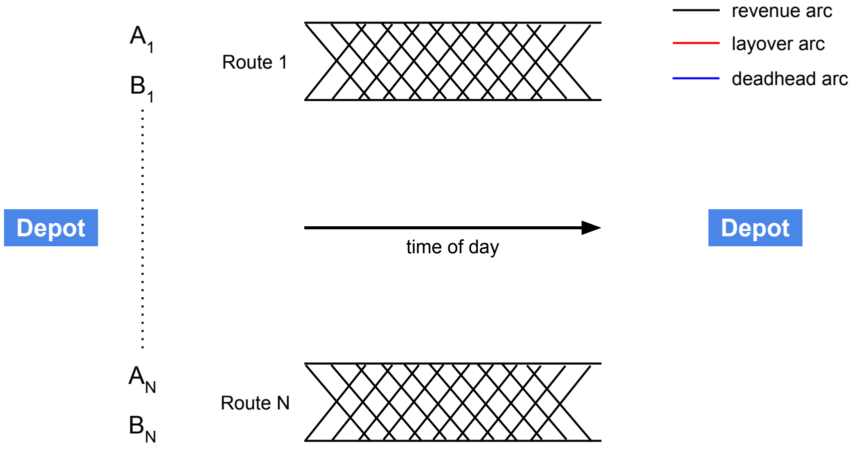
Vehicle Scheduling Problem

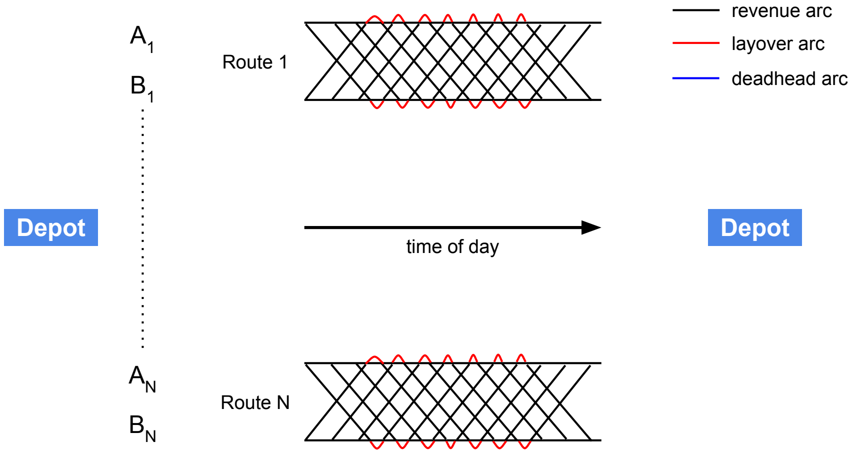
Variations

- each vehicle restricted to a single line vs. interlining permitted
- single depot vs. multi-depot
- vehicle fleet size constrained at depot level
- routes (trips) assigned to specific depot
- multiple vehicle types

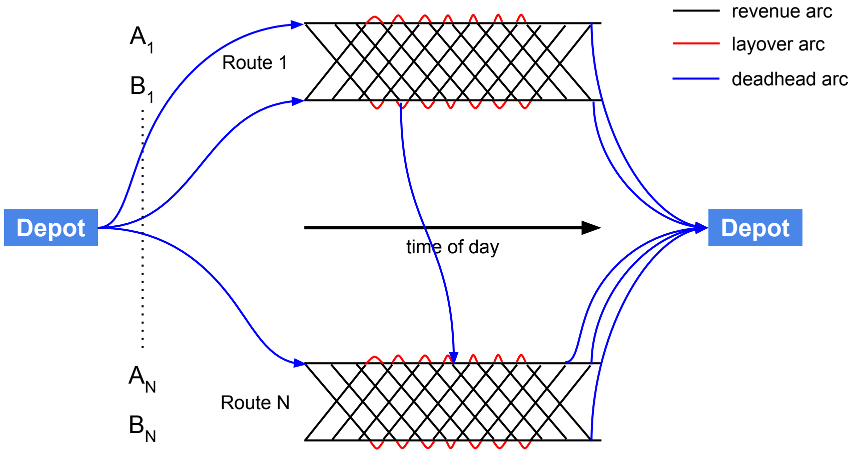
Time-Space Network Representation



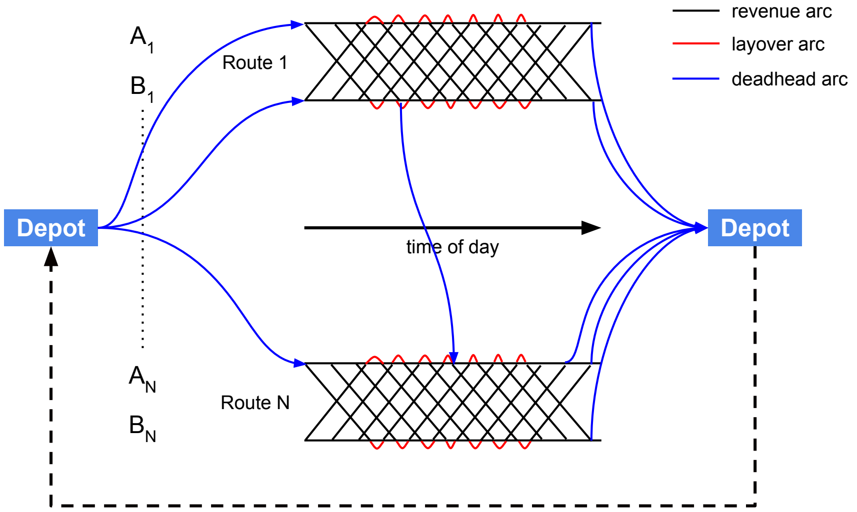
Time-Space Network Representation



Time-Space Network Representation



Time-Space Network Representation



Mathematical Modelling: Network Flows

- The vehicle scheduling problem can be modelled as a *minimum cost network flow problem*, with arcs representing trips.
- Arcs have lower and upper bound constraints on flow
  - revenue arc
  - layover arc
  - deadhead arc
- Arcs have cost
  - revenue arc
  - layover arc
  - deadhead arc

Mathematical Modelling: Network Flows

- The vehicle scheduling problem can be modelled as a *minimum cost network flow problem*, with arcs representing trips.
- Arcs have lower and upper bound constraints on flow

revenue arc	l = 1	u = 1
layover arc	l = 0	u = 1
deadhead arc	l = 0	u = 1

- Arcs have cost

revenue arc	cost irrelevant
layover arc	driver cost of extra layover time
deadhead arc	driver & vehicle cost of deadhead

Mathematical Modelling: Network Flows

- Vehicle Scheduling Problem (VSP)
  - find a set of feasible vehicle blocks such that
    - each trip in the timetable is covered exactly once
    - total cost is minimized
      - layover arcs
      - deadhead arcs
    - maximum block length
    - flow conservation
  - significant cost reductions with interlining and trip shifting heuristics
- Single Depot Vehicle Scheduling Problem (SDVSP)
  - for smaller agencies
  - solvable in polynomial time (minimum cost network flow)
- Multidepot Vehicle Scheduling Problem (MDVSP)
  - for large agencies
  - (integer multicommodity flow)
  - NP-hard (trip-depot compatibility constraints)
  - exact algorithms exist, but heuristics are used in practice
  - SDVSP used to find suboptimal solution, or as a sub-problem

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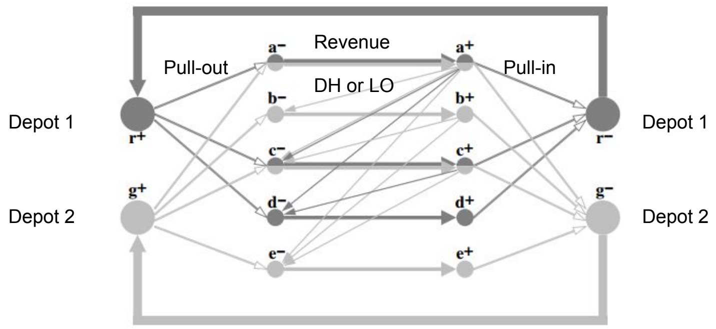
Network Representation (MDVPS)



Conservation of flow at depots

Löbel, Andreas. "Solving large-scale multiple-depot vehicle scheduling problems." In *Computer-aided transit scheduling*, pp. 193-220. Springer Berlin Heidelberg, 1999.

Heuristic Approaches

1) Define compatible trips at same terminal *k* such that trips *i* and *j* are compatible if and only if:
   - $M_k < t_{sj} - t_{ei} < 2 D_k$

   where

   - t_{sj} = starting time for trip j
   - t_{ei} = ending time for trip i
   - M_k = minimum recovery/layover time at terminal k
   - D_k = deadhead time from terminal k to depot

2) Apply restricted first-in-first-out rules at each terminal

   Step a) Start with (next) earliest arrival at terminal; if none, go to step (d)
   Step b) Link to earliest compatible trip departure; if none, return vehicle to depot and return to step (a)
   Step c) Check vehicle block length against constraint:
   i) if constraining, return vehicle to depot and return to step (a)
   ii) otherwise, return to step (b) with new trip arrival time
   Step d) Serve all remaining unlinked departures from depot

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Single Route Scheduling Practice

Results of earlier planning and scheduling analysis:

	AM Peak Period	Base Period
	6:00 - 9:00	after 9:00
Headways	20 min	30 min
Scheduled Trip Time A→B or B→A	40 min	35 min
Minimum Layover Time	10 min	10 min

Dominant direction of travel in AM Peak is A→B

Timetable and Vehicle Block Development

Depart A	Arrive B
6:00	6:40
6:20	7:00
6:40	7:20
7:00	7:40
7:20	8:00
7:40	8:20
8:00	8:40
8:20	9:00
8:40	9:20
9:00	9:35
9:30	10:05
10:00	10:35
10:30	11:05
11:00	11:35

Timetable and Vehicle Block Development

Depart A	Arrive B	Depart B	Arrive A
6:00	6:40	6:50	7:30
6:20	7:00	7:10	7:50
6:40	7:20	7:30	8:10
7:00	7:40	7:50	8:30
7:20	8:00	8:10	8:50
7:40	8:20	8:30	9:10
8:00	8:40	8:50	9:30
8:20	9:00	9:15	9:50
8:40	9:20		
9:00	9:35	9:45	10:20
9:30	10:05	10:15	10:50
10:00	10:35	10:45	11:20
10:30	11:05	11:15	11:50
11:00	11:35	11:45	12:20

Timetable and Vehicle Block Development

Veh #	Depart A	Arrive B	Depart B	Arrive A
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	6:20	7:00	7:10	7:50
	6:40	7:20	7:30	8:10
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	7:20	8:00	8:10	8:50
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	8:20	9:00	9:15	9:50
	8:40	9:20		
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x = pull out (from depot)

y = pull in (back to depot)

Timetable and Vehicle Block Development

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	8:40	9:20		
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Timetable and Vehicle Block Development

Veh #	Depart A	Arrive B	Depart B	Arrive A
1x	6:00	6:40	6:50	7:30
2x	6:20	7:00	7:10	7:50
	6:40	7:20	7:30	8:10
	7:00	7:40	7:50	8:30
	7:20	8:00	8:10	8:50
1	7:40	8:20	8:30	9:10
2y	8:00	8:40	8:50	9:30
	8:20	9:00	9:15	9:50
	8:40	9:20		
	9:00	9:35	9:45	10:20
1	9:30	10:05	10:15	10:50
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1x	6:00	6:40	6:50	7:30
2x	6:20	7:00	7:10	7:50
3x	6:40	7:20	7:30	8:10
	7:00	7:40	7:50	8:30
	7:20	8:00	8:10	8:50
1	7:40	8:20	8:30	9:10
2y	8:00	8:40	8:50	9:30
3	8:20	9:00	9:15	9:50
	8:40	9:20		
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1x	6:00	6:40	6:50	7:30
2x	6:20	7:00	7:10	7:50
3x	6:40	7:20	7:30	8:10
4x	7:00	7:40	7:50	8:30
	7:20	8:00	8:10	8:50
1	7:40	8:20	8:30	9:10
2y	8:00	8:40	8:50	9:30
3	8:20	9:00	9:15	9:50
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3x	6:40	7:20	7:30	8:10
4x	7:00	7:40	7:50	8:30
5x	7:20	8:00	8:10	8:50
1	7:40	8:20	8:30	9:10
2y	8:00	8:40	8:50	9:30
3	8:20	9:00	9:15	9:50
4y	8:40	9:20		
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Timetable and Vehicle Block Development

Block 1: Depot - A (6:00) - B (6:50) - A (7:40) - B (8:30) - A (9:30) - B (10:15) - A (11:00) - B (11:45) - . . .

Block 2: Depot - A (6:20) - B (7:10) - A (8:00) - B (8:50) - Depot

Block 3: Depot - A (6:40) - B (7:30) - A (8:20) - B (9:15) - A (10:00) - B (10:45) - . . .

Block 4: Depot - A (7:00) - B (7:50) - A (8:40) - Depot

Block 5: Depot - A (7:20) - B (8:10) - A (9:00) - B (9:45) - A (10:30) - B (11:15) - . . .

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