LECTURE #5

1.060 ENGINEERING MECHANICS II

Fluid at rest $\Rightarrow \vec{q}$ = velocity vector = $(u, v, w) = 0$

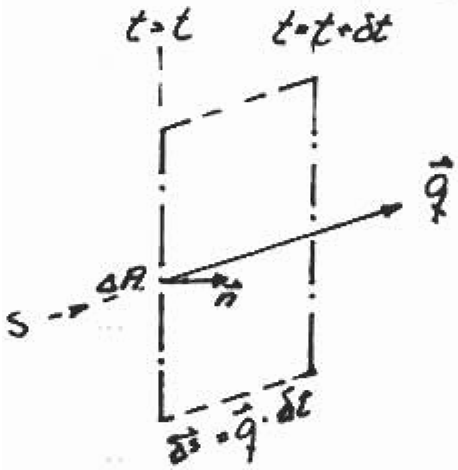
HYDROSTATICS : Been there - done that

Fluid in motion is described by its velocity field

$$\vec{q}(x,y,z,t) = \lim_{\delta t \to \text{"}0\text{"}} (\delta x_p, \delta y_p, \delta z_p)/\delta t = \lim_{\delta t \to \text{"}0\text{"}} (\vec{\delta s}/\delta t)$$

where

$\vec{\delta s}$ = infinitessimally small displacement vector along the streamline passing through (x, y, z) at time $\to t$.



ΔA = elementary area over which $\vec{q}$ and ρ are $\simeq$ constant

$\vec{n}$ = unit vector $\perp \Delta A$

$q_\perp = \vec{q} \cdot \vec{n}$ = velocity component $\perp \Delta A$

$\Delta \forall$ = volume between ΔA at t and $t + \delta t$ = $(q_\perp \delta t) \Delta A$

This volume, $\Delta \forall$, must have been supplied by the flow through ΔA at $t = t$.

Volume flow rate per unit area = Volume flux =
$(\Delta \forall / \delta t)/\Delta A = q_\perp \delta t \Delta A/(\delta t \Delta A) = q_\perp = \vec{q}\cdot\vec{n}$

Mass flow rate per unit area = Mass flux =
[Volume flux][mass/volume] = $\dot{m} = \rho q_\perp = \rho \vec{q}\cdot\vec{n}$

If flow area A is much larger than ΔA, so that neither ρ nor $\vec{q}$ can be considered constant over A, we have

Rate of Mass Flow across $A = \sum \dot{m}\,\Delta A =$

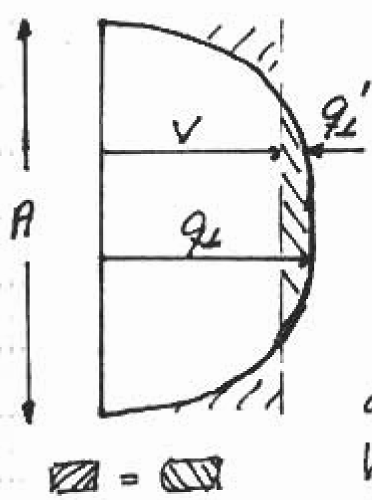
$$\dot{M} = \int_A \rho\, q_\perp\, dA = \int_A \rho\, \vec{q}\cdot\vec{n}\, dA$$

Rate of Volume Flow across $A = \sum q_\perp \Delta A =$

$$Q = \int_A q_\perp\, dA = \int_A \vec{q}\cdot\vec{n}\, dA$$

If ρ = constant

$$\dot{M} = \rho\, Q$$



Q = discharge $(m^3/s) = \int_A q_\perp\, dA = VA$

$V = \dfrac{Q}{A}$ = AVERAGE VELOCITY OVER A

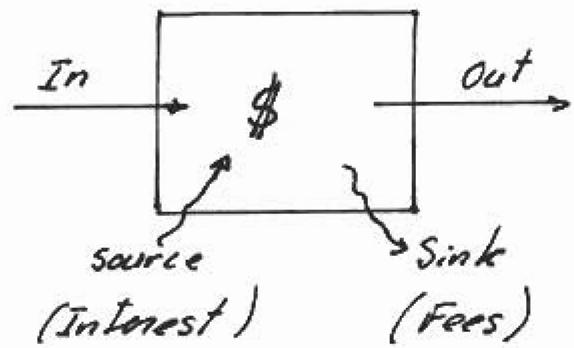
If $q_\perp = V + q_\perp'$ then $\int_A q_\perp'\, dA = 0$ and if $q_\perp' \ll V$ over most of A, V may be considered to represent $q_\perp$ quite well.

NATURE OF CONSERVATION LAWS

Conservation laws are analogous to a BANK ACCOUNT :

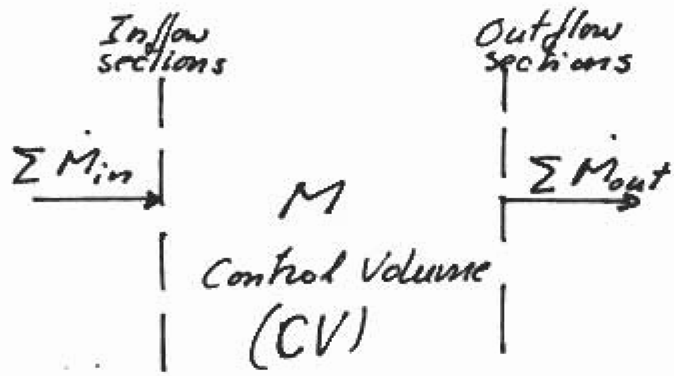
"In" minus "Out" = "Rate of Change Within"

Deposit Account Withdrawal



$$\Delta \$ = \Delta \$_{deposit} - \Delta \$_{withdrawal} + \Delta \$_{\substack{interest \\ (source)}} - \Delta \$_{\substack{Fees \\ (sink)}}$$

CONSERVATION OF MASS

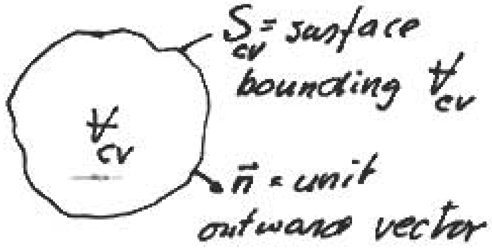


$\sum \dot{M}_{in} - \sum \dot{M}_{out}$ = Net rate of mass in =

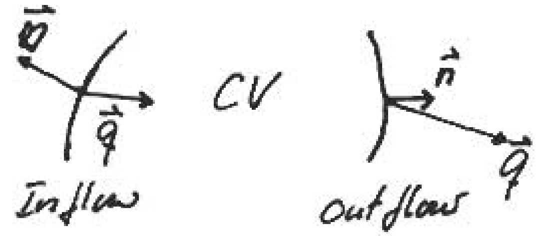
$\frac{\partial M}{\partial t}$ = Rate of change of mass within CV.

(for mass - no source or sinks)

$$\sum_{\substack{\text{inflow}\\ \text{areas}}} \Big[\int_A \rho q_\perp \, dA\Big] - \sum_{\substack{\text{outflow}\\ \text{areas}}} \Big[\int_A \rho q_\perp \, dA\Big] = \frac{\partial}{\partial t}\Big[\int_{\forall_{cv}} \rho \, d\forall\Big]$$



$$\vec{n}\cdot\vec{q} \gtrless 0 \text{ if } \begin{matrix}\text{outflow}\\ \text{Inflow}\end{matrix}$$



Compact expression:

$$\frac{\partial}{\partial t}\int_{\forall_{cv}} \rho \, d\forall + \int_{S_{cv}} \rho \vec{q}\cdot\vec{n} \, dS = 0 \qquad (1)$$

<u>CONSERVATION OF VOLUME</u> (Continuity)

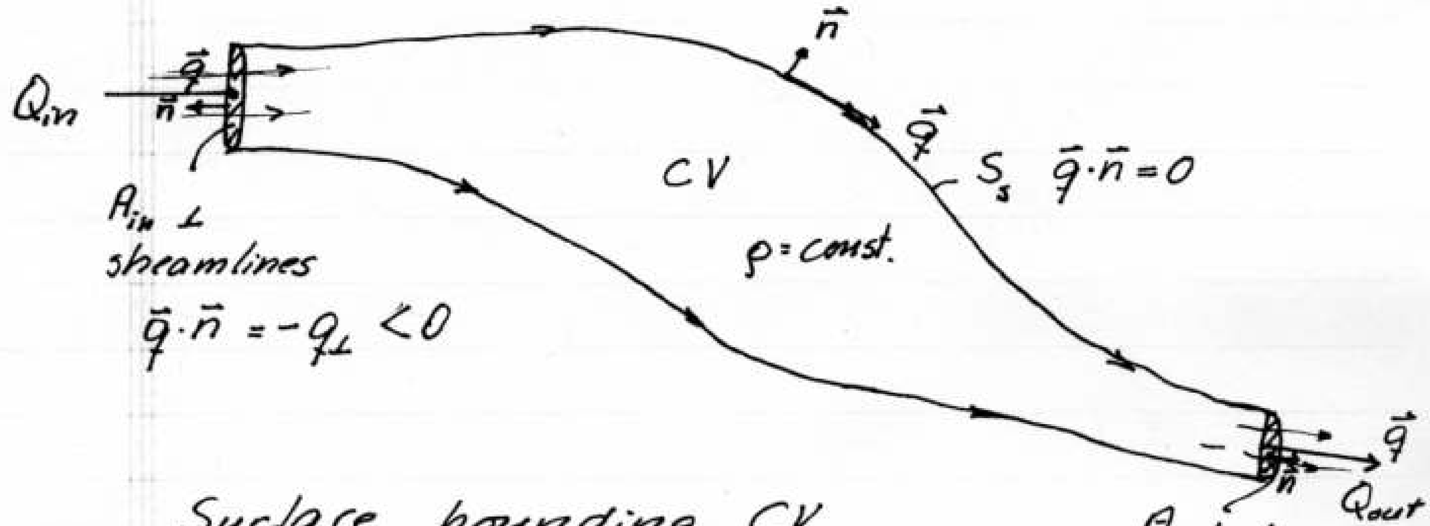
$$\sum Q_{in} - \sum Q_{out} = \frac{\partial \forall_{cv}}{\partial t} = \frac{\partial}{\partial t}\Big[\int_{\forall_{cv}} d\forall\Big]$$

or, in compact form

$$\frac{\partial \forall_{cv}}{\partial t} + \int_{S_{cv}} \vec{q}\cdot\vec{n} \, dS = 0 \qquad (2)$$

For a homogeneous fluid of constant density, ρ cancels out in (1) and it becomes (2). BUT (2) holds for <u>any incompressible</u> fluid [one whose volume remains the same regardless of Temperature and Pressure] whether ρ is constant or not.

The Stream Tube



Surface bounding CV
consists of S_s = streamline "walls"
and flow cross-sections A

$$\frac{\partial \forall_{cv}}{\partial t} + \int_A \vec{q}\cdot\vec{n}\, dA = \frac{\partial \forall_{cv}}{\partial t} + Q_{out} - Q_{in} = 0$$

If flow is steady: $\partial \forall_{cv} / \partial t = 0$ and

$$Q = Q_{in} = Q_{out} = \text{Constant along Stream Tube}$$

or

$$Q = VA = const \Rightarrow V = \frac{Q}{A}$$

V is large where A (area of stream tube) is small and vice versa.
(This should be called the da Vinci Principle!)