1.225J (ESD 205) Transportation Flow Systems

Lecture 2

Modeling Air Traffic Flows

Prof. Ismail Chabini
Prof. Amedeo R. Odoni

Lecture 2 Outline

- Background
- Factors that 'Determine' a Runway Capacity
- Minimum Time Separation between Two Arriving Aircrafts
- Arrival Capacity Model for One Runway
- Example
- Practical Issues and Model Analysis
- Other Models
- Summary

Background

- Two main types of air transportation infrastructure:
  - Airports
  - Air Traffic Control (ATC)
- Airport services and facilities:
  - Airside or Airfield (runways, taxiways, hangers)
  - Landside (terminals, parking areas, access roads)
- Models deal with issues related to:
  - Passenger terminals
  - Runway and taxiway systems
  - ATC
- Types of models:
  - Analytical (i.e., the model of this lecture)
  - Simulation (will be covered in a later lecture)

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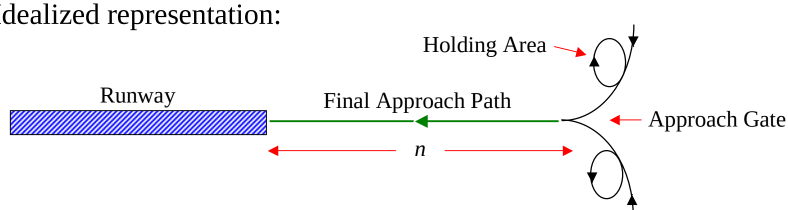
Arrival runway capacity is (partially) determined by:

- Rules of Traffic Flow:
  - Minimum separation between two successive aircraft
  - Only one aircraft can be on the runway at any time
- Aircraft Population:
  - Types: Heavy, Large/Medium, Small
  - Mix: Percentage of each type

Arrival Capacity Model of a Single Runway

- Idealized representation:



- Notation:
  - n: length of final approach path
  - i (j): type of leading (trailing) aircraft
  - v_i: ground speed of type i aircraft
  - o_i: runway occupancy time of type i aircraft
  - s_{ij}: minimum separation between two airborne aircraft
  - T_{ij}: minimum acceptable time interval between successive arrivals at the runway of type i and type j aircrafts (unknown)

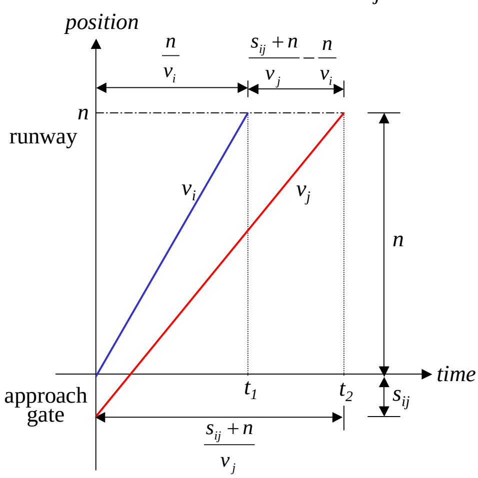
Minimum Time Separation of Two Aircrafts

- Minimum time separation is a consequence of:
  - Minimum space separation must not be violated
  - Only a single aircraft can be on the runway at a given time instance
- Question: What is the expression of minimum time separation T_{ij}?
- $T_{ij} > o_i$
- Answer:

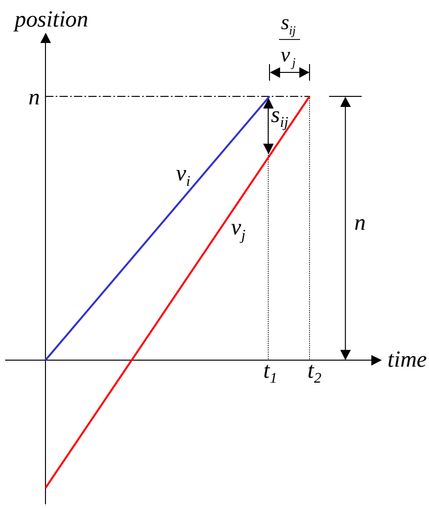
$$T_{ij} = \begin{cases} \max\left[\dfrac{n+s_{ij}}{v_j} - \dfrac{n}{v_i}, o_i\right] & \text{for } v_i > v_j \quad \text{(‘opening case’)} \\ \max\left[\dfrac{s_{ij}}{v_j}, o_i\right] & \text{for } v_i \le v_j \quad \text{(‘closing case’)} \end{cases}$$

Arrival Capacity of a Single Runway

- ‘opening case’: $v_i > v_j$



- ‘closing case’: $v_i \le v_j$



Minimum Acceptable Interarrival Time

$$T_{ij} = \begin{cases} \max\left[\dfrac{n+s_{ij}}{v_j} - \dfrac{n}{v_i}, o_i\right] & for\ v_i > v_j \quad \text{('opening case')} \\ \max\left[\dfrac{s_{ij}}{v_j}, o_i\right] & for\ v_i \le v_j \quad \text{('closing case')} \end{cases}$$

- *K*: number of aircraft types
- Number of 'type *i* aircraft followed by type *j* aircraft' pairs = K^2
- p_{ij}: probability of 'type *i* aircraft followed by type *j* aircraft' pair
- Minimum acceptable interarrival time: $E[T_{ij}] = \sum_i^K \sum_j^K p_{ij} \times T_{ij}$

Example

- Parameters (given)

i (a/c type)	p_i (prob.)	v_i (knots)	o_i (secs)
1(H)	0.2	150	70
2(L)	0.35	130	60
3(M)	0.35	110	55
4(S)	0.1	90	50

- $[s_{ij}]$ matrix (given)

$S =$

Leading aircraft \ Trailing aircraft	1(H)	2(L)	3(M)	4(S)
1(H)	4	5	5	6
2(L)	2.5	2.5	2.5	4
3(M)	2.5	2.5	2.5	4
4(S)	2.5	2.5	2.5	2.5

- $[t_{ij}]$ matrix

$T =$

Leading aircraft \ Trailing aircraft	1(H)	2(L)	3(M)	4(S)
1(H)	96	157	207	320
2(L)	60	69	107	222
3(M)	60	69	82	196
4(S)	60	69	82	100

- $[p_{ij}]$ matrix: $p_{ij} = p_i p_j$

$P =$

Leading aircraft \ Trailing aircraft	1(H)	2(L)	3(M)	4(S)
1(H)	0.04	0.07	0.07	0.02
2(L)	0.07	0.1225	0.1225	0.035
3(M)	0.07	0.1225	0.1225	0.035
4(S)	0.02	0.035	0.035	0.01

$$\Rightarrow E[T_{ij}] = \sum_i^K \sum_j^K p_{ij} \times T_{ij} = 106.3\text{sec}$$

Arrival Runway Capacity: Example

- Maximum flow rate (frequency of arrivals): $\dfrac{1}{E[T_{ij}]}$ = 33.9 arrivals/hour

  ⇨ It is called maximum theoretical capacity.

- In practice, minimum separation is: $T'_{ij} = T_{ij} + b$

- In practice, $\dfrac{1}{E[T'_{ij}]}$ is called the 'saturation capacity' or 'maximum throughput'.

- Typically, b = 10 sec ⇨ $\dfrac{1}{E[T'_{ij}]} = \dfrac{1}{116.3} = 30.9$ aircrafts/hour

Comments on Analytical Model

- The analytical model is simple, but insightful!

- It assesses impacts on capacity of changes in ATC rules and ATC operational conditions and methods

- Example of possible changes to increase capacity:
  - Reduce separation
  - Increase final approach speed
  - Length of approach path (increase? or decrease?)
  - Change mix by forbidding or pricing out small planes
  - Sequencing of aircrafts that are waiting to land (operational problem)

Analysis of the Analytical Model

- Limitations of the model:
  - v_i and o_i are random in practice, but assumed constant in the model
  - Distance between aircrafts is random in practice, but not in the model
  - The model assumes an isolated runway dedicated for landing only
- Other complications in capacity modeling:
  - Airports contain and operate under multiple runways (PS1)
  - A given set of runways is operated using multiple configurations
  - The use of a configuration depends on:
    - Level of demand
    - Weather conditions
    - Wind speed and direction
    - Traffic mix
  - A runway can be used for both arrivals and departures (PS1)

Other Models in Air Transportation

- Airport passenger terminal design
- Simulation models of the airfield
- Analysis of air delay using analytical or simulation models (see later lectures)
- Estimation of the frequency of ‘conflicts’ resolution:
  - Estimation of number of overtaking conflicts involving aircrafts flying on the same airway
  - Prediction of the number of conflicts at the intersection of two airways
  - Estimates given by these models are principal indicators of the workload of an ATC system

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