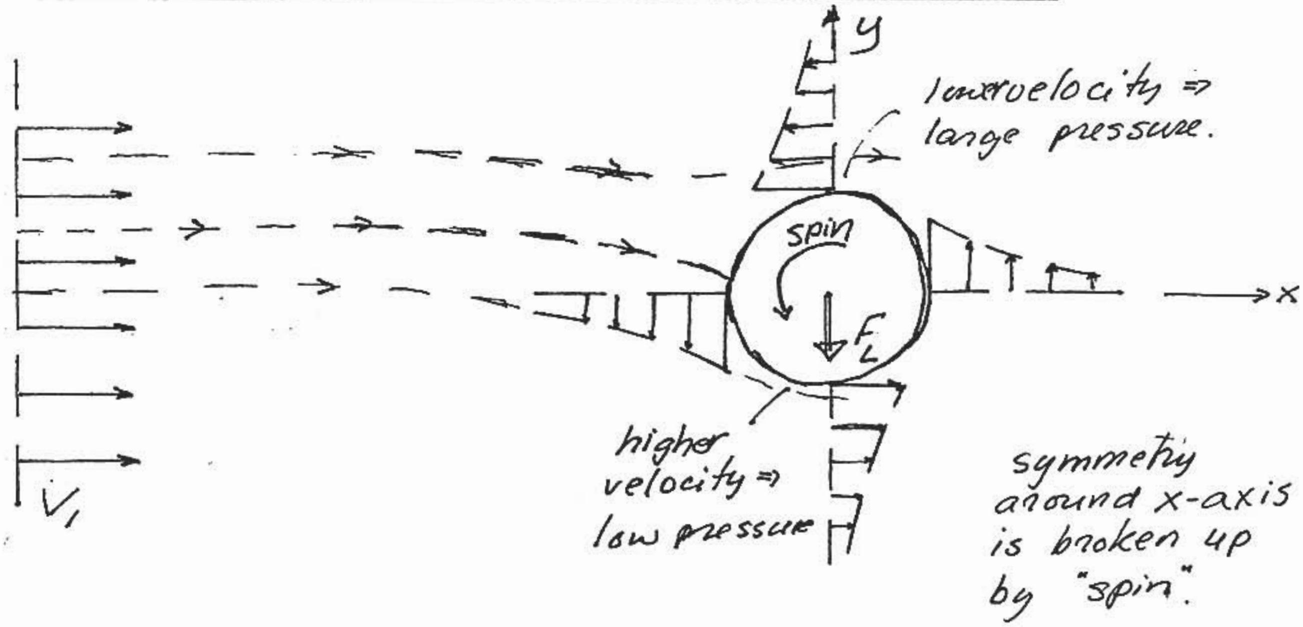
LECTURE # 21

1.060 ENGINEERING MECHANICS II

LIFT FORCE & LIFT COEFFICIENT



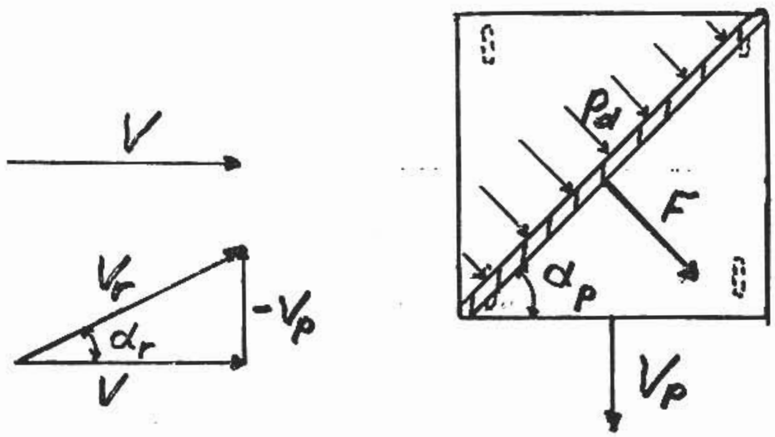
If, in addition to a uniform approach flow V_1 the fluid is "spinning", e.g. induced by the object itself rotating around an axis and dragging the surrounding fluid along, the combination of the two velocity fields will, as illustrated above, force more of the approach flow to pass below the cylinder. This breaks up the symmetry of the flow around the cylinder, and results in largera velocity at $(x, -y)$ than at (x, y). Large velocity gives, according to Bernoulli, lower pres-sure. Therefore, $p(x,-y) < p(x,y)$ and upon integration a net downward force, i.e. $\perp$ to V and therefore a "lift" force F_L results.

In general and in analogy with drag force we have

$$F_L = \frac{1}{2} \rho C_L A_\perp V^2$$

where C_L = lift coefficient.

Working the Angle



$\tan \alpha_r = V_p / V$

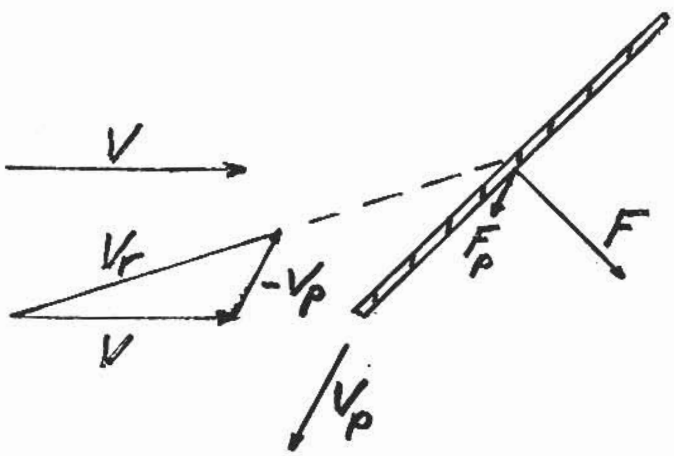
$\alpha_r < \alpha_p$:

Wind-Induced V_p

$\alpha_r > \alpha_p$:

Wind-Resisted V_p

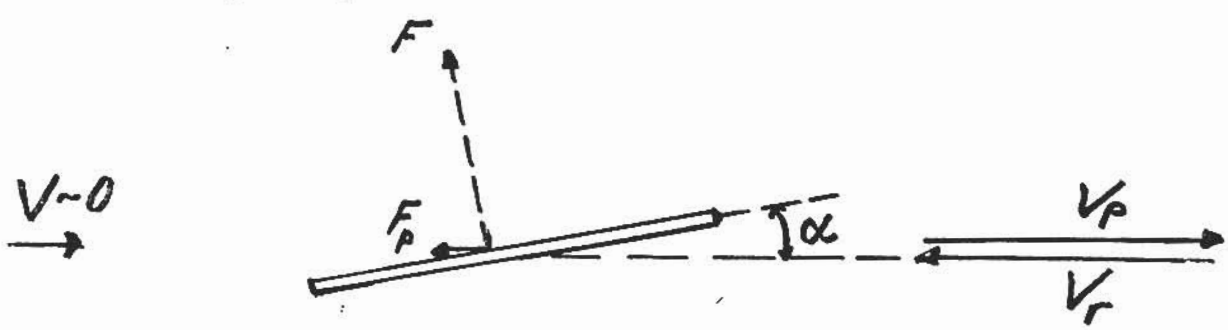
Working with the Wind



$V_p F_p > 0$

We gain Power

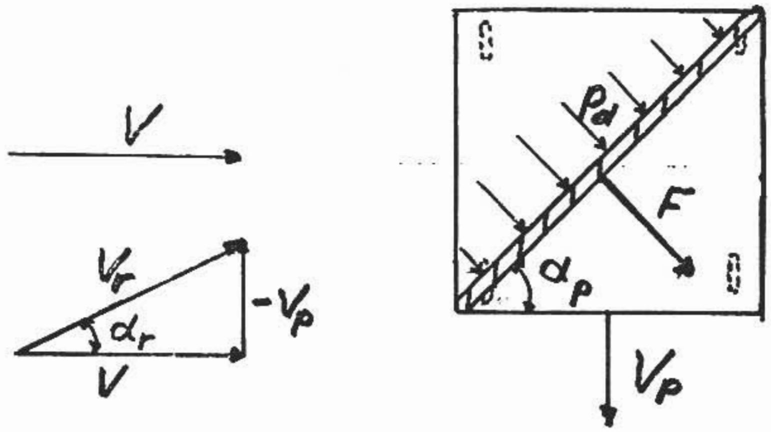
Working against the Wind



$V_p F_p < 0$

We supply Power

Working the Angle



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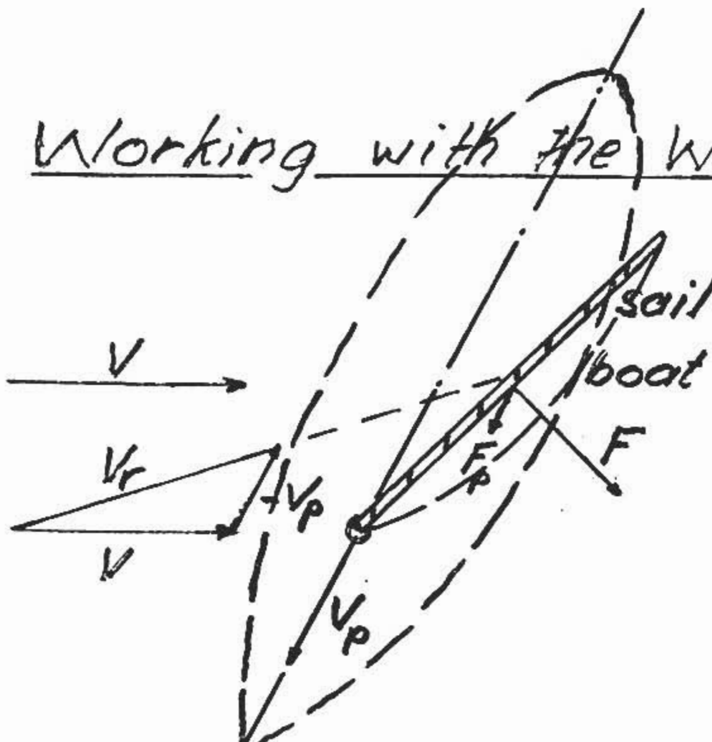
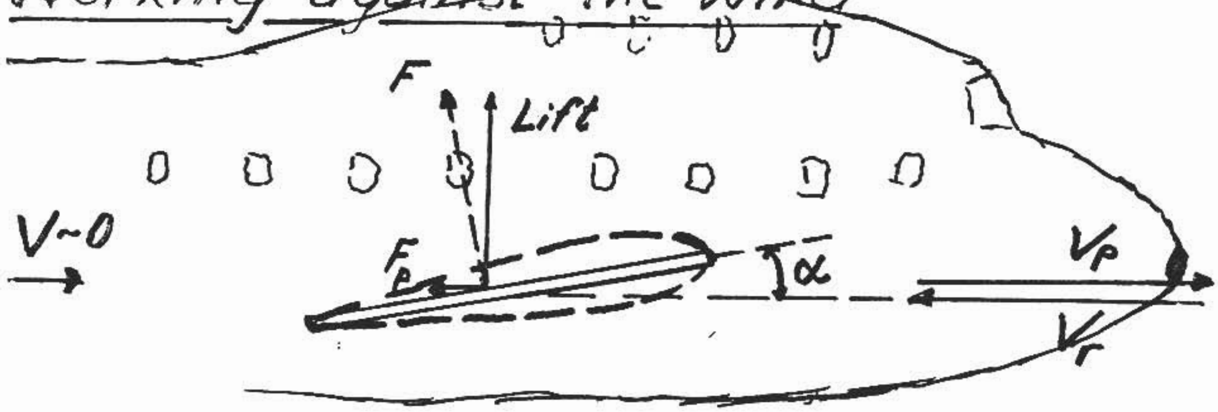
Working with the Wind



$V_p F_p > 0$

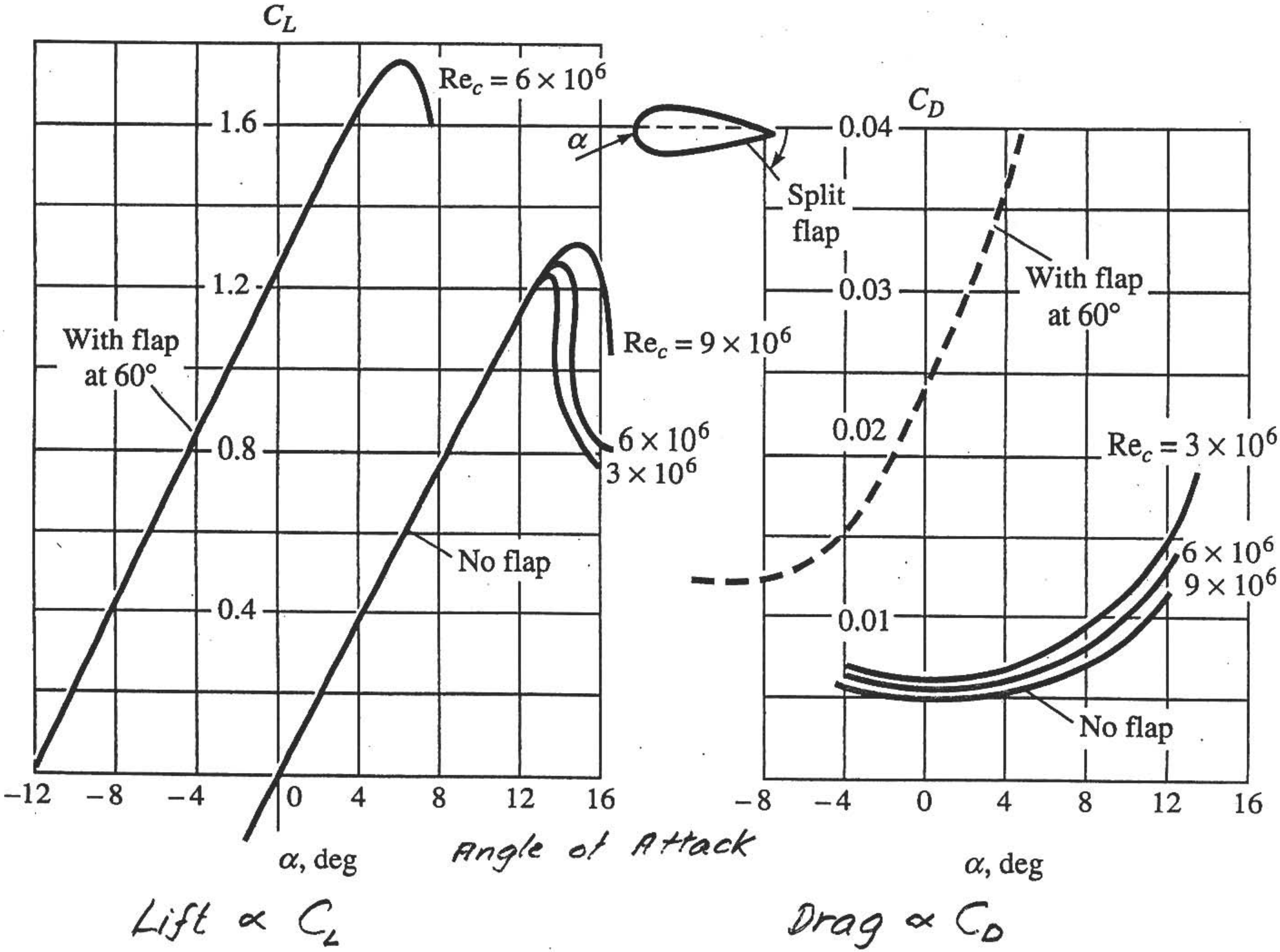
We gain Power

Working against the Wind



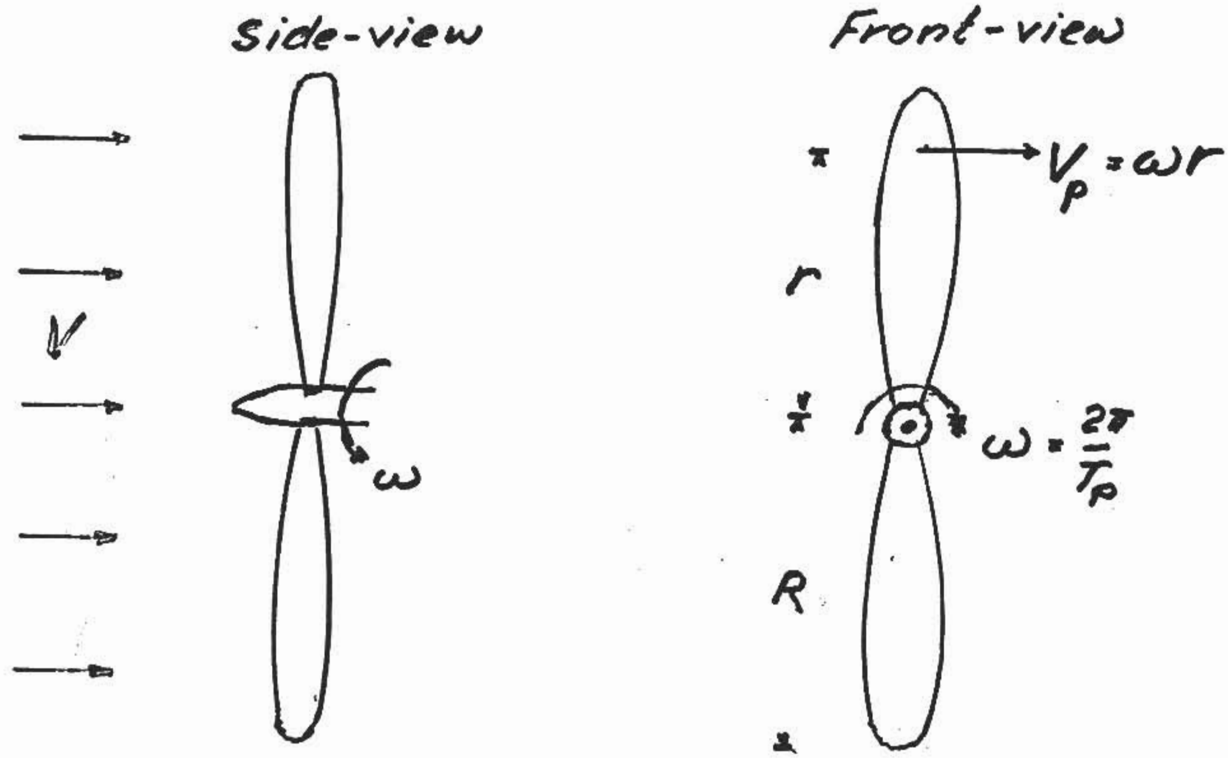
$V_p F_p < 0$

We supply Power



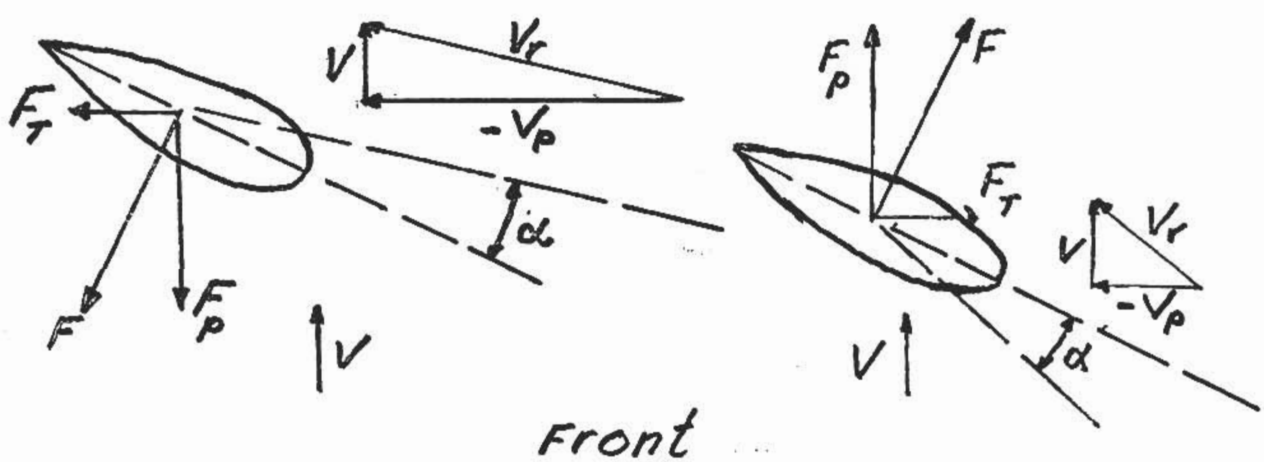
C_L
1.6
1.2
0.8
0.4
$Re_c = 6 \times 10^6$
With flap at 60°
$Re_c = 9 \times 10^6$
6×10^6
3×10^6
No flap
−12 −8 −4 0 4 8 12 16
α, deg
Angle of Attack
Lift ∝ C_L
α
Split flap
C_D
0.04
0.03
0.02
0.01
With flap at 60°
$Re_c = 3 \times 10^6$
6×10^6
9×10^6
No flap
−8 −4 0 4 8 12 16
α, deg
Drag ∝ C_D

Turbo-Power



Fan

Windmill

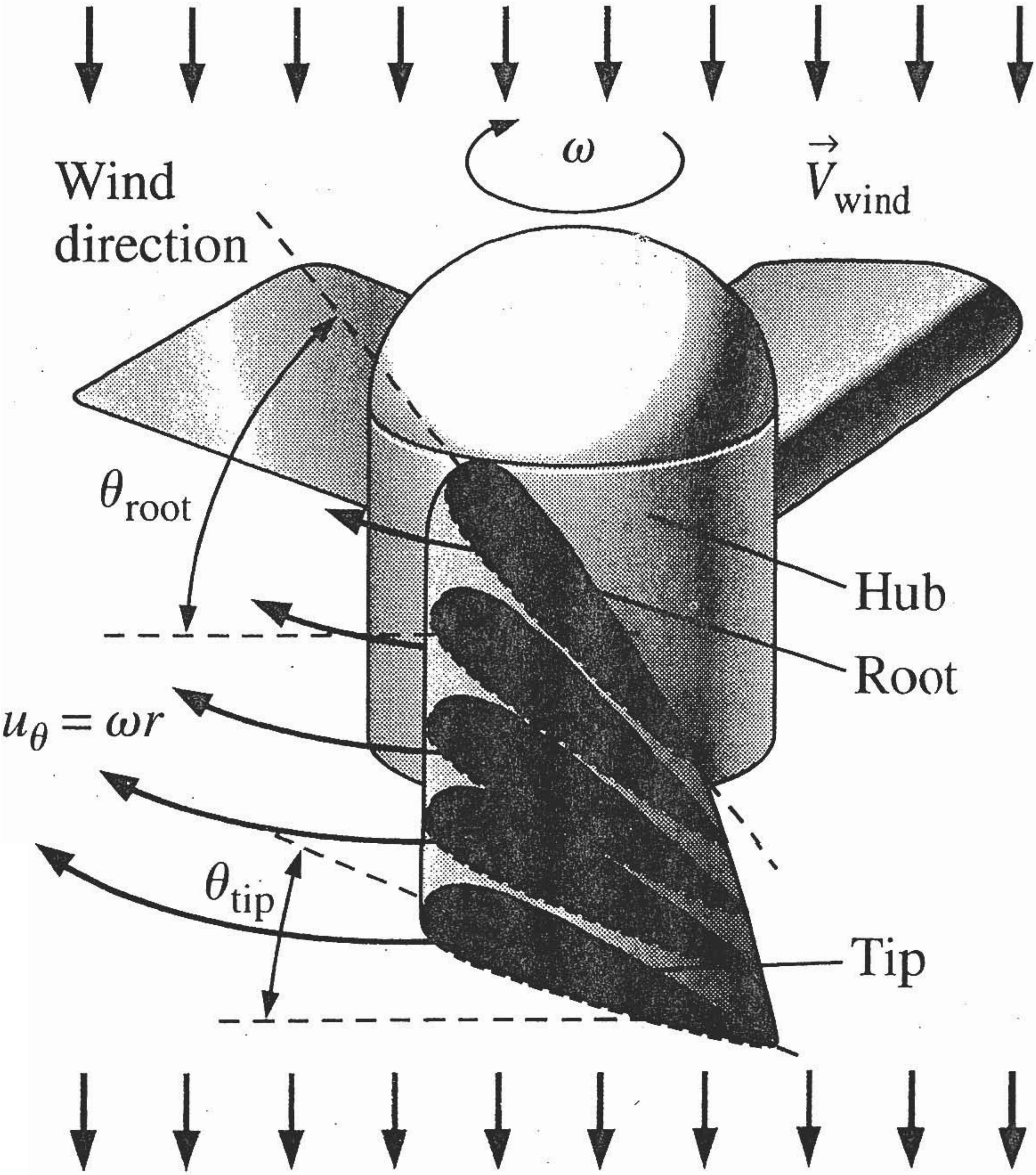


F_T against V_p

F_T with V_p

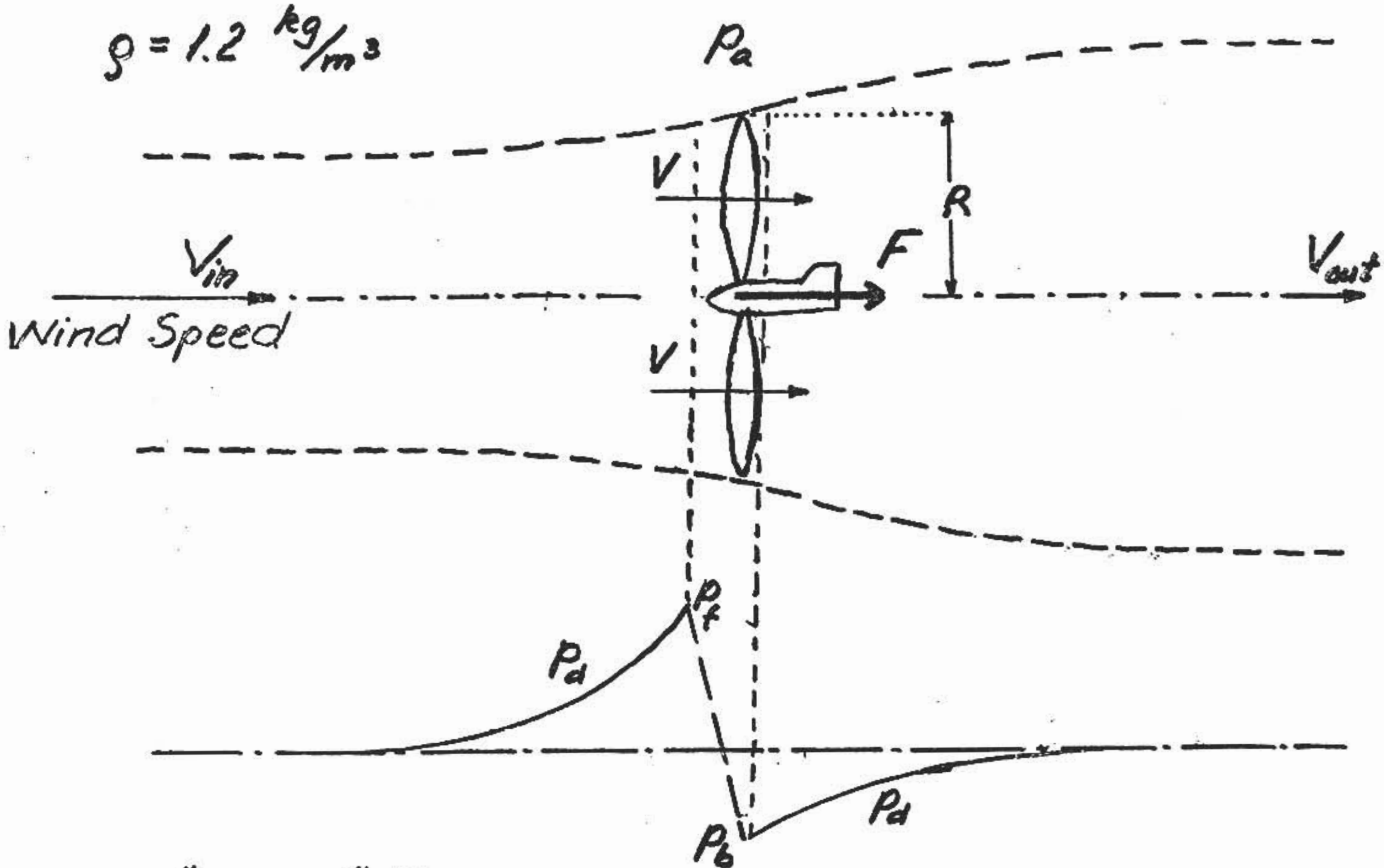
Power required to drive blade to produce flow

Flow required to drive blade to produce power



ω
$\vec{V}_{wind}$
Wind direction
θ_{root}
Hub
Root
$u_\theta = \omega r$
θ_{tip}
Tip

Ideal Wind Turbine



"Ideal" Force

$$F_{ideal} = (p_f - p_b)A = \rho V^2 A = \frac{4}{9}\rho V_{in}^2 \pi R^2$$

Ideal Power Production

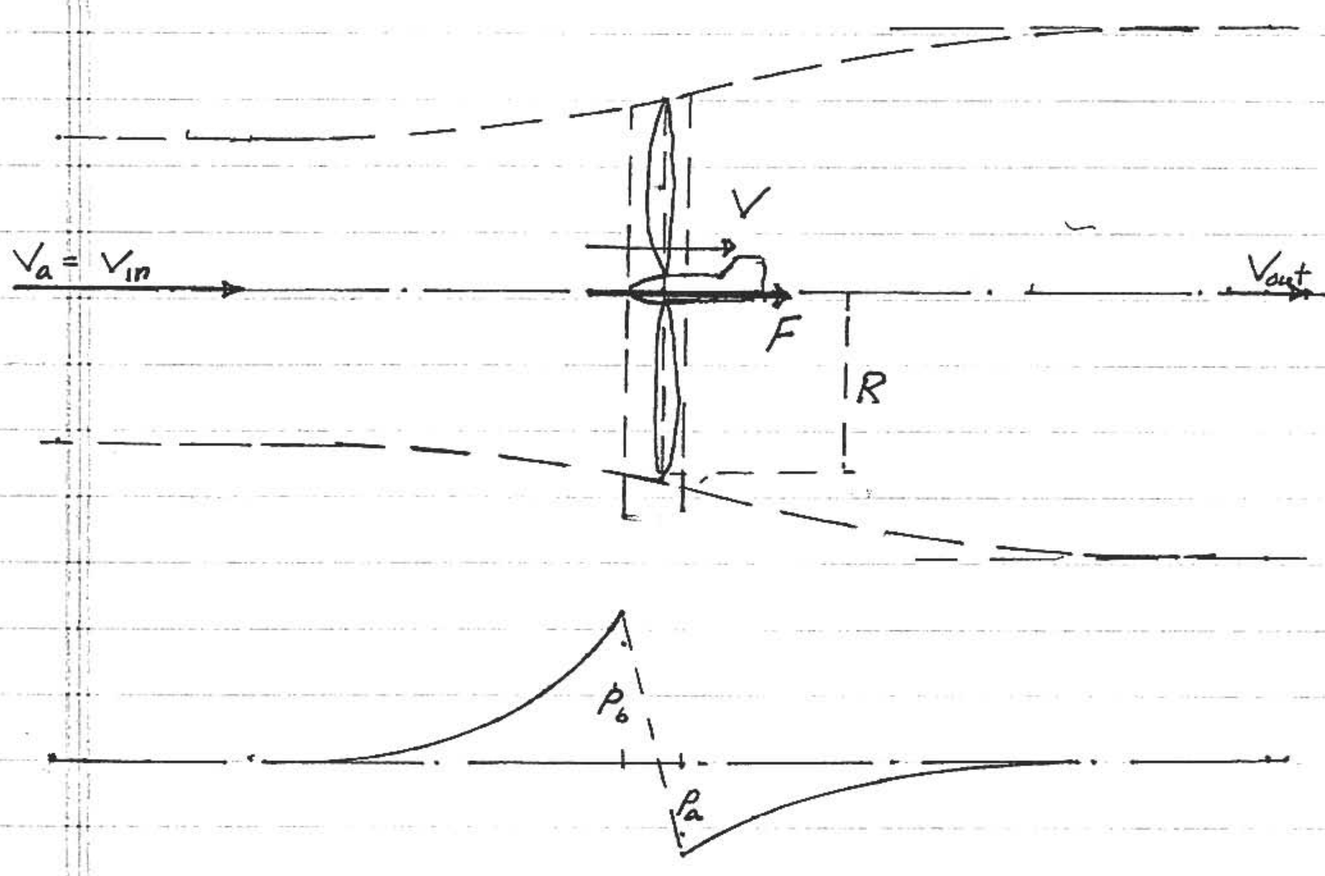
$$P_{ideal} = F_{ideal} V = \rho V^3 A = \frac{8}{27}\rho V_{in}^3 \pi R^2$$

Reality

Real Force = $F < F_{ideal}$ $\qquad F \approx 0.7 F_{ideal}$

Real Power = $P < P_{ideal}$ $\qquad P \approx 0.5 P_{ideal}$

Ideal Wind Turbine Theory



Conservation of mass for stream tube

$$\rho V_{in} A_{in} = \rho V A = \rho V_{out} A_{out} \tag{1}$$

when

$$\rho = \text{constant}$$

$$V_{in} A_{in} = V A = V_{out} A_{out} \tag{2}$$

Assumption A: No flow across stream tube walls.

Conservation of Momentum

$$\rho V_{in}^2 A_{in} = \rho V_{out}^2 A_{out} + F \qquad (3)$$

or, using (2)

$$F = \rho (VA)(V_{in} - V_{out}) = \rho V (V_{in} - V_{out}) A \qquad (4)$$

Assumption B : Pressure forces on streamtube walls and end sections balance out

Assumption C : No momentum inflow or outflow across stream tube walls.

Conservation of Energy

Bernoulli from inflow to "b" (before turbine)

$$\frac{1}{2}\rho V_{in}^2 + p_\infty = \frac{1}{2}\rho V_b^2 + p_b \qquad (5)$$

Bernoulli from outflow to "a" (after turbine)

$$\frac{1}{2}\rho V_{out}^2 + p_\infty = \frac{1}{2}\rho V_a^2 + p_a \qquad (6)$$

but mass conservation across turbine, with $A_b \simeq A_a = A$ gives

$$V_b \simeq V_a \simeq V \qquad (7)$$

So, with (7) we may subtract (6) from (5) to obtain

$$p_b - p_a = \frac{1}{2}\rho\left(V_{in}^2 - V_{out}^2\right) = \frac{1}{2}\rho\left(V_{in} - V_{out}\right)\left(V_{in} + V_{out}\right) \quad (8)$$

Assumption D: No headloss between inflow and turbine,

Assumption E: No headloss between turbine and outflow

Velocity through Turbine

From (7) it follows that

$$F = (p_b - p_a)A = \frac{1}{2}\rho\left(V_{in} - V_{out}\right)\left(V_{in} + V_{out}\right)A \quad (9)$$

by use of (8). Comparison of (9) and (4) then gives

$$V = \frac{1}{2}\left(V_{in} + V_{out}\right) \quad (10)$$

Power Loss Through Turbine

Again, accepting (7), we have

$$P = \dot{E}_b - \dot{E}_a = (p_b - p_a)VA = FV = \frac{1}{4}\rho\left(V_{in} - V_{out}\right)\left(V_{in} + V_{out}\right)^2 A$$

$$= \frac{1}{4}\rho V_{in}^3 A\left[1 - \frac{V_{out}}{V_{in}}\right]\left[1 + \frac{V_{out}}{V_{in}}\right]^2 \quad (11)$$

To maximize the power loss of the flow, which equals the power input to the turbine, we take $V_{out}/V_{in} = \alpha$, to get from (11)

$$P \propto (1-\alpha)(1+\alpha)^2$$

$$\frac{\partial P}{\partial \alpha} = (1-\alpha)2(1+\alpha) - (1+\alpha)^2 = -3\alpha^2 - 2\alpha + 1 = 0$$

or

$$\alpha = V_{out}/V_{in} = 1/3 \quad \text{for } P = P_{max} \qquad (12)$$

Thus, we have for maximum power output from (11)

$$P_{max} = \frac{1}{4}\rho V_{in}^3 A \cdot \frac{2}{3}\left(\frac{4}{3}\right)^2 = \frac{8}{27}\rho V_{in}^3 A$$

or, since V_{in} = the wind speed and $A = \pi R^2$ = area covered by the turbine blades, we have

$$P_{max} = \frac{16}{27}\left(\frac{1}{2}\rho V_{in}^2\right)\left(V_{in}\pi R^2\right) \qquad (13)$$

i.e. $16/27 = 0.593$ [known as the Betz Number] times the rate of kinetic energy transport through area of the turbines sweep.

Corresponding to (13), i.e. P_{max}, the force on the turbine is (from (9))

$$F(P_{max}) = \frac{1}{2}\rho V_{in}^2 A\left(1-\frac{1}{3}\right)\left(1+\frac{1}{3}\right) = \frac{4}{9}\rho V_{in}^2\left(\pi R^2\right) \qquad (14)$$