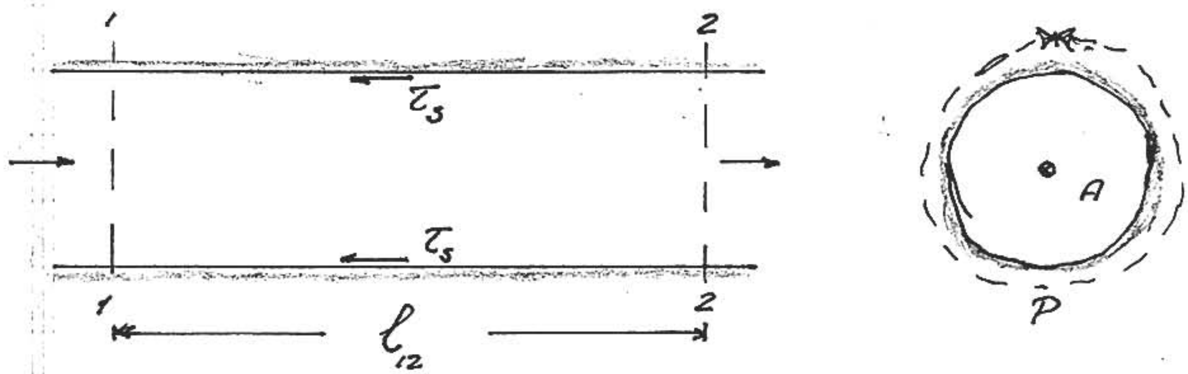
LECTURE #14

1.060 ENGINEERING MECHANICS II

BOUNDARY SHEAR STRESS AND FRICTIONAL HEADLOSS



Uniform pipe $\Rightarrow$ A = constant, P = constant
Discharge = Q = constant $\Rightarrow$ $V = Q/A$ = constant

$$H_1 - H_2 = \left(\frac{V_1^2}{2g} + \frac{p_1}{\rho g} + z_1\right) - \left(\frac{V_2^2}{2g} + \frac{p_2}{\rho g} + z_2\right) =$$

$$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right)$$ = difference in piezometric head between ① and ② = what is obtained from a manometer connected to the pipe at ① and ② [Lecture #4] =

$$\Delta H_f = \int_{s_1}^{s_2} \frac{\tau_s P}{\rho g A} ds = \frac{\tau_s P}{\rho g A} \ell_{12}$$ (since conditions are uniform along the pipe)

Dimensional Analysis

What are we looking for?

Dependent Variable : τ_s = wall shear stress

What can we change?

Independent Variables:

Fluid : ρ and ν $(=\mu/\rho)$

Pipe dimension : D = diameter [we choose O]

Fluid Velocity : V [note $Q = VA$ is known!]

Pipe wall roughness : ε

Basic Dimensions (units)

led $= D$

ted $= D/V$

med $= \rho D^3$

Independent Variables Left

$\nu \Rightarrow [\nu] = \text{led}^2/\text{ted} = D^2/(D/V) = DV$

$$\pi_1 = \frac{\nu}{DV} \quad \text{[non-dim. viscosity]}$$

$\varepsilon \Rightarrow [\varepsilon] = \text{led} = D$

$$\pi_2 = \frac{\varepsilon}{D} \quad \text{[non-dim. wall roughness]}$$

Dependent Variable

$\tau_s \Rightarrow [\tau_s]$ = Force/Area = $\text{med}(\text{led}/\text{ted}^2)/(\text{led})^2 =$
$\rho D^3(D/(D/V)^2)/D^2 = \rho V^2$

$$\pi_D = \frac{\tau_s}{\rho V^2} \quad \text{[non-dim. wall shear stress]}$$

Result of Dimensional Analysis

$$\Pi_D = \frac{\tau_s}{\rho V^2} = f_0\left(\Pi_1 = \frac{\nu}{DV} \; ; \; \Pi_2 = \frac{\varepsilon}{D}\right)$$

or with

$$\Pi_1^{-1} = \frac{DV}{\nu} = \underline{Re = \text{Reynolds Number}}$$

and

$$\Pi_2 = \frac{\varepsilon}{D} = \underline{\text{Relative Roughness}}$$

$$\underline{\tau_s} = \frac{1}{8} f(Re, \varepsilon/D)\, \rho V^2 = \underline{\frac{1}{8} f \rho V^2}$$

where

$$\underline{f} = 8 f_D = \underline{f\left(\text{Reynolds Number} = \frac{DV}{\nu}, \text{Rel. Rough.} = \frac{\varepsilon}{D}\right)} =$$

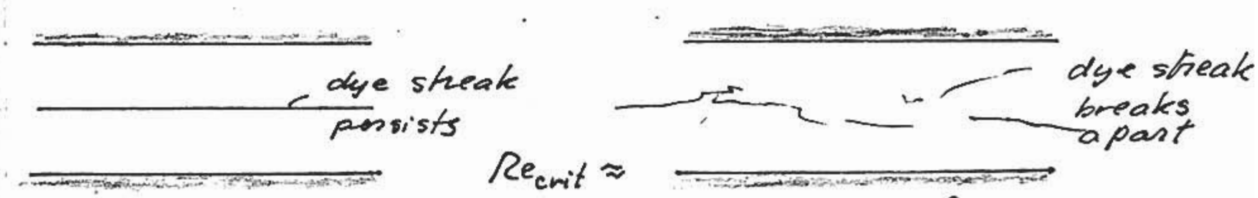
<u>The Darcy-Weisbach Friction Factor</u>

If pipe is <u>smooth</u> $\Rightarrow \varepsilon = 0$

$$\underline{f} = \underline{f\left(Re = \frac{DV}{\nu}\right)} \quad \text{for } \underline{\text{smooth}} \text{ walls}$$

<u>Reynolds</u> <u>Experiment</u>

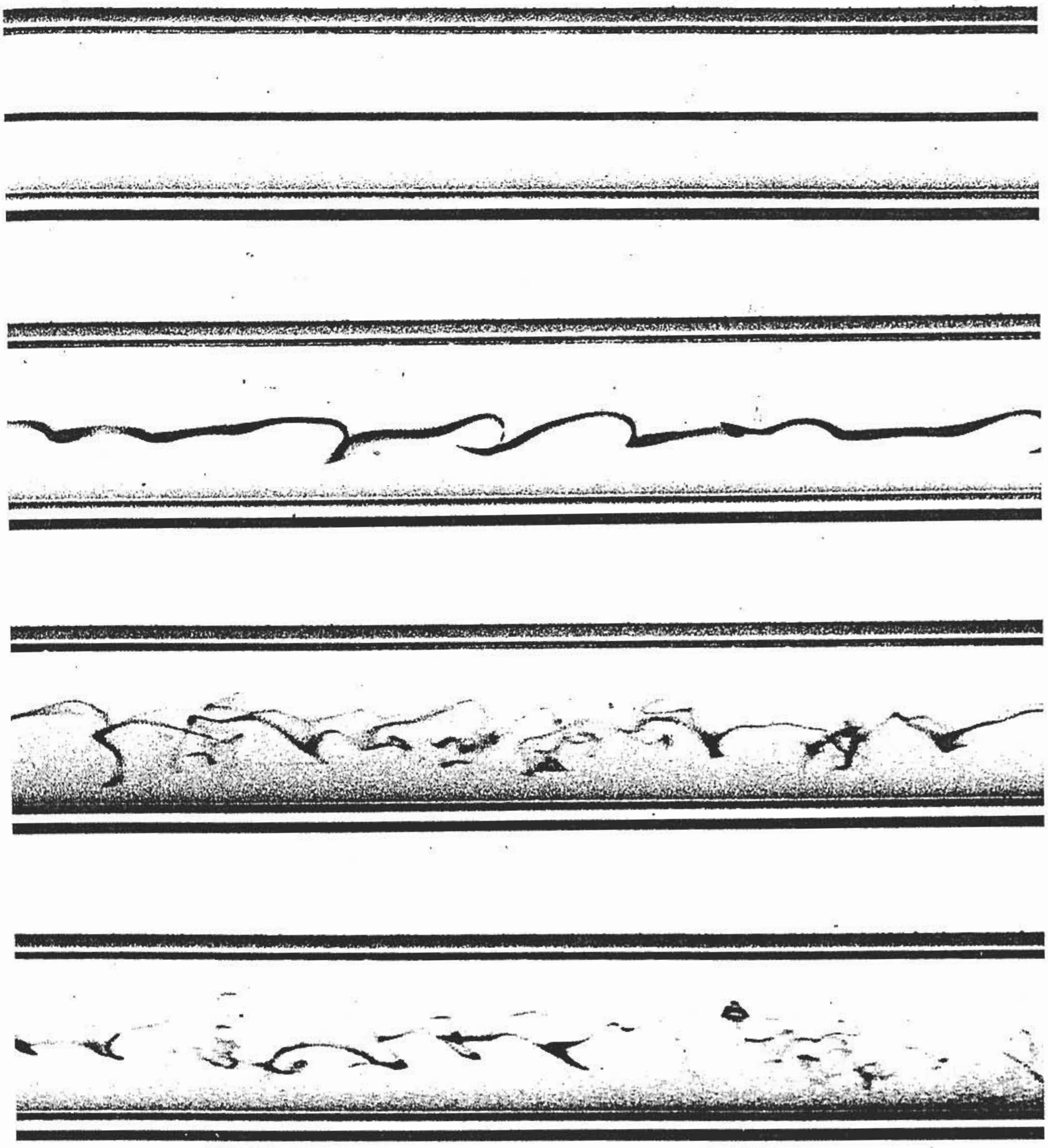
Smooth circular pipe



$Re < Re_{crit}$ LAMINAR FLOW

$Re_{crit} \approx 2 \cdot 10^3$

$Re > Re_{crit}$ TURBULENT FLOW



103. Repetition of Reynolds' dye experiment. Osborne Reynolds' celebrated 1883 investigation of stability of flow in a tube was documented by sketches rather than photography. However the original apparatus has survived at the University of Manchester. Using it a century later, N. H. Johannesen and C. Lowe have taken this sequence of photographs. In laminar flow a filament of colored water introduced at a bell-shaped entry extends undisturbed the whole length of the glass tube. Transition is seen in the second of the photographs as the speed is increased; and the last two photographs show fully turbulent flow. Modern traffic in the streets of Manchester made the critical Reynolds number lower than the value 13,000 found by Reynolds.

Transition from laminar to turbulent flow when

$$Re \simeq Re_{crit} \simeq 2 \cdot 10^3$$

For water:

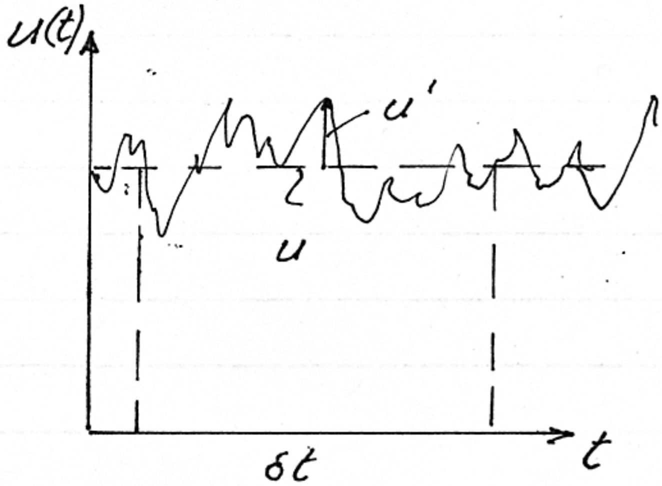
$$Re = \frac{DV}{\nu} \simeq \frac{DV}{10^{-6}\, m^2/s} = 2 \cdot 10^3 = Re_{crit}$$

$$(D.V)_{crit} \simeq 2 \cdot 10^{-3}\ m^2/s$$

If $D = 1\,cm = 0.01\,m$ [quite small]
Flow is turbulent if $V > \sim 0.2\ m/s$ [very low - it would take approximately 1 min to fill a 1 liter bottle at this flow rate, $Q = V \cdot (\pi/4) D^2 = 0.016$ liter/s]

For our fluid of choice - WATER - flows of interest to us are TURBULENT.

Thus, in principle all flows of interest to us are inherently UNSTEADY due to the chaotic nature of turbulent flows. We remove this obstacle by expanding our definition of what we consider to be "small", i.e. of a spatial or temporal scale below the scales of interest to us, by averaging over a a time interval δt larger than the time scale of turbulence, but still smaller than the temporal resolution of the flow we seek to resolve by our analysis.



$$u(t) = u + u'$$

$$u = \frac{1}{\delta t} \int_{\delta t} u(t)\, dt$$

$$\int_{\delta t} u' dt = 0$$

If u does not change over times much much larger than δt we consider the flow steady.

If u changes over times somewhat larger than δt, then we consider the flow unsteady

or

If mean value $u(t_0) = \int_{t_0}^{t_0 + \delta t} u(t) dt$ is

not a function of t_0 flow is steady, and if $u(t_0)$ varies with t_0 the flow is unsteady.