Performance Models

Outline

1. Wait time models
2. Service variation along route
3. Running time models
4. Dwell time models

Wait Time Models

Simple deterministic model

$$\mathrm{E}[W] = \frac{\mathrm{E}[h]}{2}$$

where

$\mathrm{E}[W]$ = expected waiting time

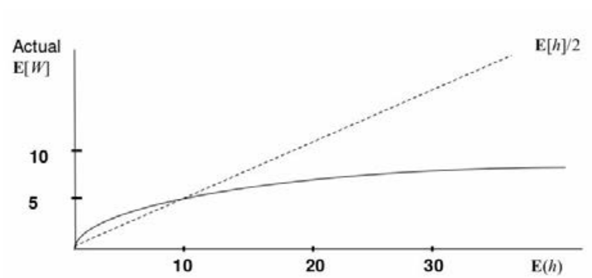
$\mathrm{E}[h]$ = expected headway

Model assumptions

- passenger arrival times are independent of vehicle departure times
- vehicles depart deterministically at equal intervals
- every passenger can board the first vehicle to arrive

Passenger Arrival Process

- Individual, group, and bulk passenger arrivals
- Passengers can be classified in terms of arrival process
  - random arrivals
  - time arrival to minimize E[W]
  - arrive with the vehicle, i.e. have W = 0



Vehicle Departure Process

Vehicle departures typically not regular and deterministic

Wait Time Model refinement:

$n(h)$ = # of passengers arriving in a headway h

$\bar{w}(h)$ = mean waiting time for passengers arriving in headway h

$g(h)$ = probability density function of headway

$$\mathrm{E}[W] = \frac{\text{Expected Total Passenger Waiting Time per Vehicle Departure}}{\text{Expected Passengers per Vehicle Departure}}$$

$$\mathrm{E}[W] = \frac{\int_0^\infty n(h) \cdot \bar{w}(h) \cdot g(h) \cdot \mathrm{d}h}{\int_0^\infty n(h) \cdot g(h) \cdot \mathrm{d}h}$$

Vehicle Departure Process

$n(h) = \lambda h$ where λ is the passenger arrival rate

$$\bar{w}(h) = \frac{h}{2}$$

$$\mathrm{E}[W] = \frac{\mathrm{E}[h^2]}{2\mathrm{E}[h]} = \frac{\mathrm{E}[h]}{2}\left[1 + \frac{\mathrm{Var}[h]}{(\mathrm{E}[h])^2}\right] = \frac{\mathrm{E}[h]}{2}[1 + c_h^2]$$

where c_h is coefficient of variation of headway

Vehicle Departure Process Examples

If $\mathrm{Var}[h] = 0$ $\quad \mathrm{E}[W] = \frac{\mathrm{E}[h]}{2}$

If vehicle departures are as in a Poisson process

$\mathrm{Var}[h] = (\mathrm{E}[h])^2 \quad \mathrm{E}[W] = \mathrm{E}[h]$

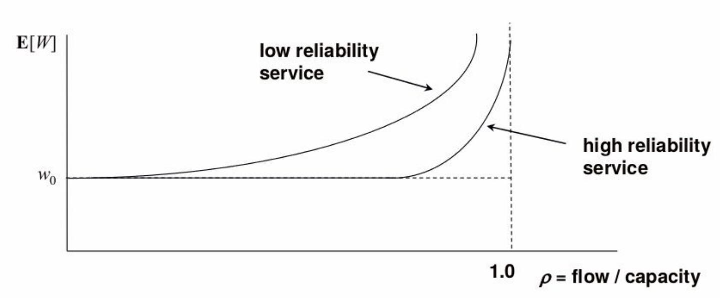
If the headway sequence is 5, 15, 5, 15, ...

$\mathrm{E}[h] = 10$

$$\mathrm{E}[W] = 2.5 \cdot 0.25 + 7.5 \cdot 0.75 = 6.25 \text{ minutes}$$

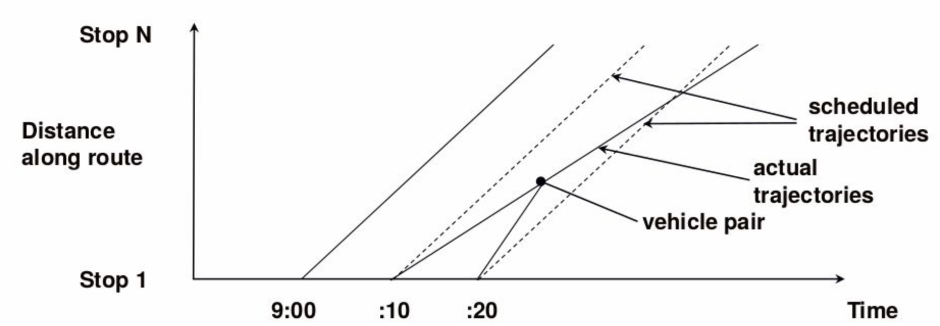
Passenger Loads Approach Vehicle Capacity

- Not all passengers can board the first vehicle to depart:

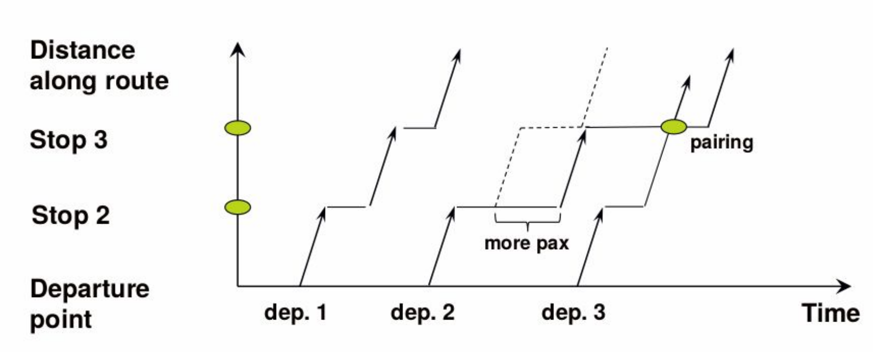


Service Variation Along Route

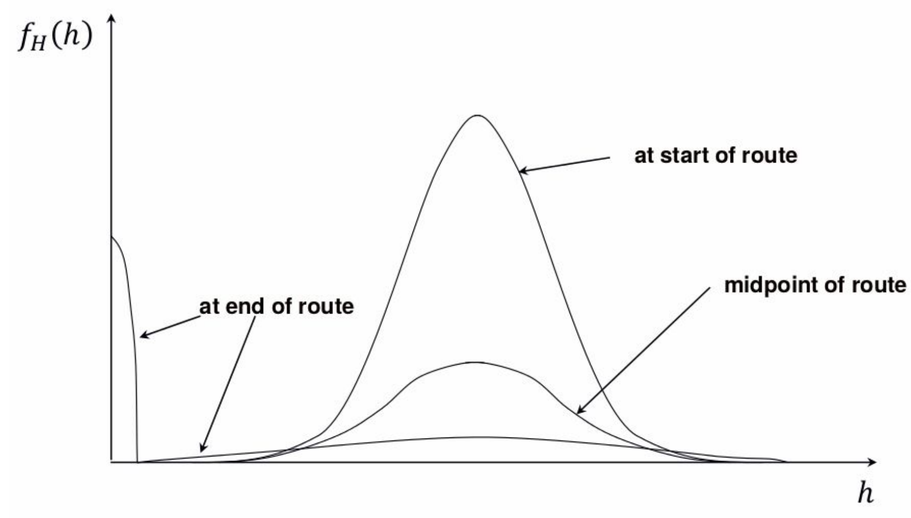
- Service may vary along route even without capacity becoming binding:
  - the headway distribution can vary along the route, affecting E[W]
  - at the limit vehicles can be paired, or bunched
  - this can also result in passenger load variation between vehicles



Service Variation Along Route



Service Variation Along Route



Factors Affecting Headway Deterioration

- Length of route
- Marginal dwell time per passenger
- Stopping probability
- Scheduled headway
- Driver behavior

Simple model $e_i = (e_{i-1} + t_i)(1 + p_{i-1} b)$

where

e_i = headway deviation (actual − scheduled) at stop *i*

t_i = travel time deviation (actual − scheduled) from stop *i* − 1 to *i*

p_i = passenger arrival rate at stop *i*

b = boarding time per passenger

Mathematical Model for Headway Variance

$$\begin{aligned}\text{var}(h_i) = {} & \text{var}(h_{i-1}) + \text{var}(\Delta t_{i-1}) + 2p_{i-1}(1-p_{i-1})(c \cdot \bar{q}_{i-1} + \ell)^2 \\ & + 2c^2 \text{var}(q_{i-1})\left[1 - \rho_q + p_{i-1}\rho_q\right](1 - p_{i+1}) \\ & + c(1-p_{i-1})^2 \cdot \text{cov}(\Delta q_{i-1}, h_{i-1})\end{aligned}$$

where:

$\text{var}(h_i)$ = headway variance at stop i

$\text{var}(\Delta t_i)$ = variance of the difference in running time between successive buses between stops $i-1$ and i

p_i = probability bus will skip stop i

c = marginal dwell time per passenger served at a stop

$\bar{q}_i$ = mean number of passengers per bus served at stop i

ℓ = the constant term of the dwell time function

$\text{var}(q_i)$ = variance of the number of passengers served per bus at stop i

ρ_q = correlation coefficient between the passengers served by successive buses at a stop

$\text{cov}(\Delta q_i, h_i)$ = covariance of the difference in number of passengers served by successive buses and the headway at stop i

* Adebisi, O., "A Mathematical Model for Headway Variance of Fixed Bus Routes." Transportation Research B, Vol. 20B, No. 1, pp 59-70 (1986).

Vehicle Running Time Models

Different levels of detail

- Very detailed, microscopic simulation
  - represents vehicle motion and interaction with other vehicles
    - buses operating in mixed traffic
    - train interaction through control system
- Macroscopic
  - identify factors which might affect running times
  - collect data and estimate model

Vehicle Running Time Models

- Running time includes dwell time, movement time, and delay time
  - dwell time is generally a function of number of passengers boarding and alighting as well as technology characteristics
  - movement time and delay depend on other traffic and control system attributes
- Typical bus running time breakdown in mixed traffic
  - 50-75% movement time
  - 10-25% stop dwell time
  - 10-25% traffic delays including traffic signals

Dwell Time Models

- Dwell Time Theory
- Bus Dwell Time Model
  - Milkovits, M.N., “Modeling the Factors Affecting Bus Stop Dwell Time: Use of Automatic Passenger Counting, Automatic Fare Counting, and Automatic Vehicle Location Data." Transportation Research Record: Journal of the Transportation Research Board, pp pp 125-130 (2008).
- Light Rail Dwell Time Model
  - Wilson, N.H.M. and T. Lin, "Dwell-Time Relationships for Light Rail Systems," Transportation Research Record #1361, 1993, pp. 296-304.
- Heavy Rail Dwell Time Model
  - Puong, A., "Dwell Time Model and Analysis for the MBTA Red Line." Internal memo, MIT, March 2000.

Dwell Time Theory

- Vehicle dwell time affects
  - system performance
  - service quality
- A critical element in vehicle bunching resulting in
  - high headway variability
  - high passenger waiting times
  - uneven passenger loads
- Dwell time impact on performance depends on:
  - stop/station spacing
  - mean dwell as proportion of trip time
  - mean headway
  - operations control procedures
- Examples
  - Commuter rail → little impact of dwell time on performance
  - Long, high-frequency bus route → major impact

Dwell Time Theory

- Dwell time depends on many factors
  - human
  - modal
  - operating policies & practices
  - weather
- For a given system we have the following possible models
  - Single door, no congestion and interference
    DOT = a + b(DONS) + c(DOFFS)
  - Single door with congestion and interference
    DOT = a + b(DONS) + c(DOFFS) + d(DONS+DOFFS)(STD)
  - Single car with m doors
    DT = max(DOT_1 , … , DOT_m)
  - Single car with m doors, with balanced flows
    DT = a + b/m(CONS) + c/m(COFFS) + d/m(CONS+COFFS)(STD)
  - n-car train
    DT = max(DT1, ..., DTn)
  - n-car train, with balanced flows
    DT = a + b/nm(TONS) + c/nm(TOFFS) + d/nm(TONS+TOFFS)(STD)

Bus Dwell Time: Prior Work

- Manually collected data
  - Limited data on infrequent events
  - Crowding
  - Do not include latest fare media
- Automatically collected data
  - Does not include fare media information
  - Poor fit of model
- Transit Capacity and Quality of Service Manual
  - Assumes a half-second penalty per passenger for crowding

Milkovits (2008)

Objective

- Develop a dwell time model using automatically collected data
- Dwell time factors
  - Boarding and alighting passengers
  - Onboard passengers
  - Fare media type
  - Alighting door selection
  - Bus type
- Minimize the unexplained variation in dwell time
- Evaluate impact on dwell time of:
  - fare media type
  - bus design
  - enforcement of rear-only alightings

Milkovits (2008)

Data Set

- Automatically collected data from Chicago Transit Authority bus network
- Non-Timepoint, Far-Side, Known Stops
- Functioning APC counters on all doors
  - Verified by non-zero counts across day
  - Minimum per-passenger dwell time of .5 seconds
- Link-in AFC transactions
  - Fare transactions that take place within the dwell time
- Data from entire month of November 2006
  - 173,750 Records
  - 2,977 Operators
  - 85 Routes
  - 927 Stops

Milkovits (2008)

Model Formulation

- Predict dominant door activity
- Segment data and compare by:
  - Bus type
  - Crowding (passengers > number of seats)
- Combine the data and test for significant differences in the estimators

Milkovits (2008)

Dwell Time Estimates – Front Door

				Adjusted R^2: 0.73
Variable	**DUMMY**	**est**	**t-stat**	**Passenger Levels**
intercept		-1.22	-26.49	All
NABI		0.53	7.81	
FON_EX		3.68	154.17	
	NOVA	0.38	10.51	
	NABI	-0.59	-11.32	
FOFF3UP		1.52	26.22	
CARDS		2.62	10.15	Open
TICKET		4.88	39.55	
	NFLYER	-0.58	-3.62	
FOFF12		2.83	104.59	
F_SENSOR		4.60	21.55	
AFC_TRANS		4.35	15.54	Crowded
FOFF12		3.52	22.54	
	NFLYER	-0.74	-3.71	
ST2_PASS		0.0011	5.56	
	NFLYER	0.0017	3.53	

Milkovits (2008)

Dwell Time Estimates – Rear Door

				Adjusted R^2: 0.37
Variable	**DUMMY**	**est**	**t-stat**	**Passenger Levels**
Intercept		1.42	22.49	All
	NABI	2.64	21.26	
ROFF		1.69	40.86	
	NOVA	0.42	7.47	
	NABI	-0.42	-5.37	
ST2_PASS		0.005	5.64	Crowded
	NOVA	0.004	2.11	
	NABI	-0.003	-3.36	

Milkovits (2008)

Bus Dwell Time Model: Key Findings

- Smart media loses benefit in crowded conditions
  - Drops from 2 second advantage in non-crowded conditions
- Crowding impact increases exponentially
- Bus attributes impact dwell time
  - Location of magnetic stripe reader (half second difference)
  - Double-wide doors
- Front door alightings may affect dwell time, while rear door alightings will happen in parallel

Milkovits (2008)

MBTA Green Line Analysis

- Branching network of 28 miles (45 km) and 70 stations
- 52-seat ALRVs operate in 1-, 2-, and 3-car trains
  - high floor, low platform configuration
  - 3 doors per car on each side
  - single side boarding/alighting
- Trunk service in central subway:
  - 10 or 14 stations on round-trip
  - 1- to 2-minute headways
  - peak flows ≈10,000 passengers/hour

Wilson and Lin (1993)

Models with Crowding Term

- One-car trains
  - DT = 12.50 + 0.55*TONS + 0.23*TOFFS + 0.0078*SUMASLS
    (8.94) (3.76) (2.03) (6.70)
    $R^2 = 0.62$
  - SUMASLS = TOFFS*AS + TONS*LS
- Two-car trains
  - DT = 13.93 + 0.27*TONS + 0.36*TOFFS + 0.0008*SUMASLS
    (7.43) (2.92) (3.79) (2.03)
    $R^2 = 0.70$

Wilson and Lin (1993)

Predicted Dwell Times

ONS	LPL	1-Car DT	2-Car DT
0	any #	12.5	13.9
10	< 53	20.3	20.2
10	150	35.6	21.0
20	< 53	28.1	26.5
20	150	58.7	28.1
30	< 53	35.9	32.8
30	150	81.8	35.1

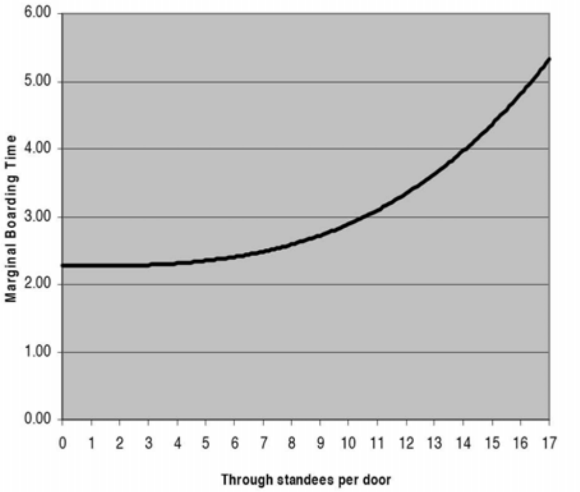
Wilson and Lin (1993)

Findings

- Dwell times for ALRVs are quite sensitive to:
  - Passenger flows
  - Passenger loads
- The crowding effect may well be non-linear.
- Dwell times for multi-car trains are different from those for one-car trains.
- The dwell time functions suggest high sensitivity of performance to perturbations
- Effective real-time operations control essential
- Running mixed train lengths dangerous
- Simulation models of high frequency, high ridership light rail lines need to include realistic dwell time functions.

Wilson and Lin (1993)

Heavy Rail Marginal Boarding Time



Puong (2000)

Heavy Rail Dwell Time Function

$$DT = 12.22 \underset{(12.82)}{} + 2.27 \cdot B_d \underset{(7.11)}{} + 1.82 \cdot A_d \underset{(9.07)}{} + 6.2\ 10^{-4} \cdot TS_d^3 B_d \underset{(4.70)}{} \quad (\bar{R}^2 = 0.89) \quad (9)$$

(12.82) (7.11) (9.07) (4.70)

where

A_d = alighting passengers *per door*,
B_d = boarding passengers *per door*, and
TS_d = through standees per door,
i.e., total through standees divided by the number of doors

Puong (2000)